Module I:

Signal & Spectra

Definition of a signal:

"A signal is a detectable physical quantity or impulse (as a voltage, current, or magnetic field strength) that varies with respect to time, space, temperature or any other independent variable or can be defined as a function x(t) of independent variable ‘ t’ by which messages or information about behaviour of a natural or artificial system can be conveyed"

Electrical signals - time varying voltages and currents - in many cases have important properties that are necessary to be measured. Sometimes it is also justified to make any of these kinds of measurements.

Power in an audio signal - as one can test an audio amplifier's output ability
Frequency - as one can use an AC tachometer to measure a motor's rpm.
Amplitude – measurement of signal strength in a communication system.

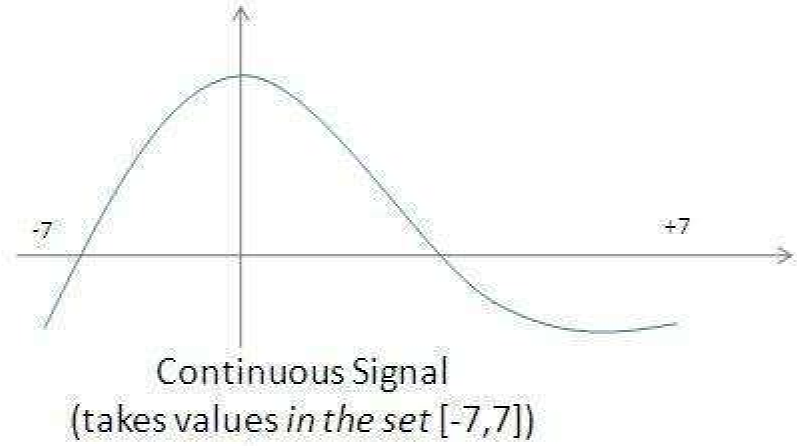
This includes the basic definition of signals. To easily understand signals & systems, we would visualize signals as simple mathematical functions.

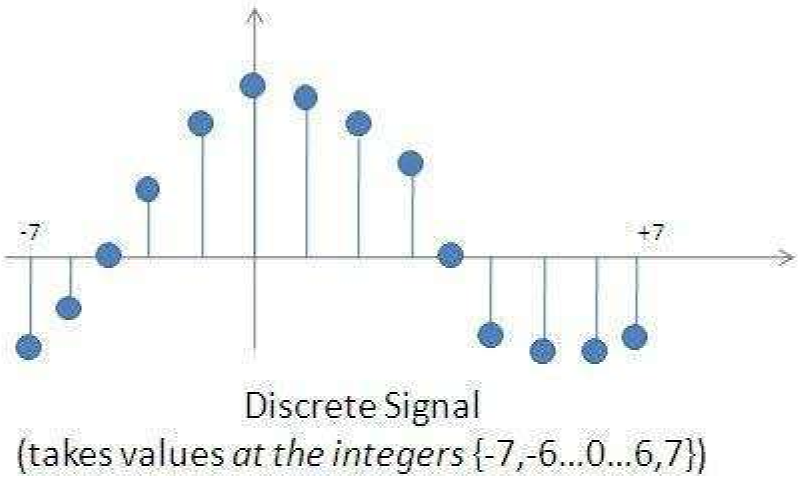
Classification of signals:

1. Continuous & Discrete Signals

Continuous signals & those defined over a set of real numbers(R) & discrete signals are those defined for discrete integers(I).
For instance, a signal (a function) having the domain [0,10] is continuous & one having

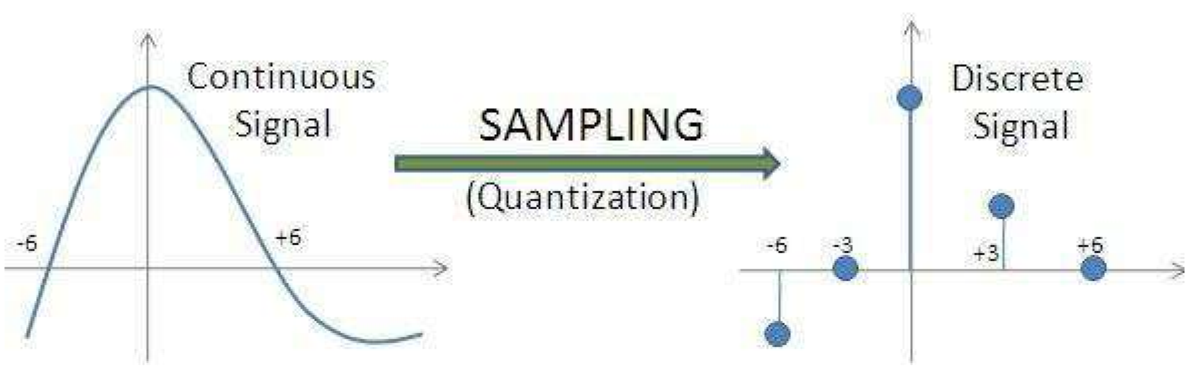
the domain {1,2,3…10} is discrete.





A Continuous Signal can be converted to a Discrete Signal using an Analog-to-Digital Converter (ADC). The conversion consists of a process called **sampling**.

The sampling process simply samples out values of the signal at certain points separated by an equal interval called the sampling period.

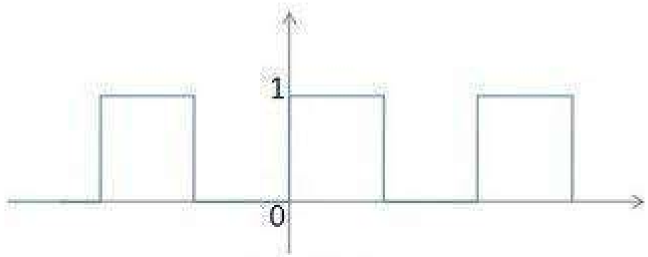


A common application of the above process is a Compact Disc (CD) which is simply a signal sampled at 44.1kHz & Quantized at 16 bits/2 bytes.

2. **Analog & Digital Signals:**

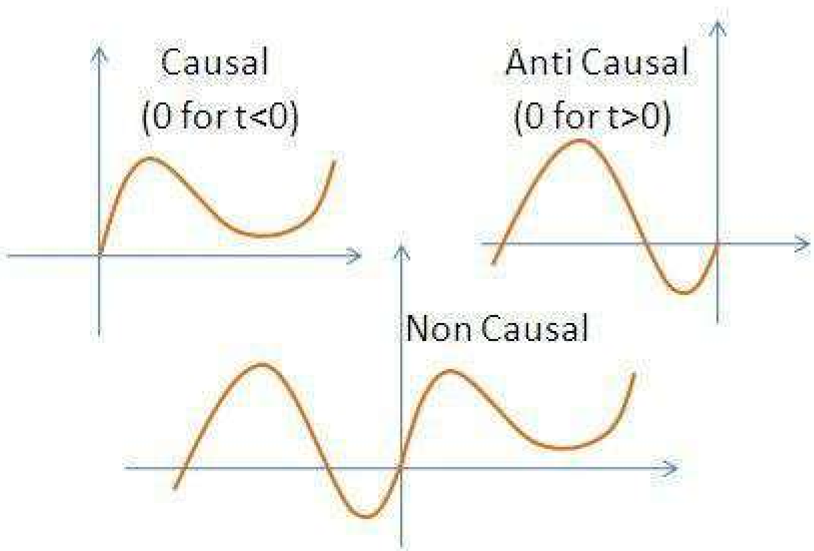
Analog signals are continuous electric signals which arise from non-electric signals. The variable of the converted signal is analogous to the non-electric time varying signal & hence, they are called analog signals. A good example is an audio (speech) signal.

Digital signal signals, take only two vales-HIGH or LOW, ON or OFF, 0 or 1, TRUE or FALSE, etc. All computers & other gadgets use digital signals to store information. (The term sometimes also refers to discrete time signals which can also take discrete values other than 0's & 1's)



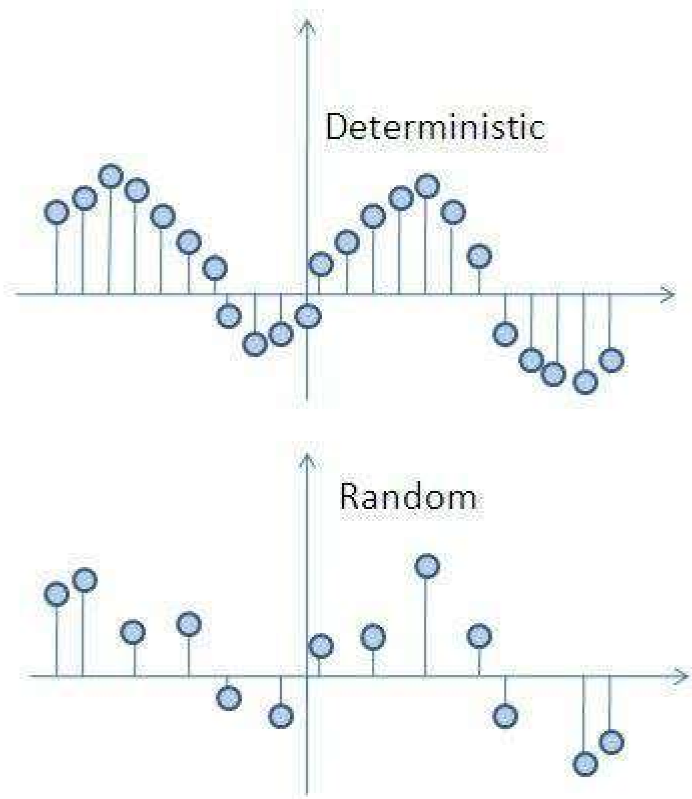
A Digital Signal

Signals are also classified as Causal and Anti causal signals: …



3. Deterministic & Random Signals.

A **random signal** takes random values & at a point on the signal, we cannot determine its value just before it or just after it. However, these values can be easily determined for a **deterministic**



signal.

Even and Odd Signal

One of characteristics of signal is symmetry that may be useful for signal analysis. Even signals are symmetric around vertical axis, and Odd signals are symmetric about origin

Even Signal:
A signal is referred to as an even if it is identical to its time - reversed counterparts; $x(t) = x(-t)$

Odd Signal:
A signal is odd if $x(t) = - x(-t)$

An odd signal must be 0 at t=0, in other words, odd signal
passes the origin.

Using the definition of even and odd signal, any signal may be decomposed into a sum of its even part, and its odd part,

Periodic Signal If the transformed signal x(t) is same as x (t+nT), then the signal is periodic. where T is fundamental period (the smallest period) of signal x(t) In discrete -time, the periodic signal is;

Aperiodic signal: Signal those do not satisfy the above condition and is not repeated over a particular interval of time.

System:

A **System** is any physical set of components that takes a signal, and produces a signal. In terms of engineering, the input is generally some electrical signal X, and the output is another electrical signal(response) Y. However, this may not always be the case. Consider a household thermostat, which takes input in the form of a knob or a switch, and in turn outputs electrical control signals for the furnace.

A main purpose of this book is to try and lay some of the theoretical foundation for future dealings with electrical signals. Systems will be discussed in a theoretical sense only.

<u>Random Variable & Processes</u>:

In probability and statistics a **random variable** or **stochastic variable** is a variable whose value is subject to variations due to chance. A random variable can take on a set of possible different values.

A random variable's possible values might represent the possible outcomes of a past experiment whose already-existing value is uncertain (for example, as a result of incomplete information or imprecise measurements). They may also conceptually represent either the results of an "objectively" random process (such as rolling a die) or the "subjective" randomness that results from incomplete knowledge of a quantity. The meaning of the probabilities assigned to the potential values of a random variable is not part of probability theory itself but is instead related to some arguments over the interpretation of probability.

The mathematical function describing the possible values of a random variable and their associated probabilities is known as a probability distribution can be discrete which , takes on any of a specified finite or countable list of values, capable with a probability mass function, characteristic of a probability distribution; or continuous, taking any numerical value in an interval or collection of intervals, The formal mathematical treatm ent of random variables is a topic in probability theory. In that context, a random variable is understood as a function defined on a sample space whose outputs are numerical values.

A random variable $X\colon \Omega \to E$ is a measurable function from the set of possible outcomes Ω to some set E. Usually, $E = \mathbb{R}$. The technical axiomatic definition requires both Ω and E to be measurable spaces.

As a real-valued function, X often describes some numerical quantity of a given event,

e.g,. the number of heads after a certain number of coin flips; the heights of diffe rent people.

When the range of X is finite or countable infinite, the random variable is called a discrete random variable and its distribution can be described by a probability mass function which assigns a probability to each val ue in the image of X. If the image is uncountable and infinite then X is called a continuous random variable. In the special case that it is absolutely continuous, its distribution can be described by a probability density function, which assigns probabilities to intervals; in particular, each individual point must necessarily have probability zero for an absolutely continuo us random variable. Not all continuous rand om variables are absolutely continuous

All random variables can be des cribed by their cumulative distribution function, which describes the probability that the random variable will be less than or equal to a certain val ue.

II. **Amplitude modulation** (**A M**) is a modulation technique used for transmitting information via a radio carrier wave. AM works by varying the strength (amplitude) of the carrier in proportion to the waveform be ing sent. That waveform may, for instance, correspond to the sounds to be reproduced by a loudspeaker, or the light intensity of televis ion pixels. This contrasts with frequency modul ation, in which the frequency of the carrier sig nal is varied, and phase modulation, in which its phase is varied, by the modulating signal.

Need for Modulation:

Ease transmission that necessit ates the decision of antenna length, Allows multiplexing of signals, Bandwidth trade-off wit h SNR

AM was the earliest modulation method used to transmit voice by radio for example it is used in portable two way radio, VHF aircraft radio and in computer modems. "AM" is often used to refer to medium wave AM radio broa dcasting.

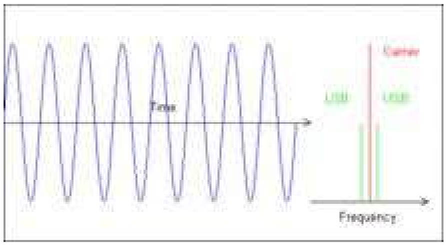
One disadvantage of all amplit ude modulation techniques (not only standard AM) is that the receiver amplifies and detects noise and electromagnetic interference in equal proportion to the signal. Increasing the received signal to noise ratio, say, by a factor of 1 0 (a 10 decibel improvement), thus would require increasing the transmitter power by a factor of 10. This is in contrast to frequency modulation (FM) and digital where the effect of such noise following demodulation is strongly reduced so long as the received signal is well above the threshold for reception. For this reason AM broadcast is not favored for music and highly fidelity broadcasting, but rather for voice communications and broadcasts (sports, news, talk radio etc.).

Another disadvantage of AM is that it is inefficient in power usage; at least two-thirds of the power is concentrated in the carrier signal. The carrier signal contains none of the original information being transmitted (voice, video, data, etc.). However its presence provides a simple means of demodulation using envelop detection, providing a frequency and phase reference to extract the modulation from the sidebands. In some modulation systems based on AM, a lower transmitter power is required through partial or total elimination of the carrier component; however receivers for these signals are more complex and costly. The receiver may regenerate a copy of the carrier frequency (usually as shifted to the intermediate frequency) from a greatly reduced "pilot" carrier (in reduced carrier- carrier or DSB-RC) to use in the demodulation process. Even with the carrier totally eliminated in double –sideband suppressed-carrier transmission, carrier regeneration is possible using a phase-locked loop. This doesn't work however for single-sideband suppressed-carrier (SSB-SC), Single sideband is nevertheless used widely in amateur radio and other voice communications both due to its power efficiency and bandwidth efficiency (cutting the RF bandwidth in half compared to standard AM). On the other hand, in MW and SW broadcasting, standard AM with the full carrier allows for reception using inexpensive receivers. The broadcaster absorbs the extra power cost to greatly increase potential audience.

An additional function provided by the carrier in standard AM, but which is lost in either single or double-sideband suppressed-carrier transmission, is that it provides an amplitude reference. In the receiver, the automatic gain control (AGC) responds to the carrier so that the reproduced audio level stays in a fixed proportion to the original modulation. On the other hand, with suppressed-carrier transmissions there is *no* transmitted power during pauses in the modulation, so the AGC must respond to peaks of the transmitted power during peaks in the modulation. This typically involves a so-called fast attack, slow decay circuit which holds the AGC level for a second or more following such peaks, in between syllables or short pauses in the program. This is very acceptable for communications radios, where compression of the audio aids intelligibility. However it is absolutely undesired for music or normal broadcast programming, where a faithful reproduction of the original program, including its varying modulation levels, is expected.

A trivial form of AM which can be used for transmitting binary data is on-off keying, the simplest form of amplitude shift keying, in which ones and zeros are represented by the presence or absence of a carrier. On-off keying is likewise used by radio amateurs to transmit Morse code where it is known as continuous wave (CW) operation, even though the transmission is not strictly "continuous."

Simplified analysis of standard AM



Left part: Modulating signal. Right part: Frequency spectrum of the resulting amplitude modulated carrier

Consider a carrier wave (sine wave) of frequency f_c and amplitude A given by:

$$c(t) = A \cdot \sin(2\pi f_c t).$$

Let $m(t)$ represent the modulatio n waveform. For this example we shall take the modulation to be simply a sine wave of a frequen cy f_m, a much lower frequency (such as an audi o frequency) than f_c:

$$m(t) = M \cdot \cos(2\pi f_m t + \phi),$$

where M is the amplitude of the modulation. We shall insist that $M<1$ so that $(1+m(t))$ is always positive. Amplitude modulation results when the carrier $c(t)$ is multiplied by the positive quantity $(1+m(t))$:

$$\begin{aligned} y(t) &= [1 + m(t)] \cdot c(t) \\ &= [1 + M \cdot \cos(2\pi f_m t + \phi)] \cdot A \cdot \sin(2\pi f_c t) \end{aligned}$$

In this simple case M is identiccal to the modulation index, discussed below. With M=0.5 the amplitude modulated signal $y(t)$ thus corresponds to the top graph (labelled "5 0% Modulation") in Figure 4.

Using prosthaphaeresis identities , $y(t)$ can be shown to be the sum of three sine waves:

$$y(t) = A \cdot \sin(2\pi f_c t) + \frac{AM}{2}\left[\sin(2\pi(f_c + f_m)t + \phi) + \sin(2\pi(f_c - f_m)t - \phi)\right].$$

Therefore, the modulated signal has three components: the carrier wave $c(t)$ wh ich is unchanged, and two pure sine waves (know n as sidebands) with frequencies slightly abov e and below the carrier frequency f_c.

Modulation Index

The AM modulation index is a measure based on the ratio of the modulation excursions of the RF signal to the level of the unmodulated carrier. It is thus defined as:

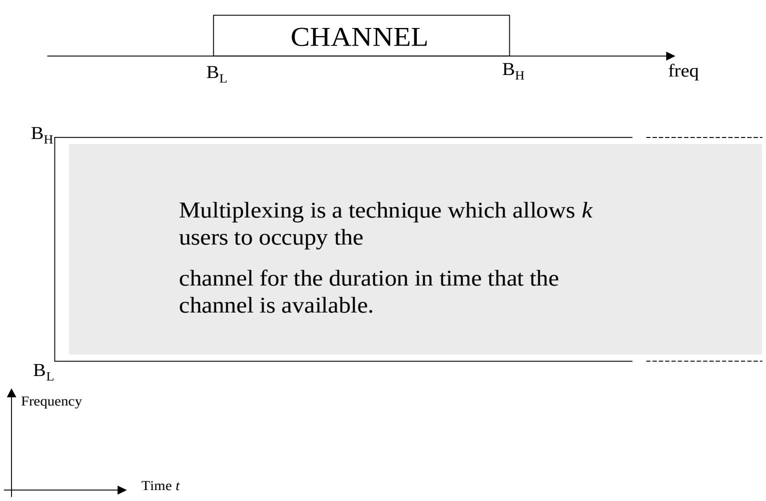
$$h = \frac{\text{peak value of } m(t)}{A} = \frac{M}{A}$$

where M and A are the m odulation amplitude and carrier amplitude, respectively; the modulation amplitude is the peak (positive or negative) change in the RF a mplitude from its unmodulated value. Modulatio n index is normally expressed as a percentage, and may be displayed on a meter connected to an AM transmitter.

TDM and FDM

Multiplexing is the name given to techniques, which allow more than one message to be transferred via the same communication channel. The channel in this context could be a transmission line, *e.g.* a twisted pair or co-axial cable, a radio system or a fibre optic system *etc*.

A channel will offer a specified bandwidth, which is available for a time t, where t may $\rightarrow \infty$. Thus, with reference to the channel there are 2 ‘degrees of freedom’, *i.e.* bandwidth or frequency and time.



Now consider a signal $v_s(t) = Amp\cos(\omega t + \phi)$. The signal is characterised by amplitude, frequency, phase and time.

Various multiplexing methods are possible in terms of the channel bandwidth and time, and the signal, in particular the frequency, phase or time. The two basic methods are:

1) **Frequency Division Multiplexing FDM**

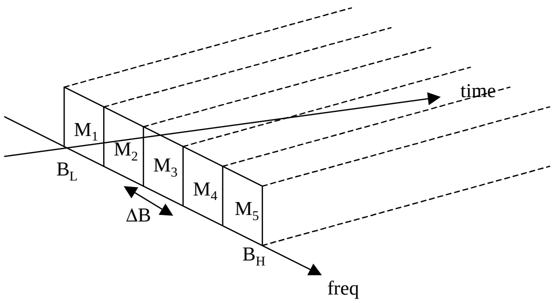
   FDM is derived from AM techniques in which the signals occupy the same physical ‘line’ but in different frequency bands. Each signal occupies its own specific band of frequencies all the time, *i.e.* the messages share the channel **bandwidth**.

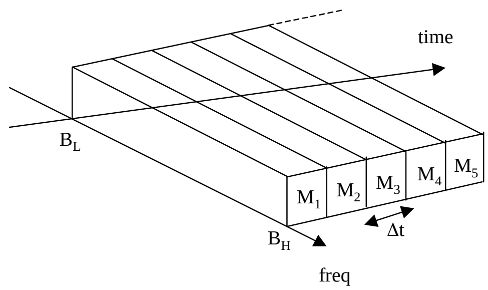
2) Time Division Multiplexing TDM

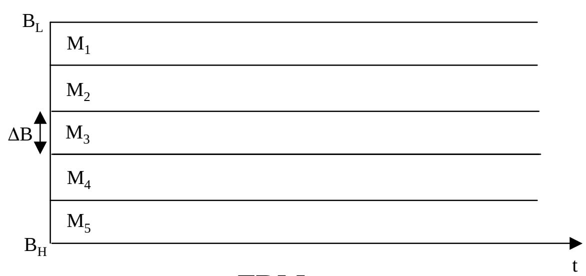
TDM is derived from sampling techniques in which messages occupy all the channel bandwidth but for short time intervals of time, *i.e.* the messages share the channel **time**.

- FDM – messages occupy **narrow** bandwidth – all the time.
- TDM – messages occupy **wide** bandwidth – for short intervals of time.

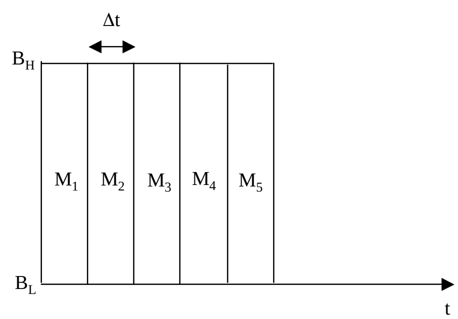
These two basic methods are illustrated below.







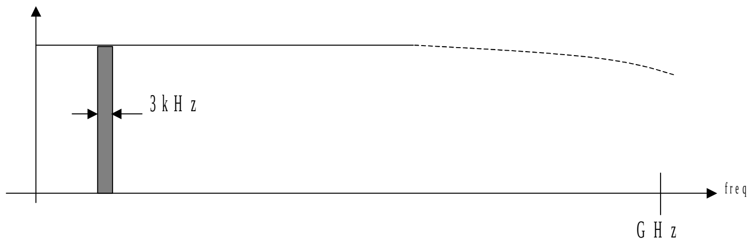
FDM



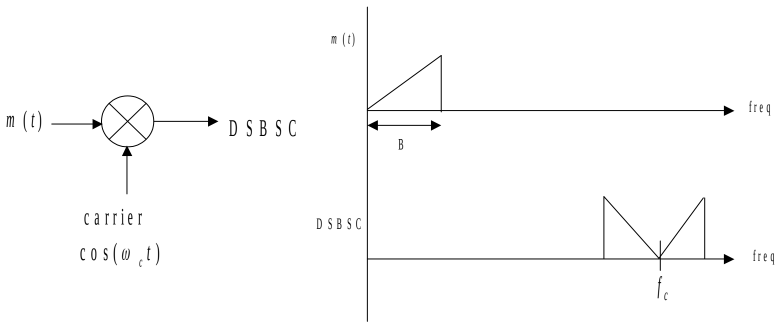
TDM

Frequency Division Multiplexing FDM

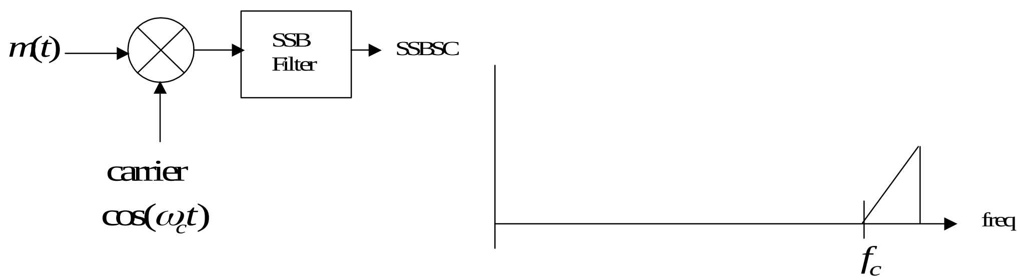
FDM is widely used in radio and television systems (*e.g.* broadcast radio and TV) and was widely used in multichannel telephony (now being superseded by digital techniques and TDM). The multichannel telephone system illustrates some important aspects and is considered below. For speech, a bandwidth of $\approx$ 3kHz is satisfactory. The physical line, *e.g.* a co-axial cable will have a bandwidth compared to speech as shown below.



From AM we have noted:

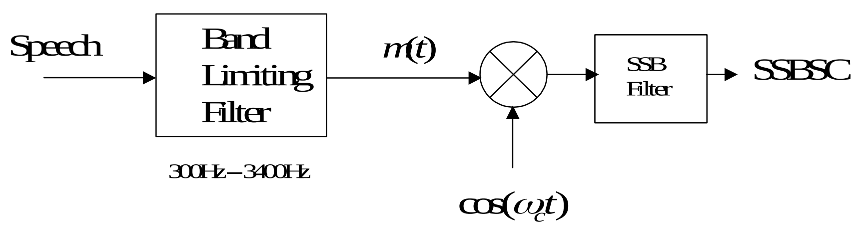


In order to use bandwidth more effectively, SSB is used *i.e.*

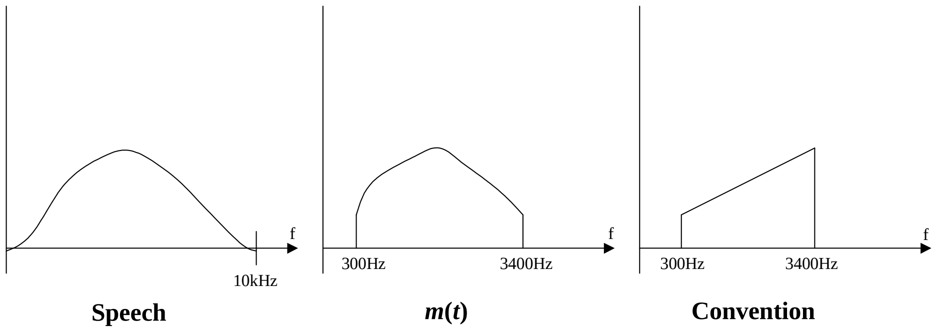


Note – the USB has been selected.

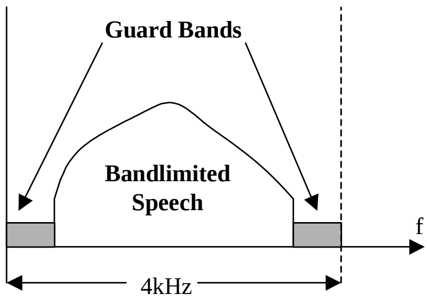
We have also noted that the message signal $m(t)$ is usually band limited, *i.e.*



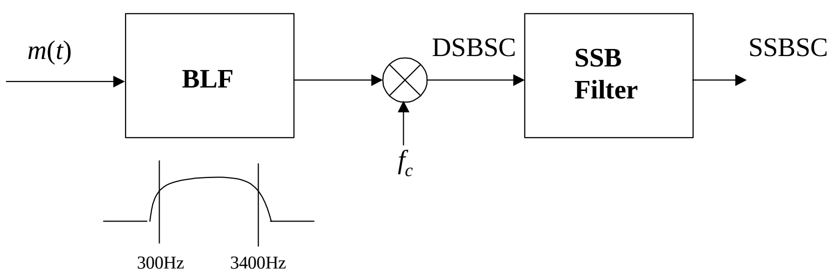
The Band Limiting Filter (BLF) is usually a band pass filter with a pass band 300Hz to 3400Hz for speech. This is to allow **guard bands** between **adjacent channels**.



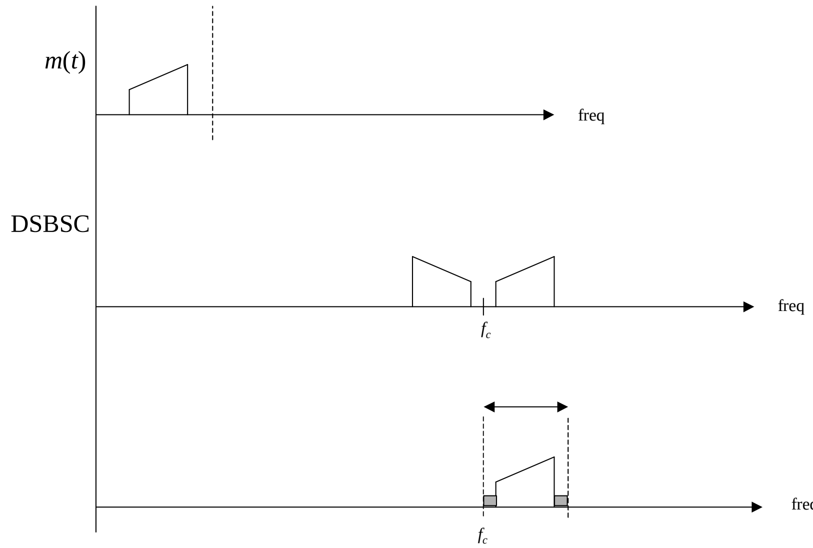
For telephony, the physical line is divided (notionally) into 4kHz bands or channels, *i.e.* the channel spacing is 4kHz. Thus we now have:



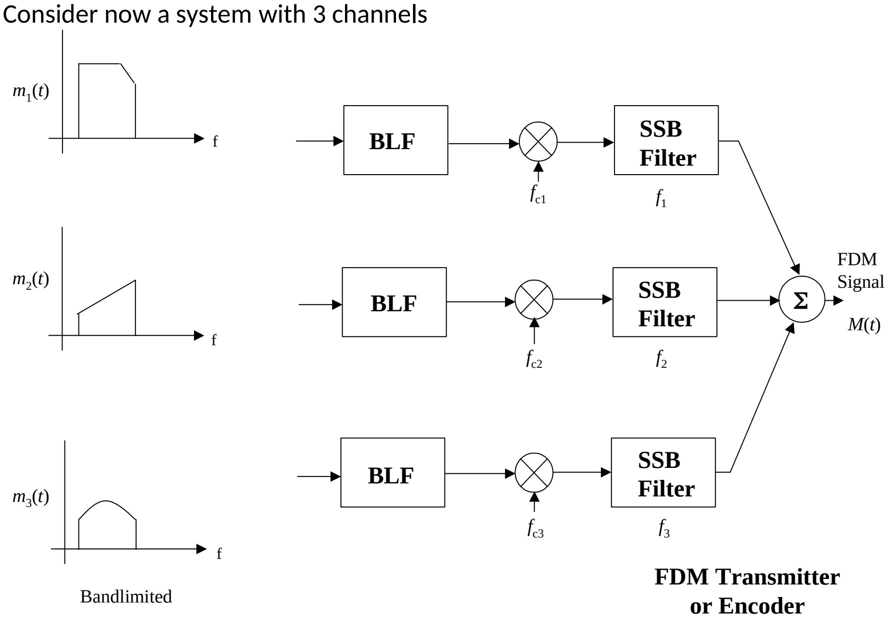
Note, the BLF does not have an ideal cut-off – the guard bands allow for filter 'roll off' in order to reduce adjacent channel crosstalk.
Consider now a single channel SSB system.



The spectra will be

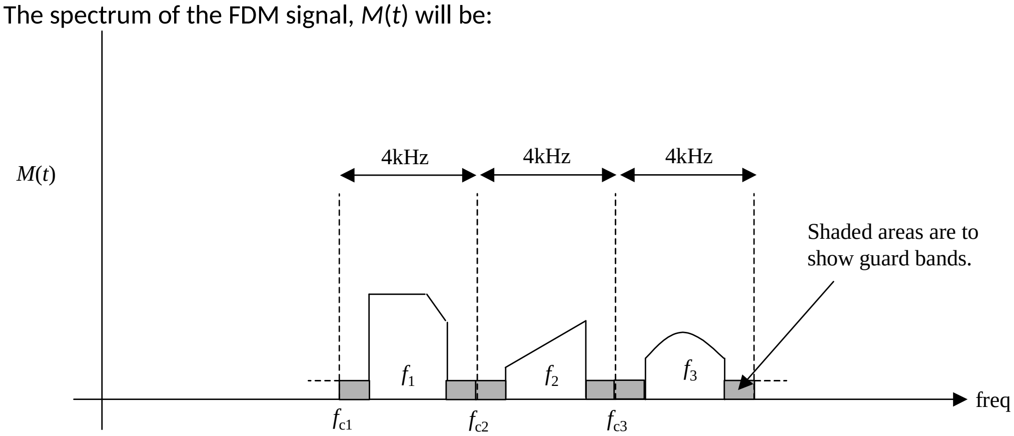


Consider now a system with 3 channels



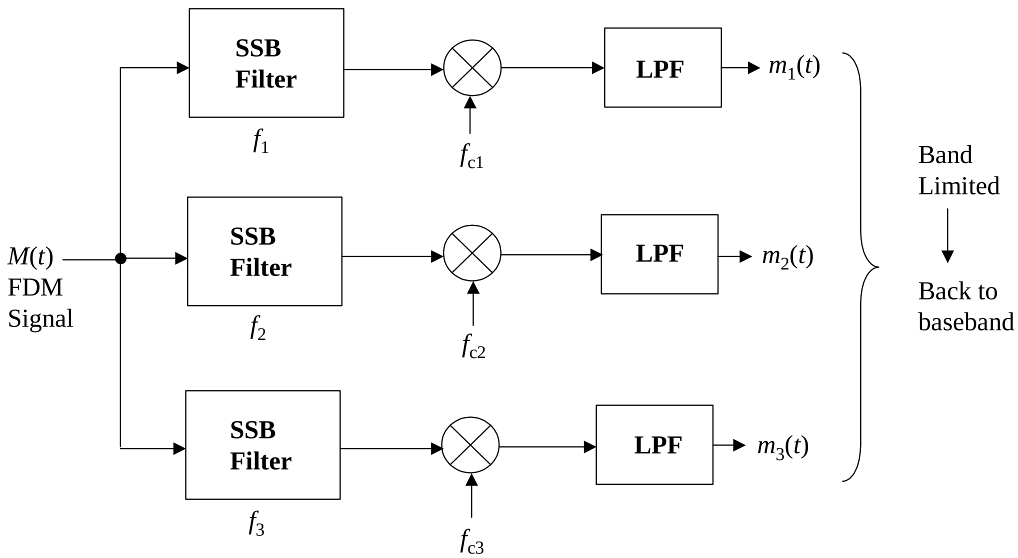
Each carrier frequency, f_{c1}, f_{c2} and f_{c3} are separated by the channel spacing frequency, in this case 4 kHz, *i.e.* $f_{c2} = f_{c1} + 4\text{kHz}$, $f_{c3} = f_{c2} + 4\text{kHz}$.

The spectrum of the FDM signal, $M(t)$ will be:



Note that the baseband signals $m_1(t)$, $m_2(t)$, $m_3(t)$ have been multiplexed into adjacent channels, the channel spacing is 4kHz. Note also that the SSB filters are set to select the USB, tuned to f_1, f_2 and f_3 respectively.

A receiver FDM decoder is illustrated below:



The SSB filters are the same as in the encoder, *i.e.* each one centred on f_1, f_2 and f_3 to select the appropriate sideband and reject the others. These are then followed by a synchronous demodulator, each fed with a synchronous LO, f_{c1}, f_{c2} and f_{c3} respectively.

For the 3 channel system shown there is 1 design for the BLF (used 3 times), 3 designs for the SSB filters (each used twice) and 1 design for the LPF (used 3 times).

A co-axial cable could accommodate several thousand 4 kHz channels, for example 3600 channels is typical. The bandwidth used is thus 3600 x 4kHz = 14.4Mhz. Potentially therefore there are 3600 different SSB filter designs. Not only this, but the designs must range from kHz to MHz.

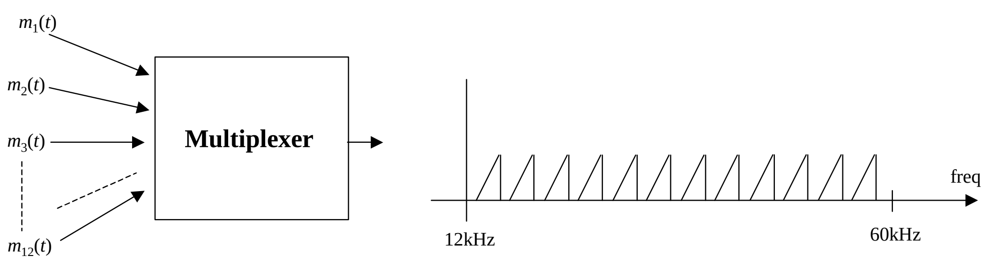
Consider also the 'Q' of the filter, where Q is defined as $Q = \dfrac{\text{centre frequency}}{\text{bandwidth}}$.

For 'designs' around say 60kHz, $Q = \dfrac{60kHz}{4kHz}$ = 15 which is reasonable. However, for designs to have a centre frequency at around say 10Mhz, $Q = \dfrac{10{,}000kHz}{4kHz}$ gives a Q = 2500 which is difficult to achieve.

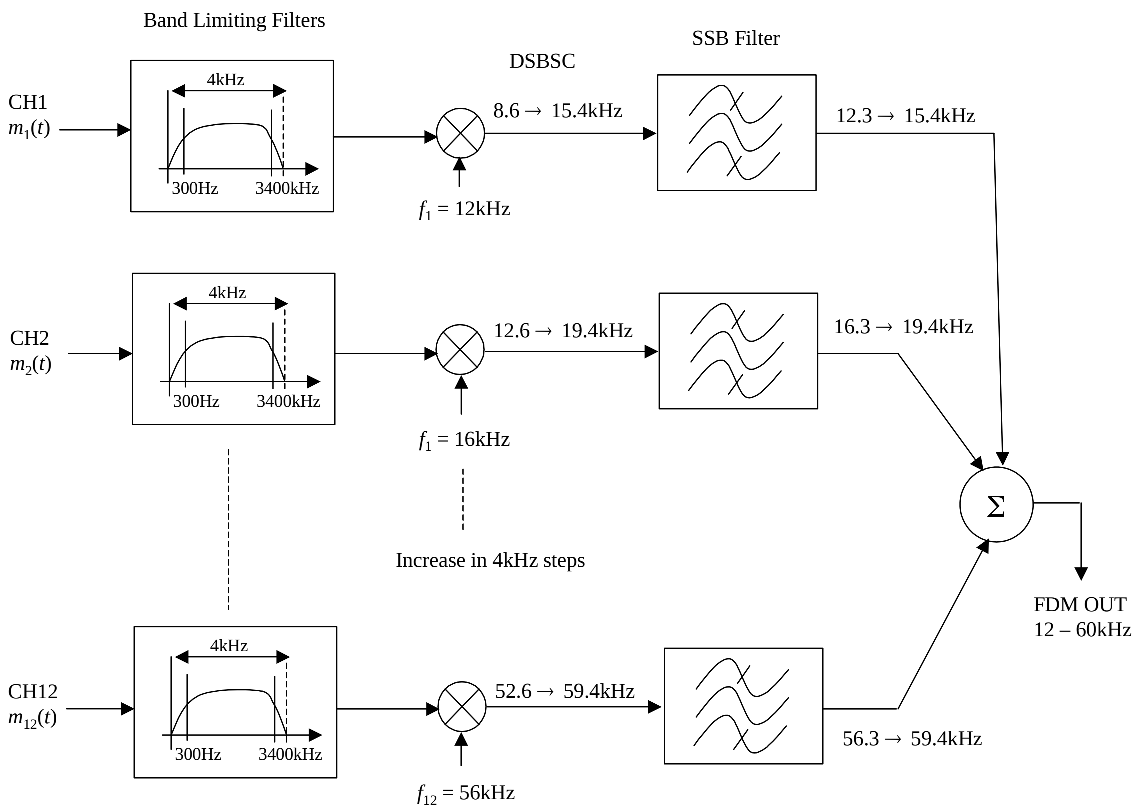
To overcome these problems, a hierarchical system for telephony used the FDM principle to form **groups**, **supergroups**, **master groups** and **supermaster groups**.

Basic 12 Channel Group

The diagram below illustrates the FDM principle for 12 channels (similar to 3 channels) to a form a basic group.

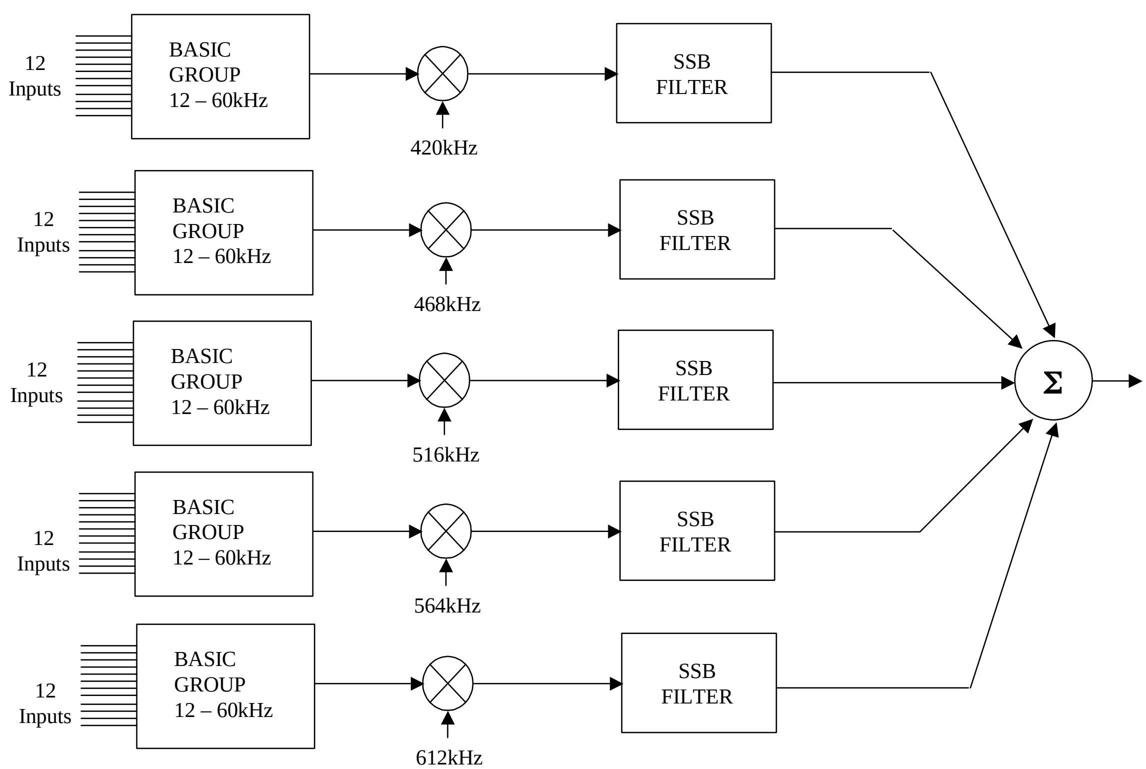


i.e. 12 telephone channels are multiplexed in the frequency band 12kHz → 60 kHz in 4kHz channels ≡ **basic group**.
A design for a basic 12 channel group is shown below:



Super Group

These basic groups may now be multiplexed to form a **super group**.



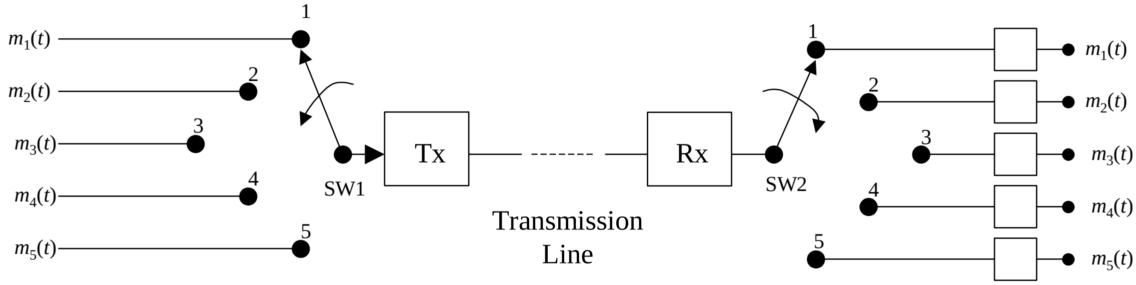
5 basic groups multiplexed to form a super group, *i.e.* 60 channels in one super group.

Note – the channel spacing in the super group in the above is 48kHz, *i.e.* each carrier frequency is separated by 48kHz. There are 12 designs (low frequency) for one basic group and 5 designs for the super group.

The Q for the super group SSB filters is $Q = \frac{612kHz}{48kHz} \approx 12$ - which is reasonable. Hence, a total of 17 designs are required for 60 channels. In a similar way, super groups may be multiplexed to form a master group, and master groups to form super master groups…

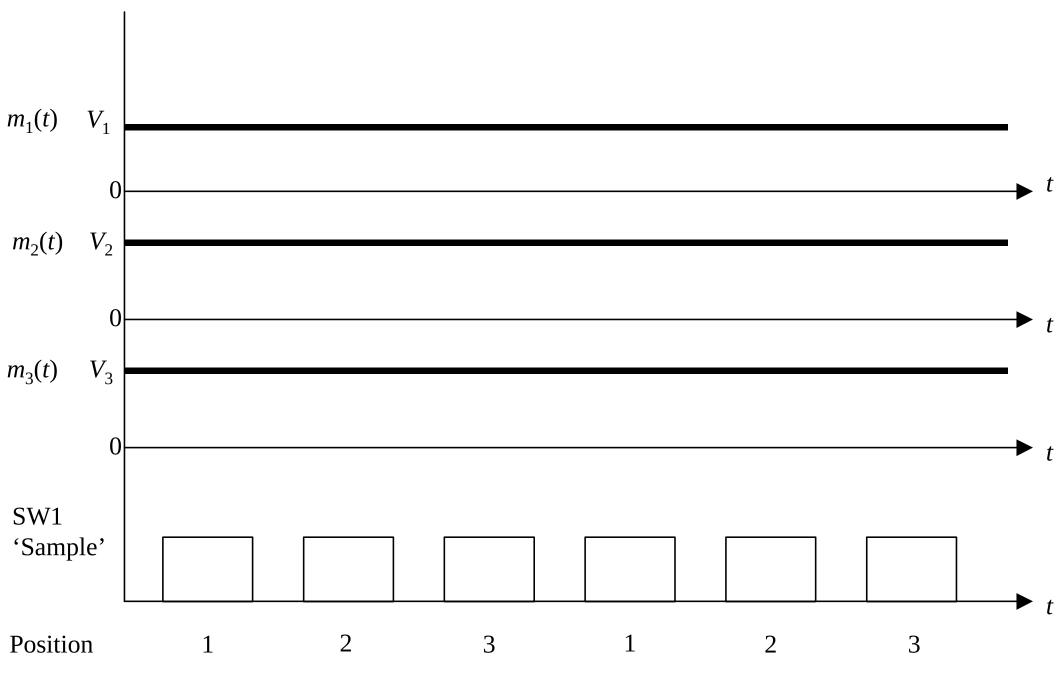
Time Division Multiplexing TDM

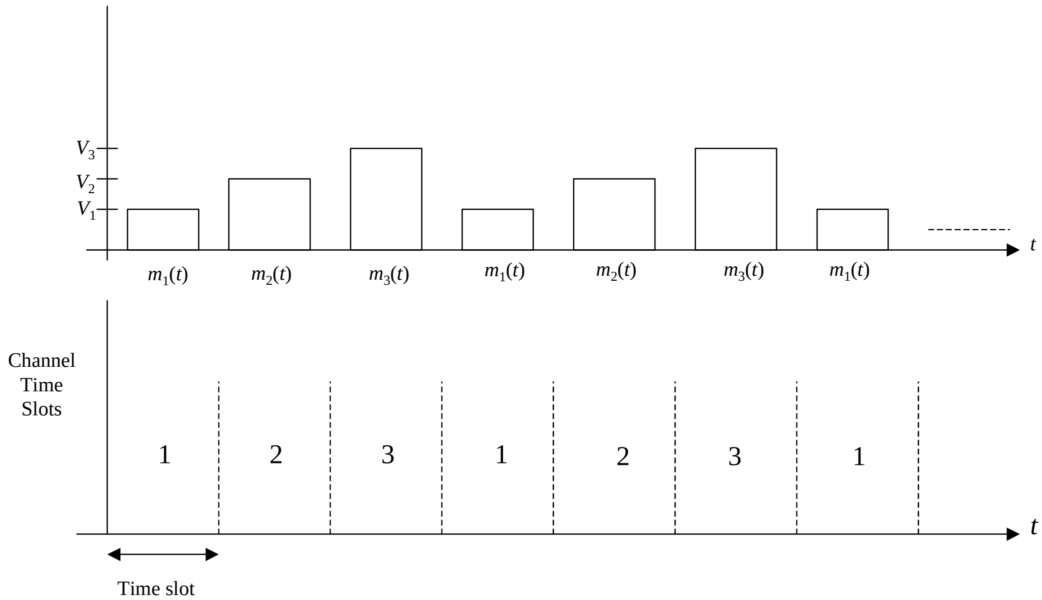
TDM is widely used in digital communications, for example in the form of pulse code modulation in digital telephony (TDM/PCM). In TDM, each message signal occupies the channel (*e.g.* a transmission line) for a short period of time. The principle is illustrated below:



Switches SW1 and SW2 rotate in synchronism, and in effect sample each message input in a sequence $m_1(t)$, $m_2(t)$, $m_3(t)$, $m_4(t)$, $m_5(t)$, $m_1(t)$, $m_2(t)$,…

The sampled value (usually in digital form) is transmitted and recovered at the 'far end' to produce output $m_1(t)$… $m_5(t)$. For ease of illustration consider such a system with 3 messages, $m_1(t)$, $m_2(t)$ and $m_3(t)$, each a different DC level as shown below.





In this illustration the samples are shown as levels, *i.e.* V_1, V_2 or V_3. Normally, these voltages would be converted to a binary code before transmission as discussed below.

Note that the channel is divided into time slots and in this example, 3 messages are time-division multiplexed on to the channel. The sampling process requires that the message signals are a sampled at a rate $f_s \geq 2B$, where f_s is the sample rate, samples per second, and B is the maximum frequency in the message signal, $m(t)$ (*i.e.* Sampling Theorem applies). This sampling process effectively produces a pulse train, which requires a bandwidth much greater than B.
Thus in TDM, the message signals occupy a wide bandwidth for short intervals of time. In the illustration above, the signals are shown as PAM (Pulse Amplitude Modulation) signals. In practice these are normally converted to digital signals before time division multiplexing.

TIME DIVISION MULTIPLEXING

A technique is called Time division multiplexing by which we may take advantage of sampling principle. Sampling of signals is needed for applications such as time sharing of a signal transmission facility, i.e. time division multiplexing (TDM) [2]. The signal samples may be used in analogue form, e.g. using pulse amplitude modulation (PAM), or they may be quantized and encoded into digital form, i.e. pulse code modulation (PCM).

Since the purpose of TDM is to exploit better the available performance of a signal transmission facility by squeezing as many channels into it as practically possible, this objective will be best achieved by using as few samples as possible per unit time per channel. For this reason it is important to know the minimum sample rate needed to avoid excessive distortion, and also to avoid placing unrealistic or costly demands on the performance of the functional elements of the system (e.g. transition bandwidth of anti aliasing and interpolation filters) [2]. These considerations apply not only to TDM systems, but in fact to all sampled data signal processing systems, which includes all digital processing of analogue data, e.g. digital filtering, digital spectrum analysis. The sampling theorem provides a theoretical basis for estimating a minimum sampling rate in these situations [4].

If two messages are sampled at the same rate but at slightly different times, then two of trains of samples can be added without mutual interaction. In Figure-1.a the signal x(t) and the corresponding PAM signal are depicted. Also in Figure-1.b, TDM of two signals is shown:

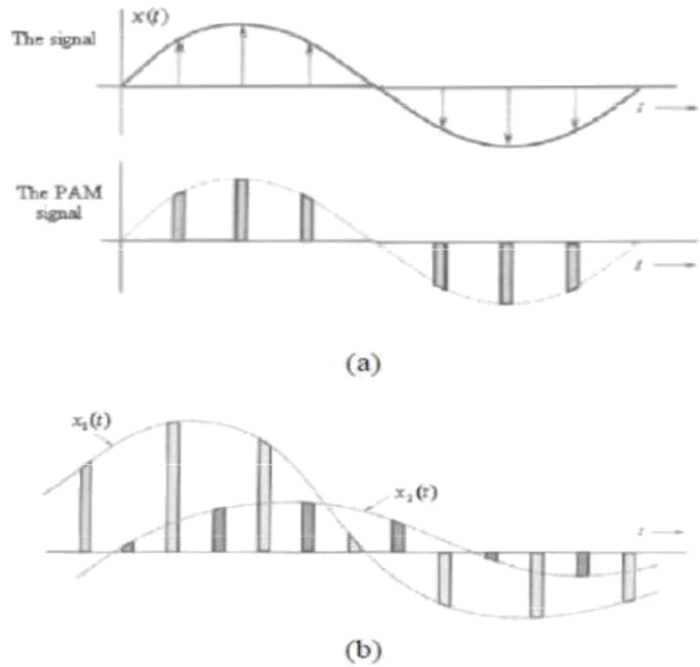
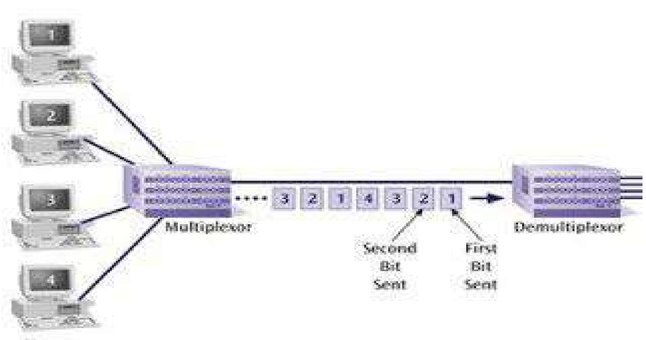


Figure 1. (a) The signal $x(t)$ and its PAM signal
(b) TDM of two signals

Concept of Time Division Multiplexing-TDM:

In TDM, sharing is accomplished by dividing available "transmission time" on a medium/channel among users. Each user of the channel is allotted a small time interval during which he transmits a message [3]. Total time available in the channel is divided, and each user is allocated a time slice. In TDM, users send message sequentially one after another. Each user can use the full channel bandwidth during the period he has control over the channel [4].



The block diagram of PAM/TDM signal production is given in Figure 2 below:

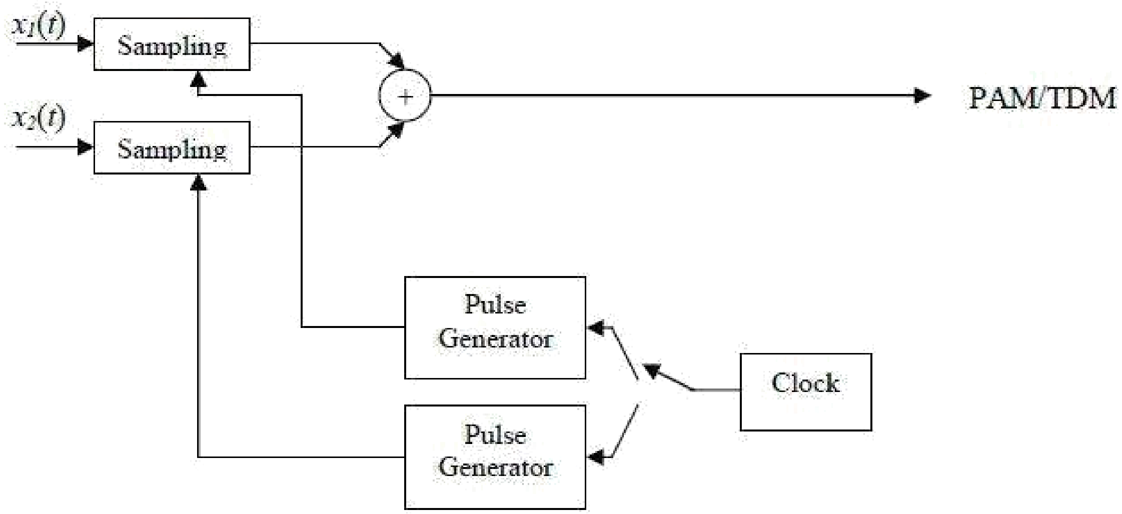


Figure 2. Production of TDM Signal

In Figure 2, if the band-widths of both signals (x1(t) and x2(t)) is 3 kHz, according to the sampling theorem each signal should be sampled with the frequency of 6 kHz. But in this case the clock frequency should be 12 kHz. The distance between the pulses is Tn =Ts/n. Here, n indicates the number of the input signals, and Ts denotes the sampling period required for one signal. The obtained TDM wave is shown in Figure 3.

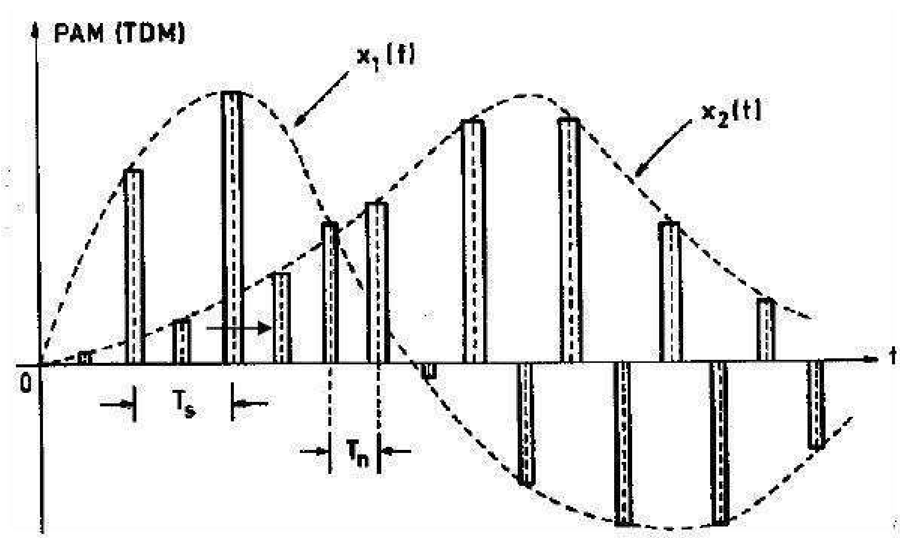
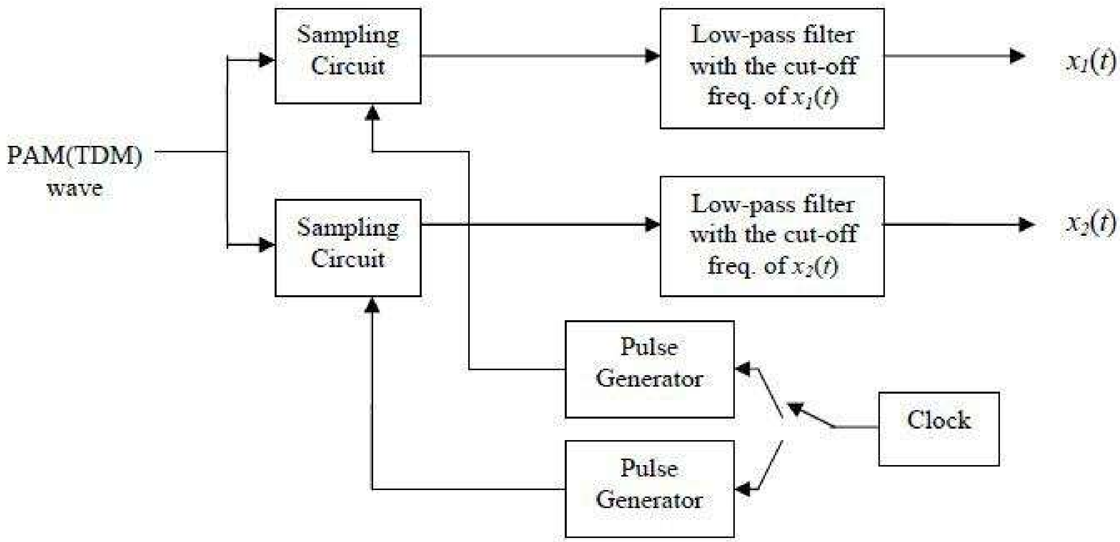


Figure 3. PAM(TDM) Wave

As considered before, if two different signals have the frequency of 3 kHz, the sampling period of each signal will be Ts=1/6000 = 166.7 µsec. Since number of input signals n=2, from Tn=Ts/n;

the distance between samples becomes Tn=Ts/2 = 83.3 μsec. Wave minimum bandwidth to transmit these samples by TDM should be; B≥166.7.10 61− =6 kHz.

A TDM receiver block diagram is shown in Figure 4 below. The most significant issue in recovery of input signals from TDM signals is the requirement for the proper synchronization between TDM transmitter and receiver. Therefore, the clock signal in the transmitter should be passed to the receiver correctly [4].



TDM RECEIVER CIRCUIT

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