UNIT -III

ANGLE MODULATION:

In this type of modulation the modulator output is of constant in amplitude, and the message signal is superimposed on the ca rrier by varying its frequency and phase togeth er is called angle [1].

Angle modulation encompasses phase modulation (PM) and frequency modu lation (FM). The phase angle of a sinusoidal carrier signal is varied according to the modulating signal. In angle modulation, the spectral comp onents of the modulated signal are not rel ated in a simple fashion to the spectrum of the modulating signal [1]. Superposition does n ot apply and the bandwidth of the modulated signal is usually much greater than the m odulating signal bandwidth. Mathematically

Consider a sinusoid $A_c \cos(2\pi f_c t + \phi_0)$, where A_c is the (constant) amplitude, f_c is the (constant) frequency in Hz and ϕ_0 is the initial phase angle. Let the sinuso id be written as $A_c \cos[\theta(t)]$ where $\theta(t) = 2\pi f_c t + \phi_0$. the case where A_c is a constant but, instead of being equal to $2\pi f_c t + \phi_0$, is a function of $m(t)$ .This leads to what is known as the angle modulated signal [2]. Two important cases of angle modulation are Frequency Modulation (FM) and Phase modulation (PM).

An important feature of FM an d PM is that they can provide much better p rotection to the message against the channel noise as compared to the linear (amplitude) modulation schemes. Also, because of their constant amplitude nature, they can withstand nonlinear distortion and amplitude fading. The price paid to achieve these benefits is the increased bandwidth requirement; that is, the transmission bandwidth of the FM or PM sign al with constant amplitude and which can provide noise immunity is much larger than $2W$, where W is the highest frequency component present in the message spectrum [2].

Now let us define PM and FM. Consider a signal s (t) given by $s(t) = A_c \cos[\theta_i(t)]$ where $\theta_i(t)$, the instantaneous angle quantity, is a function of m(t) . We define t he instantaneous frequency of the angle modulat ed wave s (t) , as

$$f_i(t) = \frac{1}{2\pi} \frac{d\theta_i(t)}{dt} \qquad (5.1)$$

(The subscript i in $\theta_i(t)$ or $f_i(t)$ is in dicative of our interest in the instantaneous b ehavior of these quantities). If $\theta_i(t) = 2\pi f_c t + \phi_0$, then $f_i(t)$ reduces to the constant f_c, w ich is in perfect agreement with our established notion of frequency of a sinusoid. This is illustrated in Fig. 5.1.

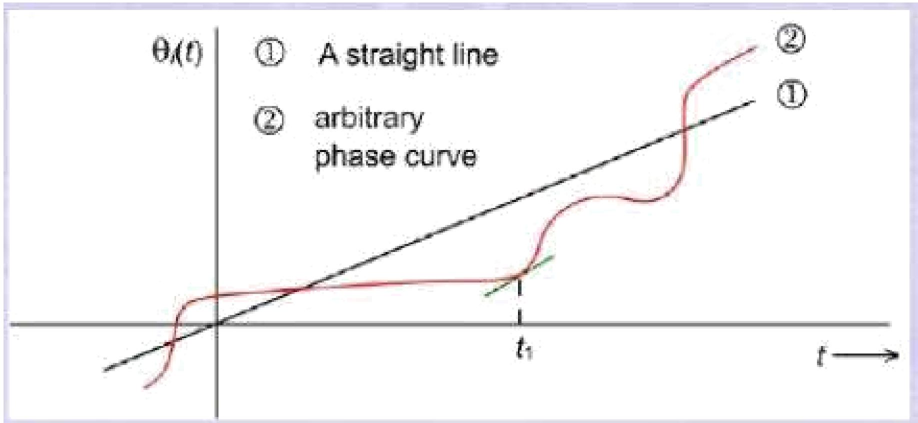


Fig.5.1 Ilustr ation of instantenious phase and frequency

Curve 1 in Fig. 5.1 depicts the p hase behavior of a constant frequency sinusoid with $\phi_0 = 0$. Hence, its phase, as a function of time is a straight line; that is $\theta_i(t) = 2\pi f_c t$. Slope of this line is a constant and is equal to the frequency of the sinusoid. Curve 2 depicts an arbitrary phase behavior; its slope changes withh time. The instantaneous frequency (in radian s per second) of this signal at t = t1 is given by th e slope of the tangent (green line) at that time [2].

a) Phase modulation:

For PM, $\theta_i(t)$ is given by

$$\theta_i(t) = 2\pi f_c t + k_p m(t) \tag{5.2}$$

The term 2πfct is the angle o f the un-modulated carrier and the constant is the phase sensitivity of the modulator wit h the units, radians per volt. (For convenience, the initial phase angle of the un-modulated carri er is assumed to be zero) [1]. Using Eq. 5.2, the phase modulated wave s (t) can be written as

$$\left[s(t)\right]_{PM} = A_c \cos\left[2\pi f_c t + k_p m(t)\right] \tag{5.3}$$

From Eq. 5.2 and 5.3, it is evident that for PM, the phase deviation of s (t) from that of the un modulated carrier phase is a linear function of the base-band message signal, m(t).The instantaneous frequency of a phase modulated signal depends on

$$\frac{d\,m(t)}{dt} = m'(t) \text{ because } \frac{1}{2\pi}\frac{d\theta_i(t)}{dt} = f_c + \frac{k_p}{2\pi} m'(t).$$

b) Frequency Modulati on:

Let us now consider the case where $f_i(t)$ is a function of *m*(*t*) ; that is,

$$f_i(t) = f_c + k_f\, m(t) \qquad (5.4)$$

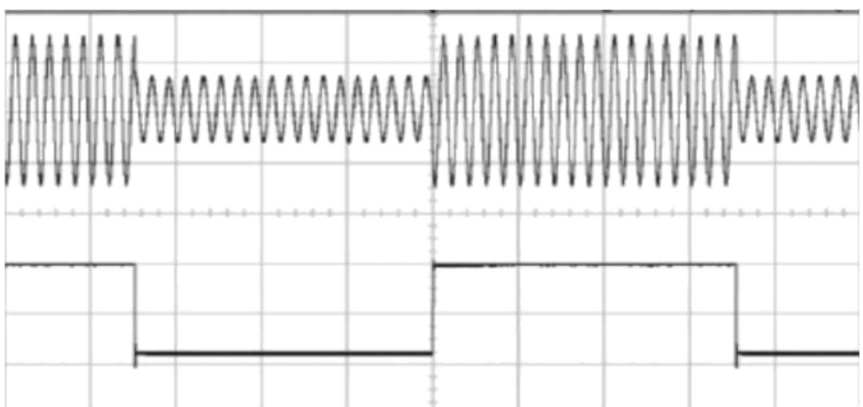
$$\theta_i(t) = 2\pi \int_{-\infty}^{t} f_i(\tau)\, d\tau \qquad (5.5)$$

$$= 2\pi f_c\, t + 2\pi k_f \int_{-\infty}^{t} m(\tau)\, d\tau \qquad (5.6)$$

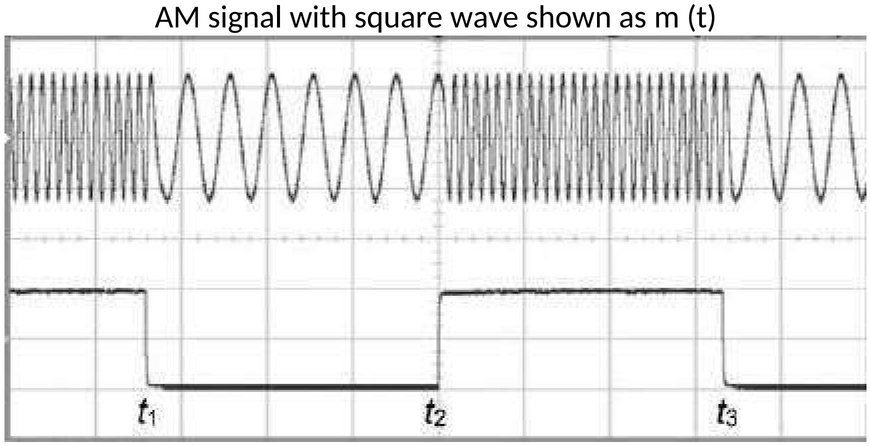
k_f is a constant, which we will identify shortly. A frequency modulated signal $s(t)$ is described in the time domain by

$$\left[s(t)\right]_{FM} = A_c \cos\left[2\pi f_c\, t + 2\pi k_f \int_{-\infty}^{t} m(\tau)\, d\tau\right] \qquad (5.7)$$

k_f is termed as the *frequency sensitivity* of the modulator with the units Hz/volt. From Eq. 5.4 we infer that for an FM signal, the instantaneous frequency



AM signal with square wave shown as m (t)



FM si gnal with square wave shown as m (t)

As both PM and FM have constant amplitude A_c, the average power of a PM or FM signal is,

$$P_{av} = \frac{A_c^2}{2},$$

regardless of the value of k_p or k_f.

TONE MODULATED FM SIG NAL:

An angle modulated signal is known as tone modulated if its amplitude arbitrarily set at unity, also referred to as an exponenti ally modulated signal [1] , has the form

$$S_m(t) = A\cos[\omega t + \theta(t)] = \text{Re}\{A\exp[j\phi(t)]\} \qquad (1)$$

Were the instantaneous phase $\phi_i(t)$ is defined as

$$\phi_i(t) = \omega t + \theta(t) \qquad (2)$$

and the instantaneous frequency of the modulated signal is defined as

$$\omega_i(t) = \frac{d}{dt}[\omega t + \theta(t)] = \omega + \frac{d(\theta(t))}{dt} \qquad (3)$$

The functions $\theta(t)$ and $\frac{d(\theta(t))}{dt}$ are referred to as the instantaneous phase and frequency deviations, respectively.

The phase deviation of the carrier $\phi(t)$ is related to the baseband message signal s(t). Depending on the nature of the relationship between $\phi(t)$ and $s(t)$ we have different forms of angle modulation [1].

$$\frac{d(\theta(t))}{dt} = k_f s(t) \qquad (4)$$

$$\phi(t) = k_f \int_{t_0}^{t} s(\lambda)d\lambda + \omega t \qquad (5)$$

Where K_f is a frequency deviation constant, (expressed in (radian/sec)/volt).

It is usually assumed that $t_0 = -\infty$ and $\phi(-\infty) = 0$.

Combining Equations-4 and 5 with Equation-1, we can express the frequency modulated signal as

$$S_m(t) = A\cos[\omega t + k_f \int_{-\infty}^{t} s(\tau)d\tau] \quad (6)$$

Fig. 1 shows a single tone (s (t) message signal), frequency modulated a carrier frequency, represented in time domain.

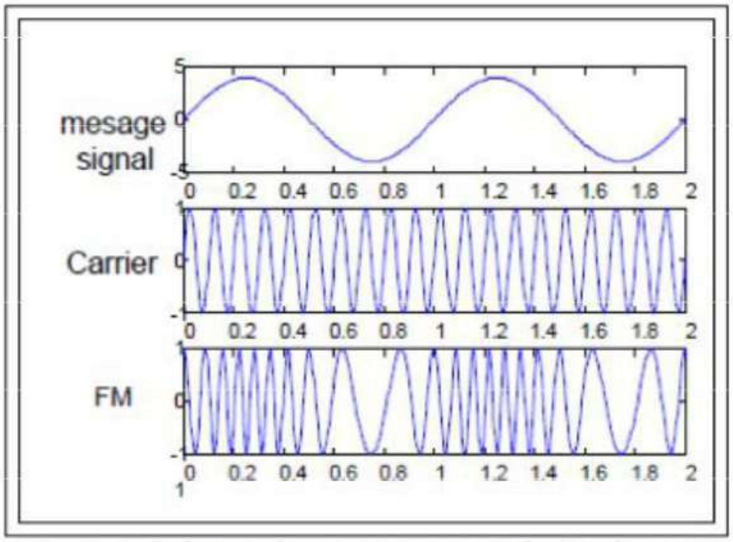


Fig 1: modulation signal (upper), carrier signal (midle), and modulated FM signal(lower)

Bessel function:

Bessel function of the first kind, is a solution of the differential equation

$$\beta^2 \frac{d^2y}{dx^2} + \beta \frac{dy}{dx} + (\beta^2 - n^2)y(\beta) = 0$$

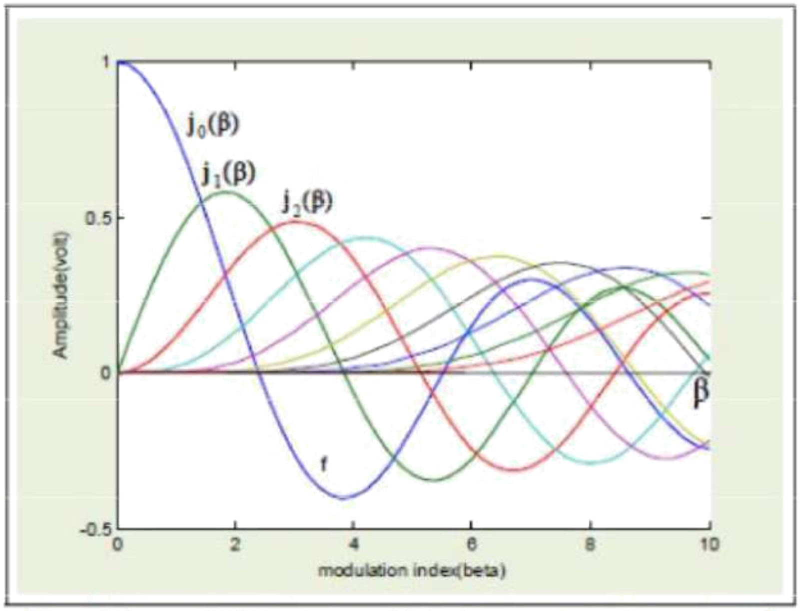


Fig. 2 Bessel function, of kind 1, and order 1 to 10

Bessel function defined for negative and positive real integers. It can be shown that for integer values of n

$$j_{-n}(\beta) = (-1)^n j_n(\beta) \tag{7}$$

$$j_{n-1}(\beta) + j_{n+1}(\beta) = \frac{2n}{\beta} j_n(\beta) \tag{8}$$

$$\sum_{n=-\infty}^{\infty} j_n^2(\beta) = 1 \tag{9}$$

A short listing of Bessel function of first kind of order n and discrete value of argument β, is shown in Table-1, and graph of the function, is shown in Fig. 2

Note that for very small β,value $j_0(\beta)$ approaches unity, while $j_1(\beta)$ to $j_n(\beta)$ approach zero.

$n \backslash \beta$	0	0.2	0.5	1	2	5	8	10
0	1.00	0.99	0.938	0.765	0.224	-0.178	0.172	-0.246
1	0	0.1	0.242	0.440	0.577	-0.328	0.235	0.043
2		0.005	0.031	0.115	0.353	0.047	-0.113	0.255
3				0.02	0.129	0.365	-0.291	0.058
4				0.002	0.034	0.391	-0.105	-0.22
5					0.007	0.261	0.186	-0.234
6						0.131	0.338	-0.14
7						0.053	0.321	0.217
8						0.018	0.223	0.318
9							0.126	0.292
10							0.061	0.208
11							0.026	0.123

Table-1 Bessel function $j_n(\beta)$

Properties of Bessel's Function:

1. Eq. -1.9 indicates that the phase relationship between the sideband components is such that the odd-order lower sidebands are reversed in phase .

2. The number of significant spectral components is a function of argument β (see Table-1). When $\beta \ll 1$, only J_0, and J_1, are significant so that the spectrum will consist of carrier plus two sideband components, just like an AM spectrum with the exception of the phase reversal of the lower sideband component.

3. A large value of β implies a large bandwidth since there will be many significant sideband components.

4. Transmission bandwidth of 98% of power always occur after $n = \beta + 1$, we note it in table-1 with underline.

5. Carrier and sidebands null many times at special values of β see table-2

Order	0	1	2	3	4	5	6
β for 1st zero	2.40	3.83	5.14	6.38	7.59	8.77	9.93
β for 2nd zero	5.52	7.02	8.42	9.76	11.06	12.34	13.59
β for 3rd zero	8.65	10.17	11.62	13.02	14.37	15.70	17.00
β for 4th zero	11.79	13.32	14.80	16.22	17.62	18.98	20.32
β for 5th zero	14.93	16.47	17.96	19.41	20.83	22.21	23.59
β for 6th zero	18.07	19.61	21.12	22.58	24.02	25.43	26.82

Table-2 Zeroes of Bessel function: Values for β when $j_n(\beta) = 0$

Power and Bandwidth of Tone modulated signal:

In the previous section we saw that a single tone modulated FM signal has an infinite number of sideband components and hence the FM spectrum seems to have infinite spectrum. Fortunately, it turns out that for any β a large portion of the power is contained in finite bandwidth. . Hence the determination of FM transmission bandwidth depends to the question of how many significant sidebands need to be included for transmission, if the distortion is to be within certain limits.

To determine FM transmission bandwidth, let us analyze the power ratio S_n, which is the fraction of the power contained in the carrier plus n sidebands, to the total power of FM signal. We search a value of number of sidebands n, for power ratio $S_n \geq 0.98$.

$$S_n = \frac{\frac{1}{2}A \sum\limits_{k=-n}^{n} j_k^2(\beta)}{\frac{1}{2}A \sum\limits_{k=-\infty}^{\infty} j_k^2(\beta)} \tag{17}$$

$$S_n \geq 0.98 \tag{18}$$

Using the value properties of Bessel function,, and Table 1, we can show that the bandwidth of FM signal B_T, depends on the number of sidebands n, and FM modulation index β. which can be expressed as

$$B_T \approx 2(\beta + 1)f_m \tag{19}$$

ARBITARY MODULATED FM SIGNAL:

Narrow Band FM:

Narrowband FM is in many ways similar to DSB or AM signals. By way of illustration let us consider the $NBFM$ signal

$$\begin{aligned} S_m(t) &= A\cos[\omega t + \phi(t)] = A\cos\omega t\cos\phi(t) - A\sin\omega t\sin\phi(t) \\ &\approx A\cos\omega t - A\phi(t)\sin\omega t \end{aligned} \tag{20}$$

Using the approximations $cos\phi = 1$ and $sin\phi \approx \phi$, when ϕ is very small. Equation-26 shows that a $NBFM$ signal contains a carrier component and a quadrature carrier linearly modulated by (a function of) the baseband signal. Since $s(t)$ is assumed to be bandlimited to f_m therefore $\phi(t)$ is also bandlimited to $f_{m,}$. Hence, the bandwidth of $NBFM$ is $2f_m$, and the $NBFM$ signal has the same bandwidth as an AM signal.

Narrow band FM modulator:

According to Equation-1.20, it is possible to generate $NBFM$ signal using a system such as the one shown in Fig-4 . The signal is integrated prior to modulation and a DSB modulator is used to generate the quadrature component of the $NBFM$ signal. The carrier is added to the quadrature component to generate an approximation to a true $NBFM$ signal.

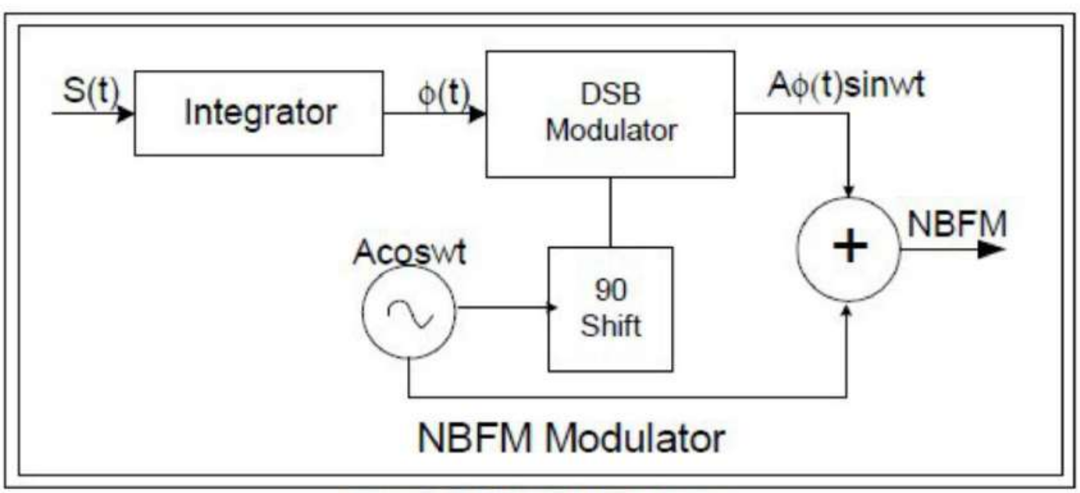


Fig 4 NBFM Modulator

Wideband FM modulator:

There are two basic methods for generating FM signals known as direct and indirect methods. The direct method makes use of a device called voltage controlled oscillator (VCO) whose oscillation frequency depends linearly on the modulation voltage.

A system that can be used for generating an FM signal is shown in Figure-5.

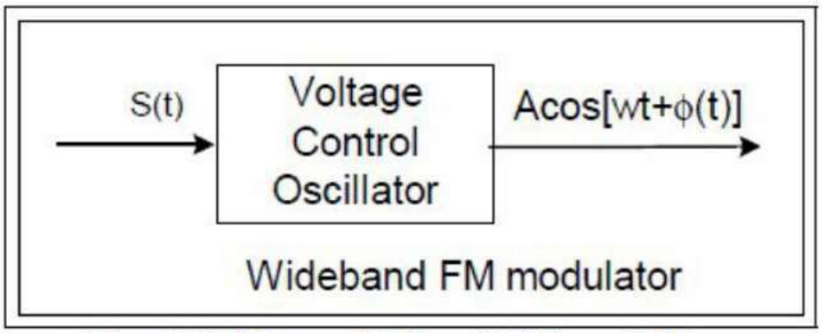


Fig. 5 VCO as wideband FM modulator

The combination of message differentiation that drive a VCO produces a PM signal. The physical device that generates the FM signal is the VCO whose output frequency depends directly on the applied control voltage of the message signal. $VCO's$ are easily implemented up to microwave frequencies.

FM MODULATORS AND DEMODULATORS:

FM MODULATOR:

The other common form of analog data transmission that is familiar to most people is called frequency modulation (FM). In the previous section, the message signal was multiplied with the carrier, modulating its amplitude in order to be transmitted (this amplitude modulation) [1] . With FM, the frequency of the carrier is modulated (varied) was the message changes. FM is what is used to transmit the majority of radio broadcasts. It is generally preferred over AM because it is less sensitive to noise, and it is in fact possible to trade bandwidth for noise performance [2]. The advantages of FM come at the expense of increased complexity in the transmitter and in the receiver. This having been said, we will see a simple FM receiver is actually no more complicated (to build) than its AM counterpart [3].

FM Generation by parameter variation method:

Figure 6.6.5 shows the block diagram of an FM transmitter. The modulator circuit uses parameter-variation method. A pre-emphasis circuit is used to reduce the effect of noise at higher audio frequencies for threshold improvement. The details about this circuit are discussed in Sec. 6.10. Like the AM transmitter, the carrier oscillator generates sub-harmonic of final carrier frequency to achieve frequency stability. A stable oscillation frequency at a lower radio frequency (say 4 MHz) is generated by an oscillator and then it is raised to the final carrier (say 96 MHz) by frequency multipliers. It should be kept in mind that more frequency stability can be obtained if the carrier oscillator operates at low radio frequency. The multiplying circuit not only increases the carrier frequency, but also the frequency deviation by the same factor. This fact is proved in Ex. 6.6.3.

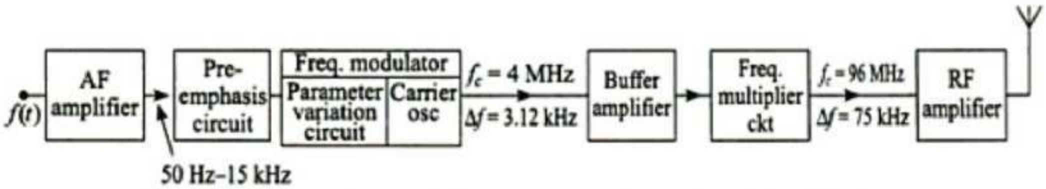


Fig. 6.6.5 *Block Diagram of an FM Transmitter using Direct Modulation*

We have seen that the direct method has the disadvantages of frequency unstability. Variations in environmental conditions (temperature, humidity), supply voltage, aging of active devices etc. can also cause frequency unstability. Therefore, FM transmitter must incorporate some auxiliary means for frequency stabilization. A stabilization scheme utilizing feedback principle is shown in Fig. 6.6.6. A stable crystal oscillator provides the reference frequency. The output of the crystal oscillator and the frequency modulator is fed into a mixture, and the difference frequency term is extracted at the output of the mixture. The mixer output then, is fed into a frequency discriminator. The frequency discriminator circuit provides an error voltage whose instantaneous value is proportional to the instantaneous frequency of its input. When the FM wave has frequency exactly equal to the assigned carrier frequency (no drift), the error signal is zero. But, whenever the transmitter carrier frequency drifts from the assigned value, a d.c. error signal of proper polarity is generated. The amplified error voltage of proper polarity is applied to a *VCO* in order to correct the transmitter frequency to the assigned value.

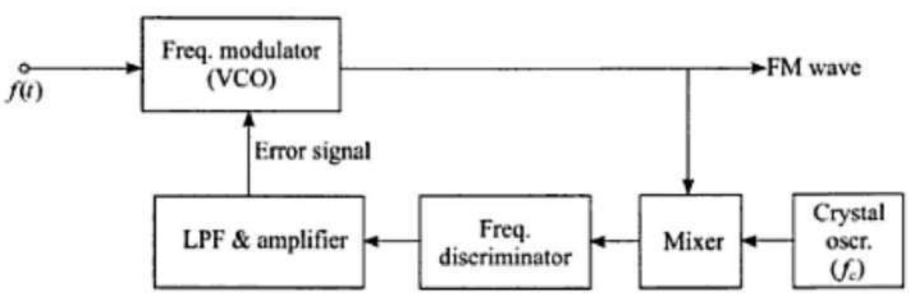


Fig. 6.6.6 *A Frequency Stabilization Scheme*

FM Generation by Armstrong's indirect method:

In the Armstrong method, frequency stability of a higher order can be obtained because the crystal oscillator can be used as a carrier generator. The basic principle of this method is to generate a narrowband FM (NBFM) indirectly by using the phase-modulation technique, and then converting this NBFM to a wideband FM (WBFM), as shown in Fig. 6.6.7. The distortion is low in NBFM as the modulation index is small. The phase modulation is preferred because of its easy generation schemes. The generation of NBFM is illustrated in Fig. 6.2.1. The multiplier circuit, apart from multiplying the carrier frequency, also increases the frequency deviation (refer Ex. 6.6.3), and thus the NBFM (small deviation) is converted into WBFM (large deviation).

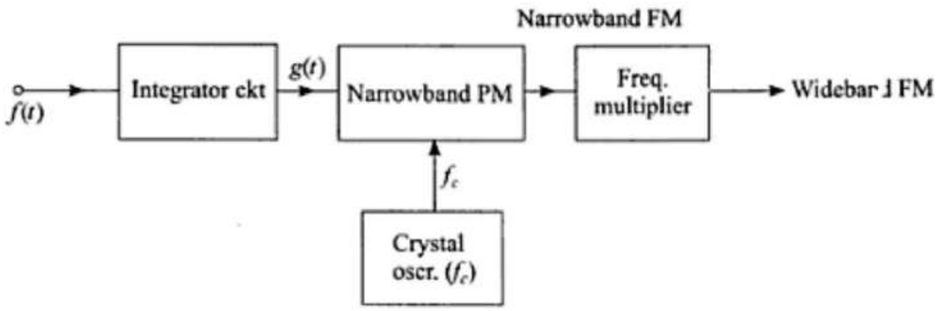


Fig. 6.6.7 *Armstrong Method for FM Generation*

FM DEMODULATOR:

An FM demodulator is required to produce an output voltage that is linearly proportional to the input frequency variation. One way to realize the requirement is to use discriminators- devices which distinguish one frequency from another, by converting frequency variations into amplitude variations. The resulting amplitude changes are detected by an envelope detector, just as done by AM detector [4].

$$S_m(t) = A\cos[\omega t + k_f \int_{-\infty}^{t} s(\tau)d\tau]$$

the discriminator output will be

$$y_d(t) = k_d k_f s(t)$$

Where K_d is the discriminator constant. The characteristics of an ideal discriminator are shown in Fig. 6. Discriminator can be realized by using a filter in the stop band region, in a linear range, assuming that the filter is differentiation in frequency domain [4].

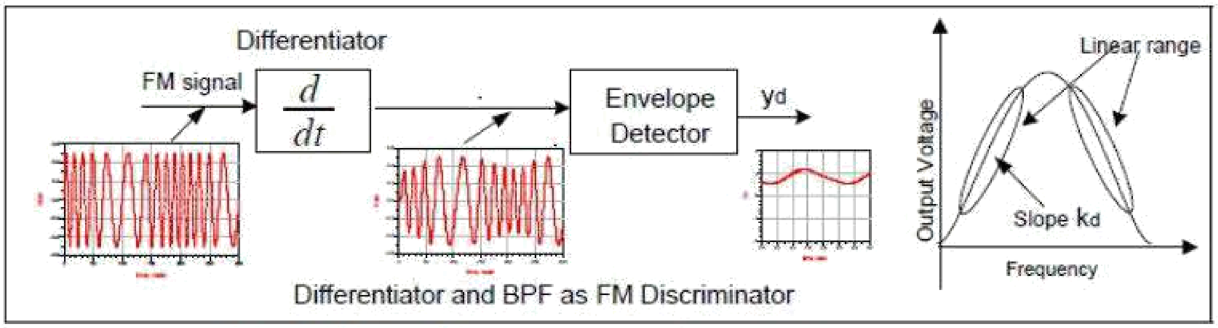


Fig 6 Ideal and real frequency demodulator

An approximation to the ideal discriminator characteristics can be obtained by the use of a differentiation followed by an envelope detector (see Figure-6) . If the input to the differentiator is Sm(t), then the output of the differentiator is

$$S_m'(t) = -A[\omega + k_f s(t)] \sin[\omega t + \phi(t)] \qquad (21)$$

With the exception of the phase deviation $\phi(t)$, the output of the differentiator is both amplitude and frequency modulated. Hence envelope detection can be used to recover the message signal. The baseband signal is recovered without any distortion if $\mathrm{Max}\{k_f s(t)\} = 2\pi\Delta f < \omega,$ which is easily achieved in most practical systems.

vi) outside the allowed spectrum may not be able to cause much of adjacent channel interference. Since the energy content in the higher harmonic is very small which is not very important.

vii) Hence the performance of the compatible SSB system with ordinary AM receivers being used for reception which is much more superior than conventional AM [4].

i) [1].

ANALOG TO DIGITAL

NOISY CHANNEL:

i) When a signal is transmitted over a long distance communication channel like radio, and when the signal arrives at the receiver it will greatly attenuated and combined with noise and electrical disturbances are also present in the channel receiver. As a result the received signal may not be distinguishable against its background of noise [4],

ii) If a signal is transmitted over a small distance communication channel like in wires or coaxial cable, which does not provide complete freedom from crosstalk disturbances and electrical noise.

iii) Hence the receiver noise is often noise source of largest power.

To resolve this problem is simply by raising the strength of the signal level at the transmitting end so high a level that in spite of the attenuation the received signal substantially overrides the noise [4]. This problem can be solved by a **repeater** (amplifier in a communication channel).

ROLE OF REPEATER

i) An amplifier at the receiver will not help the above situation, since at this point both signal and noise levels will be increased together. But a **repeater (repeater is the term used for an amplifier in a communication channel)** is located at the mid point of the long communications path [4]. The repeater will raise the signal level, and it will raise the level of only the noise introduced in the first half of the communication path.