UNIT I

Small Signal High Frequency Transistor Amplifier models

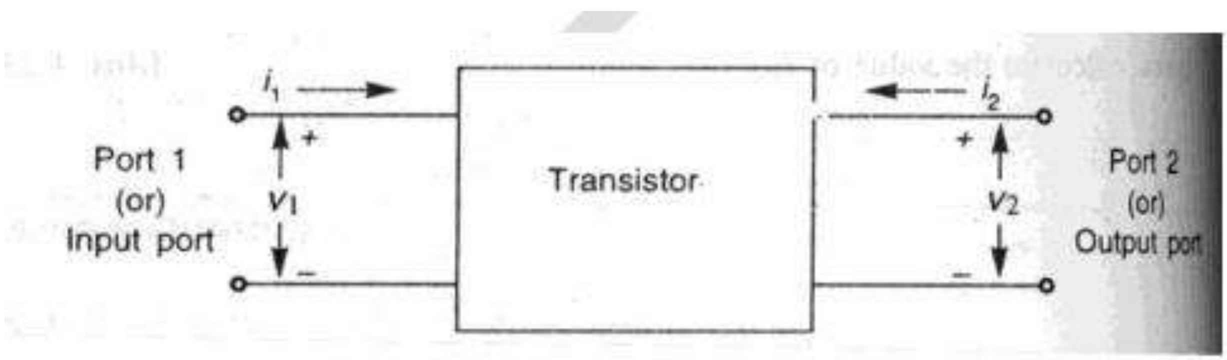
BJT: Transistor at high frequencies, Hybrid- π common emitter transistor model, Hybrid π conductances, Hybrid π capacitances, validity of hybrid π model, determination of high-frequency parameters in terms of low-frequency parameters , CE short circuit current gain, current gain with resistive load, cut-off frequencies, frequency response and gain bandwidth product. **FET:** Analysis of common Source and common drain Amplifier circuits at high frequencies.

Introduction:

Electronic circuit analysis subject teaches about the basic knowledge required to design an amplifier circuit, oscillators etc .It provides a clear and easily understandable discussion of designing of different types of amplifier circuits and their analysis using hybrid model, to find out their parameters. Fundamental concepts are illustrated by using small examples which are easy to understand. It also covers the concepts of MOS amplifiers, oscillators and large signal amplifiers.

Two port devices & Network Parameters:

A transistor can be treated as a two-part network. The terminal behavior of any two-part network can be specified by the terminal voltages V1& V2at parts 1 & 2 respectively and current i1and i2, entering parts 1 & 2, respectively, as shown in figure.



Of these four variables V1, V2, i1and i2, two can be selected as independent variables and the remaining two can be expressed in terms of these independent variables. This leads to various two part parameters out of which the following three are more important.

1. Z –Parameters (or) Impedance parameters
2. Y –Parameters (or) Admittance parameters
3. H –Parameters (or) Hybrid parameters

Hybrid parameters (or) h –parameters:

The equivalent circuit of a transistor can be dram using simple approximation by retaining its essential features. These equivalent circuits will aid in analyzing transistor circuits easily and rapidly.

If the input current i_1 and output Voltage V_2 are takes as independent variables, the input voltage V_1 and output current i_2 can be written as

$$\mathbf{V_1 = h_{11}\, i_1 + h_{12}\, V_2}$$
$$\mathbf{i_2 = h_{21}\, i_1 + h_{22}\, V_2}$$

The four hybrid parameters h11, h12, h21and h22 are defined as follows:

h11= [V1/ i1] with V2= 0 Input Impedance with output part short circuited.
h22= [i2/ V2] with i1= 0 Output admittance with input part open circuited.
h12= [V1/ V2] with i1= 0 reverse voltage transfer ratio with input part open circuited.
h21= [i2/ i1] with V2= 0 Forward current gain with output part short circuited

The dimensions of h–parameters are as follows:

h11-Ω

h22–mhos

h12, h21 –dimension less.

As the dimensions are not alike, (i.e.) they are hybrid in nature, and these parameters are called as hybrid parameters.

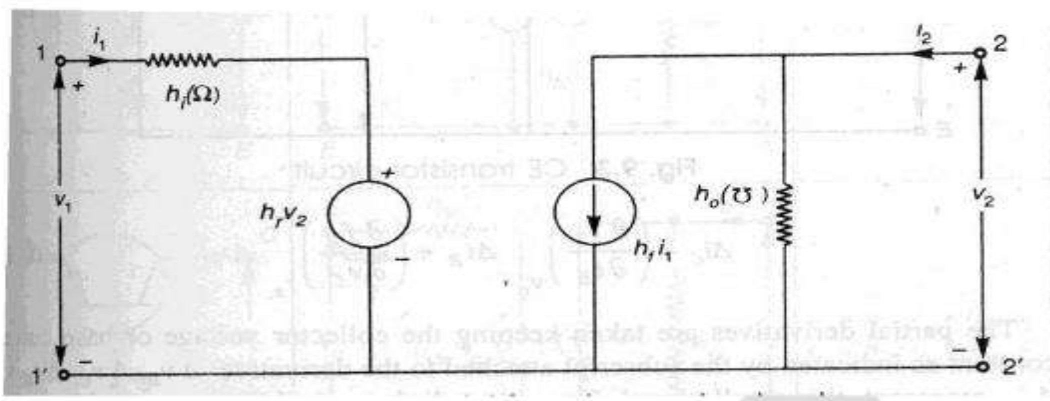
h11 = input; h 22 = output;
h21= forward transfer; h22 = Reverse transfer.

Notations used in transistor circuits:

h_{ie}= h11e= Short circuit input impedance
h_{oe}= h22e= Open circuit output admittance
h_{re} = h12e= Open circuit reverse voltage transfer ratio
h_{fe}= h21e= Short circuit forward current Gain.

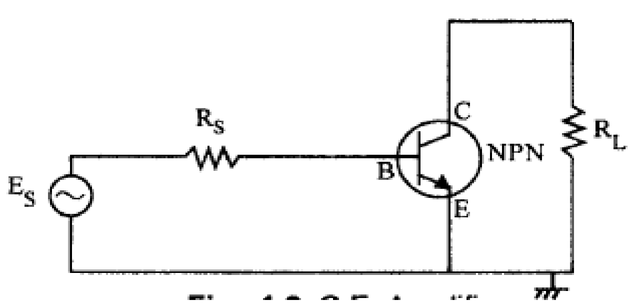
The Hybrid Model for Two-port Network:

$$V1 = h11\ i1 + h12V2$$
$$I2 = h21i1 + h22V2$$
$$V1 = h_1 i1 + h_r V2$$
$$I2 = h_f i1 + h_0 V2$$

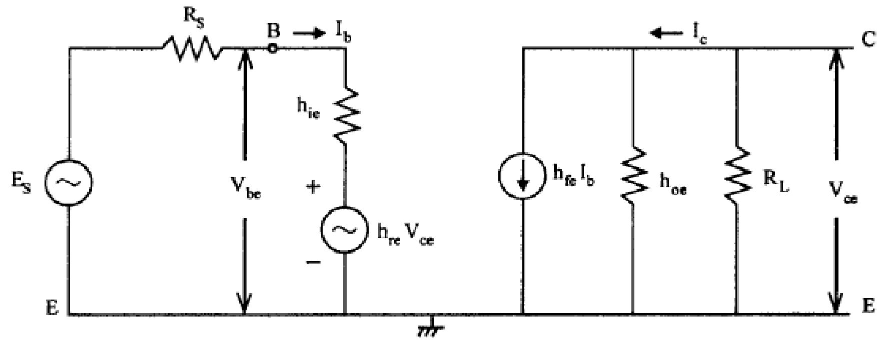


Common Emitter Amplifier

Common Emitter Circuit is as shown in the Fig. The DC supply, biasing resistors and coupling capacitors are not shown since we are performing an AC analysis.



E_s is the input signal source and R_s is its resistance. The h-parameter equivalent for the above circuit is as shown in Fig.



$$h_{ie} = \left.\frac{V_{be}}{I_b}\right|_{V_{ce}=0} \qquad h_{re} = \left.\frac{V_{be}}{V_{ce}}\right|_{I_b=0}$$

$$h_{oe} = \left.\frac{I_c}{V_{ce}}\right|_{I_b=0} \qquad h_{fe} = \left.\frac{I_c}{I_b}\right|_{V_{ce}=0}$$

The typical values of the h-parameter for a transistor in Common Emitter configuration are,

$$h_{ie} = \frac{V_{be}}{I_b}$$

$$h_{ie} = \frac{0.2V}{50\times10^{-6}} = 4K\Omega$$

$$h_{fe} = I_c/I_b \simeq 100\,.$$

$$h_{fe} >> 1 \simeq \beta$$

$$h_{re} = \frac{V_{be}}{V_{ce}}$$

$$h_{re} = 0.2 \times 10^{-3}.$$

$$h_{oe} = \frac{I_c}{V_{ce}}.$$

$$h_{oe} = 8\ \mu℧$$

$$R_i = h_{ie} - \frac{h_{fe}h_{re}}{h_{oe} + \frac{1}{R_L}}$$

$$R_i = h_{ie} - \frac{h_{fe}h_{re}}{h_{oe} + \frac{1}{R_L}}$$

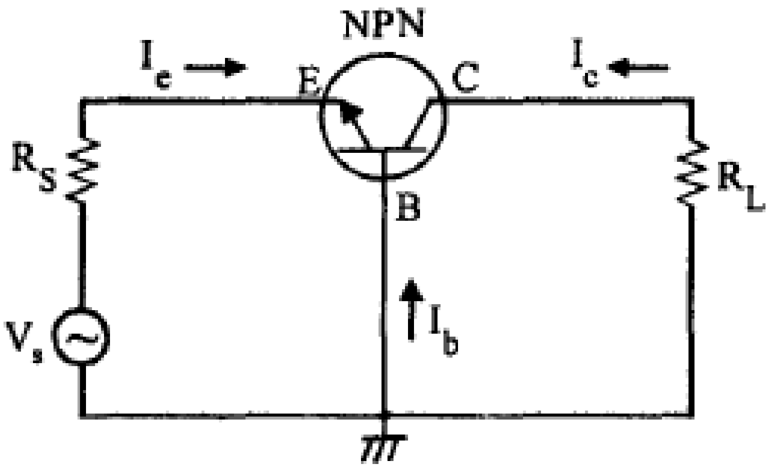
$$R_o = \frac{1}{h_{oe} - \left(\frac{h_{re}h_{fe}}{h_{ie} + R_s}\right)}$$

$$A_i = \frac{-h_{fe}}{1 + h_{oe}R_L}$$

$$A_V = \frac{-h_{fe}R_L}{h_{ie} + R_L(h_{ie}h_{oe} - h_{fe}h_{re})}$$

Common Base Amplifier

Common Base Circuit is as shown in the Fig. The DC supply, biasing resistors and coupling capacitors are not shown since we are performing an AC analysis.



$$h_{ib} = \left.\frac{V_{eb}}{I_e}\right|_{V_{cb}=0}$$

$$h_{fb} = \left.\frac{I_c}{I_e}\right|_{V_{cb}=0} = -0.99 \text{ (Typical Value)}$$

$$I_c < I_e \qquad \therefore \qquad h_{fb} < 1$$

$$h_{ob} = \left.\frac{I_c}{V_{cb}}\right|_{I_e=0} = 7.7 \times 10^{-8} \text{ mhos (Typical Value)}$$

$$h_{rb} = \left.\frac{V_{eb}}{V_{cb}}\right|_{I_e = 0} = 37 \times 10^{-6} \quad \text{(Typical Value)}$$

$$R_i = h_{ib} - \frac{h_{fb} . h_{rb}}{h_{ob} + \frac{1}{R_L}}$$

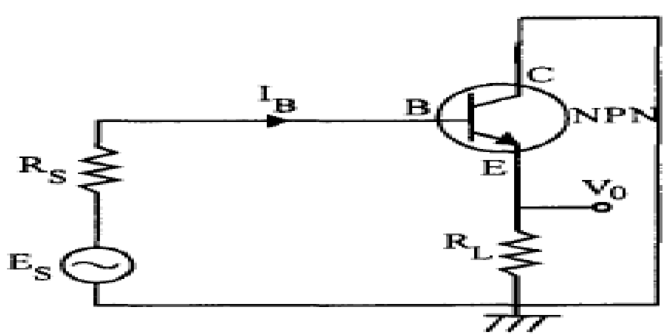
$$R_o' = \frac{1}{h_{ob} - \frac{h_{rb} h_{fb}}{h_{ib} + R_S}}$$

$$A_I = \frac{-h_{fb}}{1 + h_{ob} R_L}$$

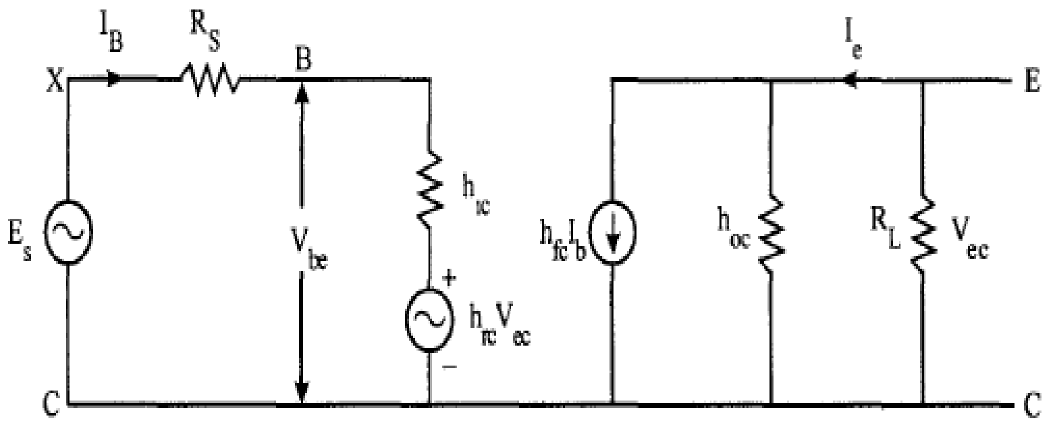
$$A_v = \frac{-h_{fb} R_L}{h_{ib} + R_L (h_{ib} h_{ob} - h_{fb} h_{rb})}$$

Common Collector Amplifier

Common Collector Circuit is as shown in the Fig. The DC supply, biasing resistors and coupling capacitors are not shown since we are performing an AC analysis



The h-parameter model is shown below



$$h_{ic} = \left.\frac{V_{bc}}{I_b}\right|_{V_{ec}=0} = 2{,}780\ \Omega \text{ (Typical Value)}$$

$$h_{oc} = \left.\frac{I_e}{V_{ec}}\right|_{I_b=0} = 7.7 \times 10^{-6} \text{ mhos (Typical Value)}$$

$$h_{fc} = -\left.\frac{I_e}{I_b}\right|_{V_{ce}=0} = 100 \text{ (Typical value)}$$

$\because\ I_e >> I_b.$

$$h_{rc} = \left.\frac{V_{bc}}{V_{ec}}\right|_{I_b=0};$$

Transistors at High Frequencies

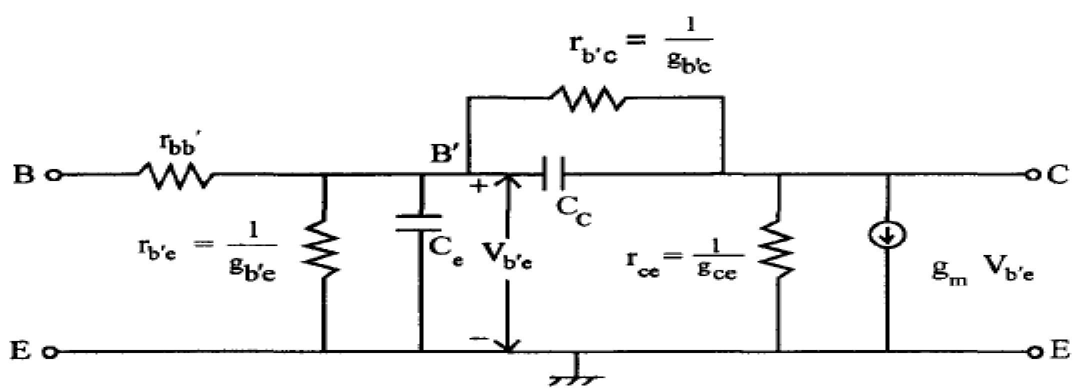
At low frequencies it is assumed that transistor responds instantaneously to changes in the input voltage or current i.e., if you give AC signal between the base and emitter of a Transistor amplifier in Common Emitter configuraii6n and if the input signal frequency is low, the output at the collector will exactly follow the change in the input (amplitude etc.,). If '1' of the input is high (MHz) and the amplitude of the input signal is changing the Transistor amplifier will not be able to respond.

It is because; the carriers from the emitter side will have to be injected into the collector side. These take definite amount of time to travel from Emitter to Base, however small it may be. But if the input signal is varying at much higher speed than the actual time taken by the carries to respond, then the Transistor amplifier will not respond instantaneously. Thus, the junction capacitances of the transistor, puts a limit to the highest frequency signal which the transistor can handle. Thus depending upon doping area of the junction etc, we have transistors which can respond in AF range and also RF range.

To study and analyze the behavior of the transistor to high frequency signals an equivalent model based upon transmission line equations will be accurate. But this model will be very complicated to analyze. So some approximations are made and the equivalent circuit is simplified. If the circuit is simplified to a great extent, it will be easy to analyze, but the results will not be accurate. If no approximations are made, the results will be accurate, but it will be difficult to analyze. The desirable features of an equivalent circuit for analysis are simplicity and accuracy. Such a circuit which is fairly simple and reasonably accurate is the Hybrid-pi or Hybrid-π model, so called because the circuit is in the form of π.

Hybrid - π Common Emitter Transconductance Model

For Transconductance amplifier circuits Common Emitter configuration is preferred. Why? Because for Common Collector (hrc< 1). For Common Collector Configuration, voltage gain Av < 1. So even by cascading you can't increase voltage gain. For Common Base, current gain is hib< 1. Overall voltage gain is less than 1. For Common Emitter, hre>>1. Therefore Voltage gain can be increased by cascading Common Emitter stage. So Common Emitter configuration is widely used. The Hybrid-x or Giacoletto Model for the Common Emitter amplifier circuit (single stage) is as shown below.



Analysis of this circuit gives satisfactory results at all frequencies not only at high frequencies but also at low frequencies. All the parameters are assumed to be independent of frequency.

Where B' = internal node in base
$r_{bb'}$ = Base spreading resistance
$r_{b'e}$ = Internal base node to emitter resistance
r_{ce} = collector to emitter resistance
C_e = Diffusion capacitance of emitter base junction
$r_{b'c}$ = Feedback resistance from internal base node to collector node
g_m = Transconductance
C_C= transition or space charge capacitance of base collector junction

Circuit Components

B' is the internal node of base of the Transconductance amplifier. It is not physically accessible. The base spreading resistance rbb is represented as a lumped parameter between base B and internal node B'. $g_{mVb'e}$ is a current generator. Vb'e is the input voltage across the emitter junction. If Vb'e increases, more carriers are injected into the base of the transistor. So the increase in the number of carriers is proportional to Vb'e. This results in small signal current since we are taking into account changes in Vb'e. This effect is represented by the current generator g_mVb'e. This represents the current that results because of the changes in Vb'e' when C is shorted to E.

When the number of carriers injected into the base increase, base recombination also increases. So this effect is taken care of by gb'e. As recombination increases, base current increases. Minority carrier storage in the base is represented by C_e the diffusion capacitance.

According to Early Effect, the change in voltage between Collector and Emitter changes the base width. Base width will be modulated according to the voltage variations between Collector and Emitter. When base width changes, the minority carrier concentration in base changes. Hence the current which is proportional to carrier concentration also changes. I_E changes and I_C changes. This feedback effect [IE on input side, I_C on output side] is taken into account by connecting gb'e between B', and C. The conductance between Collector and Base is g_{ce}.C_c represents the collector junction barrier capacitance.

Hybrid - *n* Parameter Values

Typical values of the hybrid-n parameter at I_C = 1.3 rnA are as follows:

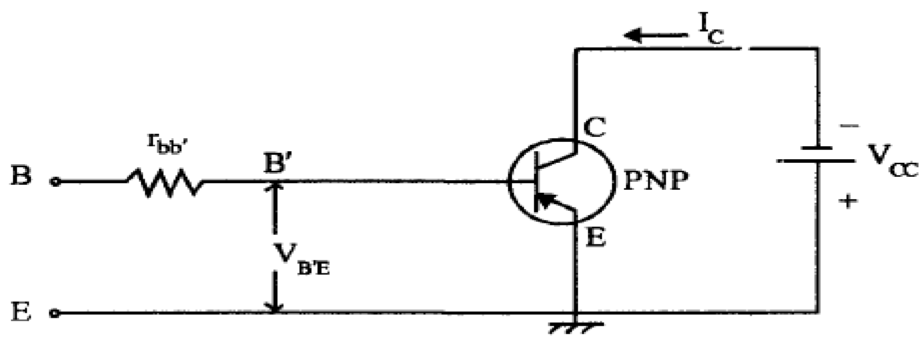
$$g_m = 50 \text{ mA/v}$$

rbb' = 100 Ω
rb'e = 1 kΩ
ree = 80 kΩ
Cc = 3 pf
Ce = 100 pf
rb'c = 4 MΩ

These values depend upon:
1. Temperature 2. Value of I_C

Determination of Hybrid-x Conductances

1. Trans conductance or Mutual Conductance (g_m)



The above figure shows PNP transistor amplifier in Common Emitter configuration for AC purpose, Collector is shorted to Emitter.

$$I_C = I_{C0} - \alpha_0 \cdot I_E$$

I_{CO} opposes I_E. I_E is negative. Hence $I_C = I_{CO} - \alpha_0 IE$ α_0 is the normal value of α at room temperature.

In the hybrid - π equivalent circuit, the short circuit current = $g_m Vb'e$

Here only transistor is considered, and other circuit elements like resistors, capacitors etc are not considered.

$$g_m = \left.\frac{\partial I_C}{\partial V_{b'e}}\right|_{V_{CE} = K}$$

Differentiate (l) with respect to Vb'e partially. I_{CO} is constant

$$g_m = 0 - \alpha_0 \frac{\partial I_E}{\partial V_{b'e}}$$

For a PNP transistor, Vb'e = $-V_E$ Since, for PNP transistor, base is n-type. So negative voltage is given

$$g_m = \alpha_0 \frac{\partial I_E}{\partial V_E}$$

If the emitter diode resistance is r_e then

$$r_e = \frac{\partial V_E}{\partial I_E}$$

$$g_m = \frac{\alpha_0}{r_e}$$

$$r = \frac{\eta . V_T}{I} \quad \eta = 1, \qquad I = I_E \qquad r = \frac{V_T}{I_E}$$

$$g_m = \frac{\alpha_0 . I_E}{V_T} \qquad \alpha_0 \simeq 1, \qquad I_E \simeq I_C$$

$$I_E = I_{C0} - I_C$$

$$g_m = \frac{I_{C0} - I_C}{V_T}$$

Neglect I_{C0}

$$g_m = \frac{|I_C|}{V_T}$$

g_m is directly proportional to I_C is also inversely proportional to T. For PNP transistor, I_C is negative

$$\boxed{g_m = \frac{-I_C}{V_T}}$$

At room temperature i.e. T=300^{0}K

$$g_m = \frac{|I_C|}{26}, \; I_C \text{ is in mA.}$$

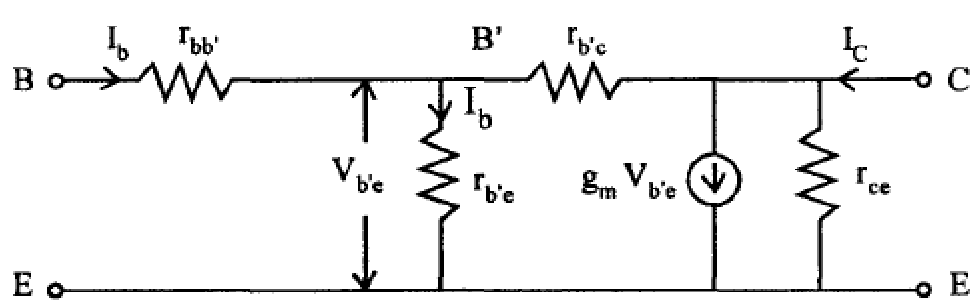
If $\quad I_C = 1.3$ mA, $g_m = 0.05$ A/V

If $\quad I_C = 10$ mA, $g_m = 400$ mA/V

Input Conductance (gb'e):

At low frequencies, capacitive reactance will be very large and can be considered as Open circuit. So in the hybrid-π equivalent circuit which is valid at low frequencies, all the capacitances can be neglected.

The equivalent circuit is as shown in Fig.



The value of rb'c» rb'e (Since Collector Base junction is Reverse Biased)So I_b flows into rb'e only. [This is lb' (IE - I_b)will go to collector junction]

$$V_{b'e} \simeq I_b \cdot r_{b'e}$$

The short circuit collector current,

$$I_C = g_m \cdot V_{b'e}; \qquad V_{b'e} = I_b \cdot r_{b'e}$$

$$I_C = g_m \cdot I_b \cdot r_{b'e}$$

$$h_{fe} = \left.\frac{I_C}{I_B}\right|_{V_{CE}} = g_m \cdot r_{b'e}$$

$$\boxed{r_{b'e} = \frac{h_{fe}}{g_m}}$$

$$g_m = \frac{|I_C|}{V_T}$$

$$r_{b'e} = \frac{h_{fe} \cdot V_T}{|I_C|}$$

$$g_{b'e} = \boxed{\frac{|I_C|}{h_{fe} V_T}} \text{ or } \boxed{\frac{g_m}{h_{fe}}}$$

Feedback Conductance ($g_{b'c}$)

hre = reverse voltage gain, with input open or $I_b = 0$
hre =Vb'e/Vce = Input voltage/Output voltage

$$h_{re} = \frac{r_{b'e}}{r_{b'e} + r_{b'c}}$$

[With input open, i.e., $I_b - 0$, V_{ce} is output. So it will get divided between $r_{b'e}$ and $r_{b'c}$ only]

or $$h_{re}(r_{b'e} + r_{b'c}) = r_{b'e}$$

$$r_{b'e}[1 - h_{re}] = h_{re} r_{b'c}$$

But $$h_{re} \ll 1$$

$\therefore$ $$r_{b'e} = h_{re} \; r_{b'c}; \quad r_{b'c} = \frac{r_{b'e}}{h_{re}}$$

or $$\boxed{g_{b'c} = h_{re} \; g_{b'e}} \quad \frac{1}{r_{b'c}} = g_{b'c} \; = \frac{h_{re}}{r_{b'e}}$$

$$h_{re} = 10^{-4}$$

$\therefore$ $$r_{b'c} \gg r_{b'e}$$

Base Spreading Resistance (r bb')

The input resistance with the output shorted is hie. If output is shorted, i.e., Collector and Emitter arejoined; $r_{b'e}$ is in parallel with $r_{b'c}$.

$$h_{ie} = r_{bb'} + r_{b'e}$$

$$\boxed{r_{bb'} = h_{ie} - r_{b'e}}$$

$$h_{ie} = r_{bb'} + r_{b'e}$$

$$r_{b'e} = \frac{h_{fe} . V_T}{|I_C|}$$

$$h_{ie} = r_{bb'} + \frac{h_{fe} . V_T}{|I_C|}$$

Output Conductance (g_{ce})

This is the conductance with input open circuited. In h-parameters it is represented as hoe. For $I_b = 0$, we have,

$$I_C = \frac{V_{ce}}{r_{ce}} + \frac{V_{ce}}{r_{b'c} + r_{b'e}} + g_m \; V_{b'e}$$

$$h_{re} = \frac{V_{b'e}}{V_{ce}} \qquad \therefore \qquad V_{b'e} = h_{re} . V_{ce}$$

$$I_C = \frac{V_{ce}}{r_{ce}} + \frac{V_{ce}}{r_{b'c} + r_{b'e}} + g_m . h_{re} . V_{ce}$$

$$h_{oe} = \frac{1}{r_{ce}} + \frac{1}{r_{b'c}} + g_m \cdot h_{re}$$

$$= g_{ce} + g_{b'c} + g_m h_{re}$$

$$g_{b'e} = \frac{g_m}{h_{fe}}$$

$$g_m = g_{b'e} \cdot h_{fe}$$

$$h_{re} = \frac{r_{b'e}}{r_{b'e} + r_{b'c}} \approx \frac{r_{b'e}}{r_{b'c}} = \frac{g_{b'c}}{g_{b'e}}$$

$$h_{oe} = g_{ce} + g_{b'c} + g_{b'e} h_{fe} \cdot \frac{g_{b'c}}{g_{b'e}}$$

$$g_{ce} = h_{oe} - (1 + h_{fe}) \cdot g_{b'c}$$

$$h_{fe} >> 1, \; 1 + h_{fe} \approx h_{fe}$$

$$\boxed{g_{ce} = h_{oe} - h_{fe} \cdot g_{b'c}}$$

$$g_{b'c} = h_{re} \cdot g_{b'e}$$

$$g_{ce} = h_{oe} - h_{fe} \cdot h_{re} \cdot g_{b'e}$$

Hybrid - π Capacitances

In the hybrid - **π** equivalent circuit, there are two capacitances, the capacitance between the Collector Base junction is the C_c or $C_{b'e}$'. This is measured with input open i.e., $I_E = 0$, and is specified by the manufacturers as C_{Ob}. 0 indicates that input is open. Collector junction is reverse biased.

$$C_C \ \alpha \ \frac{1}{(V_{CE})^n}$$

$$n = \frac{1}{2} \quad \text{for abrupt junction}$$

$$= 1/3 \quad \text{for graded junction.}$$

C_e = Emitter diffusion capacitance C_{De} + Emitter junction capacitance C_{Te}

C_T = Transition capacitance.

C_D = Diffusion capacitance.

$C_{De} >> C_{Te}$

$C_e \simeq C_{De}$

$C_{De} \ \alpha \ I_E$ and is independent of Temperature T.

Validity of hybrid-π model

The high frequency hybrid Pi or Giacoletto model of BJT is valid for frequencies less than the unit gain frequency.

High frequency model parameters of a BJT in terms of low frequency hybrid parameters

The main advantage of high frequency model is that this model can be simplified to obtain low frequency model of BJT. This is done by eliminating capacitance's from the high frequency model so that the BJT responds without any significant delay (instantaneously) to the input signal. In practice there will be some delay between the input signal and output signal of BJT which will be very small compared to signal period (1/frequency of input signal) and hence can be neglected. The high frequency model of BJT is simplified at low frequencies and redrawn as shown in the figure below along with the small signal low frequency hybrid model of BJT.

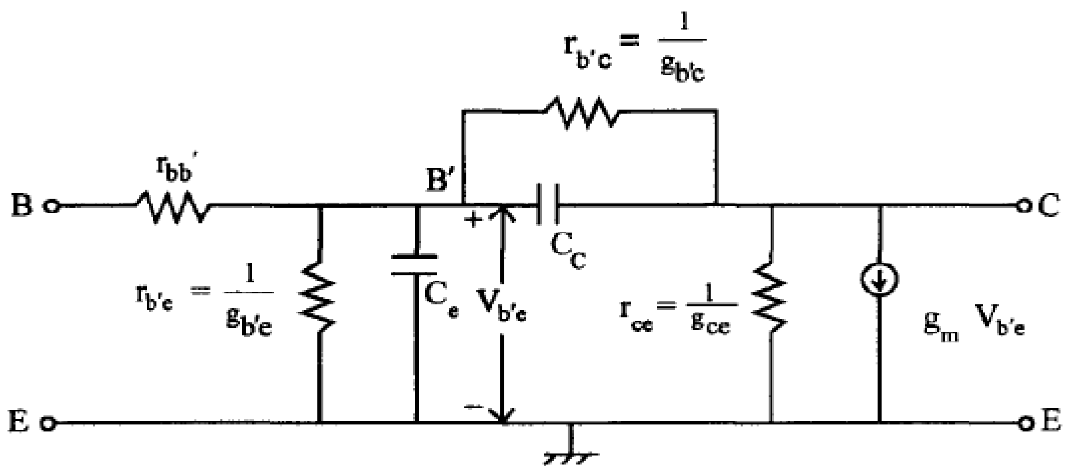


Fig. high frequency model of BJT at low frequencies

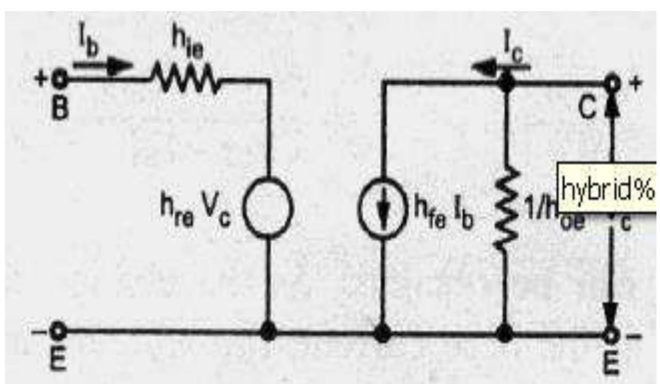


Fig hybrid model of BJT at low frequencies

The High frequency model parameters of a BJT in terms of low frequency hybrid parameters are given below:

Transconductance $g_m = I_c/V_t$
Internal Base node to emitter resistance $r_{b'e} = h_{fe}/ g_m = (h_{fe} * V_t)/ I_c$
Internal Base node to collector resistance $r_{b'e} = (h_{re} * r_{b'c}) / (1- h_{re})$ assuming $h_{re} << 1$ it reduces to $r_{b'e} = (h_{re} * r_{b'c})$
Base spreading resistance $r_{bb'} = h_{ie} - r_{b'e} = h_{ie} - (h_{fe} * V_t)/ I_c$
Collector to emitter resistance $r_{ce} = 1 / (h_{oe} - (1+ h_{fe})/r_{b'c})$

Collector Emitter Short Circuit Current Gain

Consider a single stage Common Emitter transistor amplifier circuit. The hybrid-1t equivalent circuit is as shown:

$$I_L = -g_m V_{b'e}$$

$$V_{b'e} = \frac{I_i}{g_{b'e} + j\omega(C_e + C_c)}$$

A_i under short circuit condition is,

$$A_i = \frac{I_L}{I_i} = \frac{-g_m}{g_{b'e} + j\omega(C_e + C_c)}$$

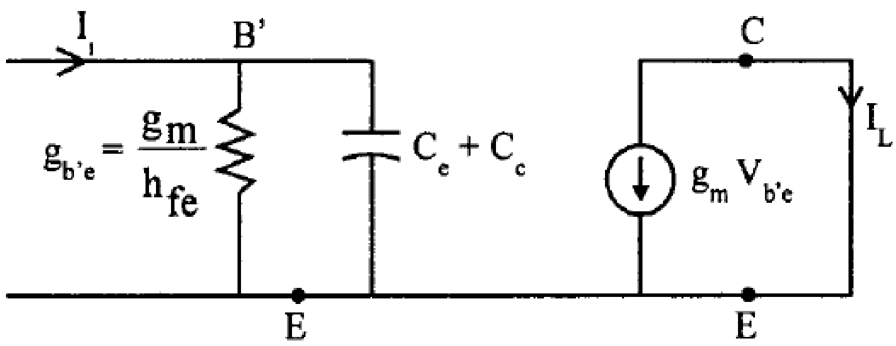
But $$g_{b'e} = \frac{g_m}{h_{fe}}, \quad C_e + C_c \simeq C_e$$

$$C_e = \frac{g_m}{2\pi f_T}$$

$$= \frac{-g_m}{\frac{g_m}{h_{fe}} + \frac{j\,2\pi . g_m . f}{2\pi f_T}}$$

$\therefore$ $$A_i = \frac{-1}{\frac{1}{h_{fe}} + j\left(\frac{f}{f_T}\right)}$$

If the output is shorted i.e. $R_L = 0$, what will be the flow response of this circuit? When$R_L = 0$, $V_o = 0$. Hence $A_v = 0$. So the gain that we consider here is the current gain I_L/Ic. The simplified equivalent circuit with output shorted is,



A current source gives sinusoidal current Ic. Output current or load current is IL· $g_{b'c}$ isneglected since $g_{b'c} \ll g_{b'e}$, g_{ce} is in shunt with short circuit R = 0. Therefore g_{ce} disappears. The current is delivered to the output directly through Ce and $g_{b'c}$ is also neglected since this will be very small.

$$I_L = -g_m V_{b'e}$$

$$V_{b'e} = \frac{I_i}{g_{b'e} + j\omega(C_e + C_c)}$$

A_i under short circuit condition is,

$$A_i = \frac{I_L}{I_i} = \frac{-g_m}{g_{b'e} + j\omega(C_e + C_c)}$$

But $$g_{b'e} = \frac{g_m}{h_{fe}}, \quad C_e + C_c \simeq C_e$$

$$C_e = \frac{g_m}{2\pi f_T}$$

$$= \frac{-g_m}{\frac{g_m}{h_{fe}} + \frac{j\,2\pi . g_m . f}{2\pi f_T}}$$

$\therefore$ $$A_i = \frac{-1}{\frac{1}{h_{fe}} + j\left(\frac{f}{f_T}\right)}$$

$$= \frac{-h_{fe}}{1 + j\, h_{fe}\left(\frac{f}{f_T}\right)}$$

$$A_i = \frac{-h_{fe}}{1 + j\left(\frac{f}{f_\beta}\right)}$$

$$\frac{f_T}{h_{fe}} = f_\beta$$

$$|A_i| = \frac{h_{fe}}{\sqrt{1 + \left(\frac{f}{f_\beta}\right)^2}}$$

Where $$f_\beta = \frac{g_{b'e}}{2\pi(C_e + C_C)}$$

$$g_{b'e} = \frac{g_m}{h_{fe}}$$

$\therefore$ $$f_\beta = \frac{g_m}{h_{fe}\, 2\pi(Ce + C_c)}$$

At $f = f_\beta$, $A_i = \frac{1}{\sqrt{2}}$ $= 0.707$ of h_{fe}.

Current Gain with Resistance Load:

$$f_T = f_\beta \, . \, h_{fe} = \frac{g_m}{2\pi(C_e + C_c)}$$

Considering the load resistance R_L
$V_{b'e}$ is the input voltage and is equal to V_1
V_{ce} is the output voltage and is equal to V_2

$$K_2 = \frac{V_{ce}}{V_{b'e}}$$

This circuit is still complicated for analysis. Because, there are two time constants associated with the input and the other associated with the output. The output time constant will be much smaller than the input time constant. So it can be neglected.

K = Voltage gain. It will be >> 1

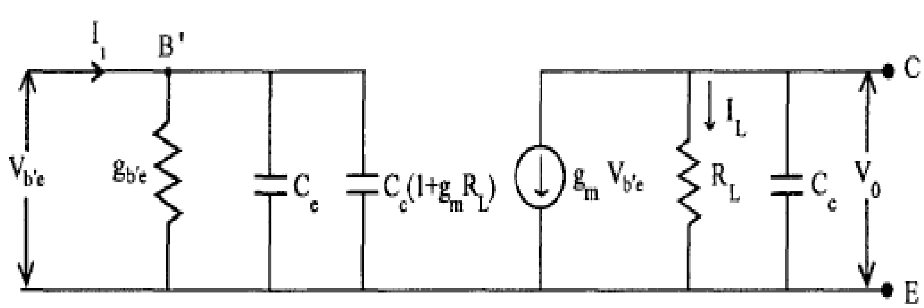
$$g_{b'c}\left(\frac{K-1}{K}\right) \simeq g_{b'c}$$

$$g_{b'c} < g_{ce} \quad \because \quad r_{b'c} \simeq 4\ M\Omega, \qquad r_{ce} = 80\ K \ \text{(typical values)}$$

So $g_{b'c}$ can be neglected in the equivalent circuit. In a wide band amplifier RL will not exceed 2KΩ. If R_L is small f_H is large.

$$f_H = \frac{1}{2\pi C_s (R_C \| R_L)}$$

Therefore g_{ce} can be neglected compared with R_L. Therefore the output circuit consists of current generator gm $V_{b'e}$ feeding the load R_L so the Circuit simplifies as shown in Fig.



$$K = \frac{V_{ce}}{V_{b'e}} = -g_m R_L; \qquad g_m = 50 \text{ mA\V}, \qquad R_L = 2K\Omega \text{ (typical values)}$$

$$K = -100$$

Miller's Theorem

It states that if an impedance Z is connected between the input and output terminals, of a network, between which there is voltage gain, K, the same effect can be had by removing Z and connecting an impedance Z_i at the input =Z/(1-K) and Zo across the output = ZK/(K-1)

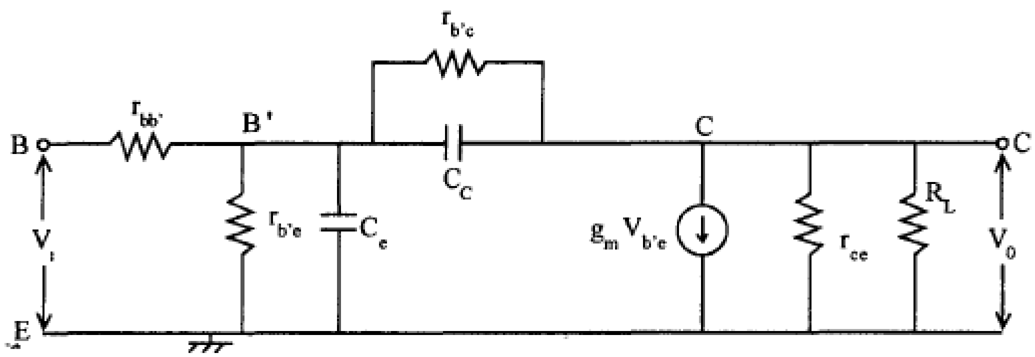


Fig. High frequency equivalent circuit with resistive load R_L

Therefore high frequency equivalent circuit using Miller's theorem reduces to

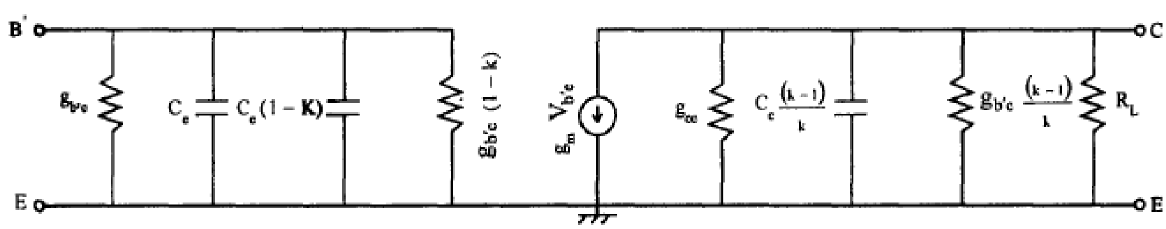


Fig. Circuit after applying Millers' Theorem

$$K = \frac{V_{ce}}{V_{b'e}}$$

$$V_{ce} = -I_c \cdot R_L$$

$$K = \frac{-I_C . R_L}{V_{b'e}}$$

$$\frac{I_C}{V_{b'e}} = g_m$$

$$K = -g_m . R_L$$

The Parameters f_T

f_T is the frequency at which the short circuit Common Emitter current gain becomes unity.

The Parameters f_β

$$A_i = 1, \quad \text{or} \quad \frac{h_{fe}}{\sqrt{1+\left(\frac{f_T}{f_\beta}\right)^2}} = 1$$

$$f = f_T, \quad A_i = 1$$

$$h_{fe} = \sqrt{1+\left(\frac{f_T}{f_\beta}\right)^2}$$

$$(h_{fe})^2 = 1 + \left(\frac{f_T}{f_\beta}\right)^2 \cong \left(\frac{f_T}{f_\beta}\right)^2$$

$$h_{fe} \simeq \frac{f_T}{f_\beta} \text{ when } Ai = 1$$

$$\boxed{f_T \simeq h_{fe} \cdot f_\beta}$$

$$f_\beta = \frac{g_m}{h_{fe}\{C_e + C_c\}}$$

$$f_T = f_\beta \cdot h_{fe} = \frac{g_m}{2\pi(C_e + C_c)}$$

$$C_e >> C_c$$

$$\boxed{f_T \simeq \frac{g_m}{2\pi C_e}}$$

$$A_i = \frac{-g_m}{g_{b'e} + j\omega(C_e + C_c)}$$

Dividing by $g_{b'e}$, Numerator and Denominator,

$$A_i = \frac{-g_m / g_{b'e}}{1 + \dfrac{j2\pi f (C_e + C_c)}{g_{b'e}}}$$

we know that $g_{b'e} = \dfrac{g_m}{h_{fe}}$

$\therefore$ $\dfrac{g_m}{g_{b'e}} = h_{fe}$

$$A_i = \frac{-h_{fe}}{1 + jf\left[\dfrac{2\pi(C_e + C_C)}{g_{b'e}}\right]}$$

But we know that $A_i = \dfrac{-h_{fe}}{1 + j\dfrac{f}{f_\beta}}$

Comparing, $f_\beta = \dfrac{g_{b'e}}{2\pi(C_e + C_c)} = \dfrac{g_m}{h_{fe} . 2\pi (C_e + C_c)}$ $\because$ $g_{b'e} = \dfrac{g_m}{h_{fe}}$

$\therefore$ $\boxed{f_\beta = \dfrac{g_m}{h_{fe} . 2\pi (C_e + C_c)}}$

$$\boxed{f_T = \frac{g_m}{2\pi (C_e + C_c)}}$$

Gain - Bandwidth (B.W) Product

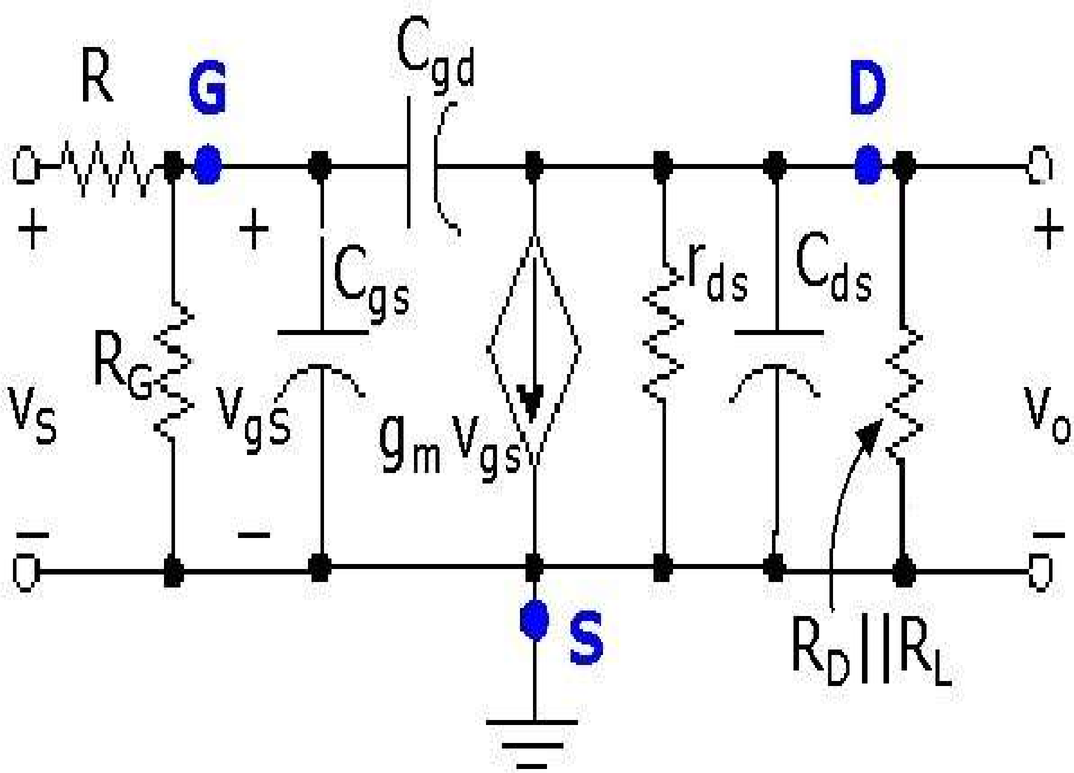
This is a measure to denote the performance of an amplifier circuit. Gain - B. W product is also referred as Figure of Merit of an amplifier. Any amplifier circuit must have large gain and large bandwidth. For certain amplifier circuits, the mid band gain Am maybe large, but not Band width or Vice - Versa. Different amplifier circuits can be compared with thus parameter.

FET: Analysis of common Source and common drain Amplifier circuits at high frequencies.

Just like for the BJT, we could use the original small signal model for low frequency analysis–the only difference was that external capacitances had to be kept in the circuit. Also just like the BJT, for high frequency operation, the internal capacitances between each of the device's terminals can no longer be ignored and the small signal model must be modified. Recall that for high frequency operation, we're stating that external capacitances are so large (in relation to the internal capacitances) that they may be considered short circuits.

High frequency response of Common source amplifier

The JFET implementation of the common-source amplifier is given to the left below, and the small signal circuit in corporating the high frequency FET model is given to the right below. As stated above, the external coupling and bypass capacitors are large enough that we can model them as short circuits for high frequencies.



We may simplify the small signal circuit by making the following approximations and observations:

1. R_{ds} is usually larger than $R_D||R_L$, so that the parallel combination is dominated by $R_D||R_L$ and r_{ds} may be neglected. If this is not the case, a single equivalent resistance, $r_{ds}||R_D||R_L$ may be defined.

2. The Miller effect transforms C_{gd} into separate capacitances seen in the input and output circuits as

$$C_{M1} = C_{gd}(1 - A_v) \quad (input\ circuit)$$
$$C_{M2} = C_{gd}\left(1 - \frac{1}{A_v}\right) \quad (output\ circuit).$$

3. C_{ds} is very small, so the impedance contribution of this capacitance may be considered to be an open circuit and may be ignored.

$$C_{in} = C_{gs} + C_{M1} = C_{gs} + C_{gd}(1 - A_v).$$

4. The parallel capacitances in the input circuit, C_{gs} and C_{M1}, may be combined to a single equivalent capacitance of value

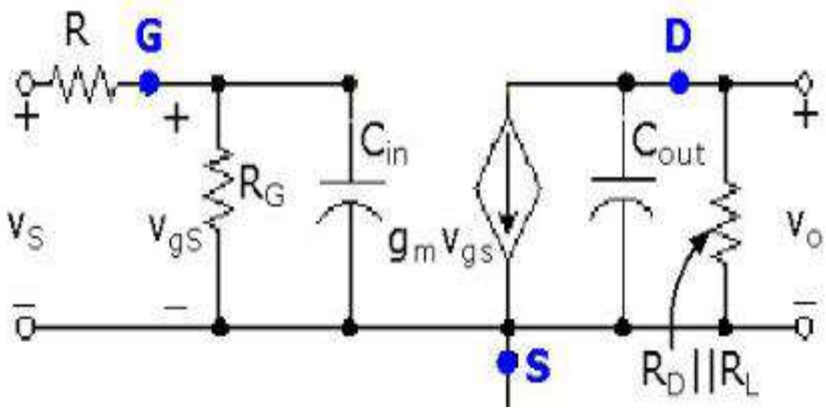
$$C_{in} = C_{gs} + C_{M1} = C_{gs} + C_{gd}(1 - A_v).$$

5. Similarly, the parallel capacitances in the output circuit, C_{ds} and C_{M2}, may be combined to a single equivalent capacitance of value

$$C_{out} = C_{ds} + C_{M2} = C_{ds} + C_{gd}\left(1 - \frac{1}{A_v}\right),$$

Where $A_v = -g_m(R_D||R_L)$ for a common-source amplifier.

Setting the input source, v_S, equal to zero allows us to define the equivalent resistances seen by C_{in} and C_{out}(the Method of Open Circuit Time Constants).Note that, with v_S=0, the dependent current source also goes to zero (opens) and the input and output circuits are separated.



$$R_{Cin} = R \,||\, R_G .$$

$$R_{Cout} = R_D \,||\, R_L .$$

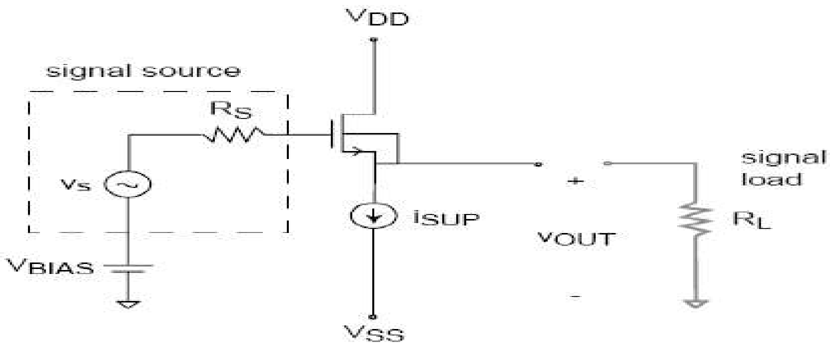
$$\tau_{Cin} = C_{in} R_{Cin} ; \qquad \tau_{Cout} = C_{out} R_{Cout} ,$$

$$\omega_H = \frac{1}{\dfrac{1}{\omega_{Cin}} + \dfrac{1}{\omega_{Cout}}} = \frac{1}{\tau_{Cin} + \tau_{Cout}} = \frac{1}{C_{in}R_{Cin} + C_{out}R_{Cout}} = \frac{1}{C_{in}(R \,||\, R_G) + C_{out}(R_D \,||\, R_L)}$$

Generally, the input is going to provide the dominant pole, so the high frequency cut off is given by

$$\omega_H = \frac{1}{C_{in}(R \,||\, R_G)} ; \qquad f_H = \frac{\omega_H}{2\pi} = \frac{1}{2\pi C_{in}(R \,||\, R_G)} .$$

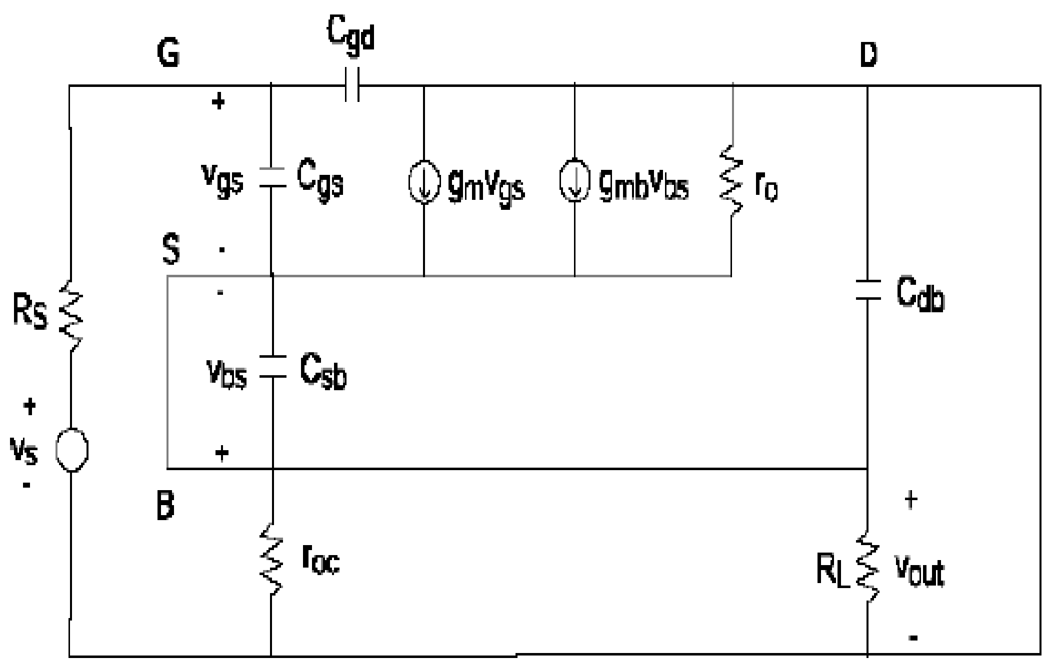
High frequency response of Common source amplifier

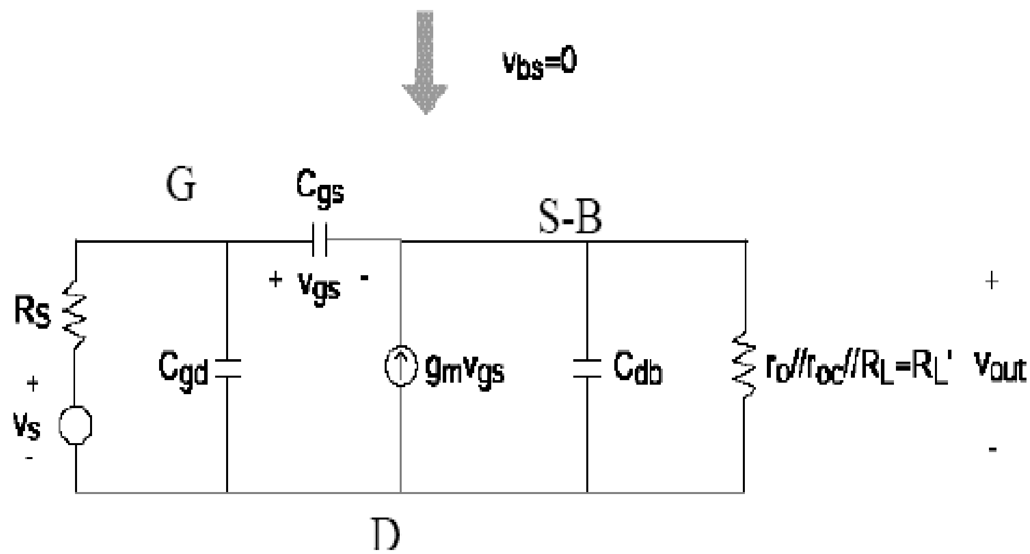


Characteristics ofCDAmplifier:

- Voltagegain ≈1
- Highinputresistance
- Lowoutputresistance
- Goodvoltage buffer

High frequency small signal model





$$A_{vC_{gs}} = \frac{R_L}{R_{out} + R_L} = \frac{g_m R_L}{1 + g_m R_L}$$

$$C_M = C_{gs}\left(1 - A_{vC_{gs}}\right) = C_{gs}\left(1 - \frac{g_m R_L}{1 + g_m R_L}\right) = \frac{C_{gs}}{1 + g_m R_L}$$

$$C_T = C_{gs}\left(1 - \frac{g_m R_L}{1 + g_m R_L}\right) + C_{gd}$$

$$R_T = R_S \parallel R_{in} = R_S$$

$$R_{C_{db}} = R_{out} \parallel R_L = \frac{R_{out} R_L}{R_{out} + R_L} = \frac{R_L}{1 + g_m R_L}$$

$$\omega_{3dB} \approx \frac{1}{R_S\left(\frac{C_{gs}}{1+g_m R_L} + C_{gd}\right) + C_{db}\frac{R_L}{1+g_m R_L}}$$

If R_S is not too high, bandwidth can be rather high and approach ω_T