

$$S \cos 50 + Q \sin \beta = Q$$

$$\Rightarrow S \cos 50 = Q(1 - \cos \beta)$$

$$\Rightarrow Q \frac{\sin \beta}{\sin 50} \cos 50 = Q(1 - \cos \beta)$$

$$\Rightarrow \cot 50 = \frac{1 - \cos \beta}{\sin \beta}$$

$$\Rightarrow 0.839 \sin \beta = 1 - \cos \beta$$

Squaring both sides,

$$0.703 \sin^2 \beta = 1 + \cos^2 \beta - 2 \cos \beta$$

$$0.703(1 - \cos^2 \beta) = 1 + \cos^2 \beta - 2 \cos \beta$$

$$0.703 - 0.703 \cos^2 \beta = 1 + \cos^2 \beta - 2 \cos \beta$$

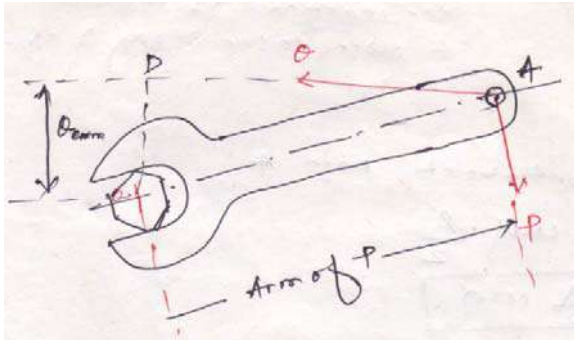
$$\Rightarrow 1.703 \cos^2 \beta - 2 \cos \beta + 0.297 = 0$$

$$\Rightarrow \cos^2 \beta - 1.174 \cos \beta + 0.297 = 0$$

$$\Rightarrow \beta = 63.13^\circ$$

Method of moments

Moment of a force with respect to a point:



- Considering wrench subjected to two forces P and Q of equal magnitude. It is evident that force P will be more effective compared to Q, though they are of equal magnitude.
- The effectiveness of the force as regards its tendency to produce rotation of a body about a fixed point is called the moment of the force with respect to that point.
- Moment = Magnitude of the force $\times$ Perpendicular distance of the line of action of force.
- Point O is called moment centre and the perpendicular distance (i.e. OD) is called moment arm.
- Unit is N.m

Theorem of Varignon:

The moment of the resultant of two concurrent forces with respect to a centre in their plane is equal to the algebraic sum of the moments of the components with respect to some centre.

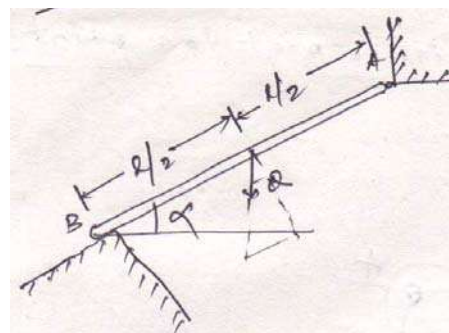
Problem 1:

A prismatic bar of AB of length l is hinged at A and supported at B. Neglecting friction, determine the reaction R_b produced at B owing to the weight Q of the bar.

Taking moment about point A,

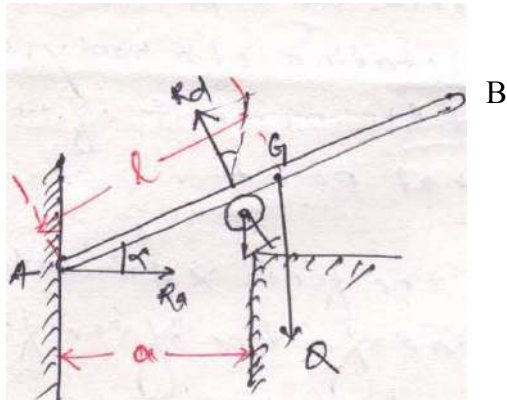
$$R_b \times l = Q \cos \alpha \cdot \frac{l}{2}$$

$$\Rightarrow R_b = \frac{Q}{2} \cos \alpha$$



Problem 2:

A bar AB of weight Q and length $2l$ rests on a very small frictionless roller at D and against a smooth vertical wall at A. Find the angle α that the bar must make with the horizontal in equilibrium.



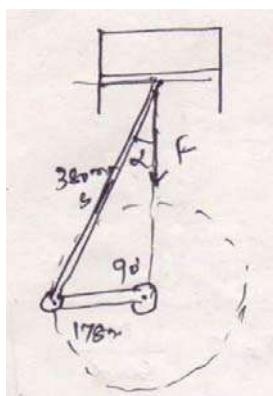
Resolving vertically,
 $R_d \cos \alpha = Q$

Now taking moment about A,

$$\begin{aligned} \frac{R_d \cdot a}{\cos \alpha} - Q \cdot l \cos \alpha &= 0 \\ \Rightarrow \frac{Q \cdot a}{\cos^2 \alpha} - Q \cdot l \cos \alpha &= 0 \\ \Rightarrow Q \cdot a - Q \cdot l \cos^3 \alpha &= 0 \\ \Rightarrow \cos^3 \alpha &= \frac{Q \cdot a}{Q \cdot l} \\ \Rightarrow \alpha &= \cos^{-1} \sqrt[3]{\frac{a}{l}} \end{aligned}$$

Problem 3:

If the piston of the engine has a diameter of 101.6 mm and the gas pressure in the cylinder is 0.69 MPa. Calculate the turning moment M exerted on the crankshaft for the particular configuration.



Area of cylinder

$$A = \frac{\pi}{4} (0.1016)^2 = 8.107 \times 10^{-3} \text{ m}^2$$

Force exerted on connecting rod,

$$\begin{aligned} F &= \text{Pressure} \times \text{Area} \\ &= 0.69 \times 10^6 \times 8.107 \times 10^{-3} \\ &= 5593.83 \text{ N} \end{aligned}$$

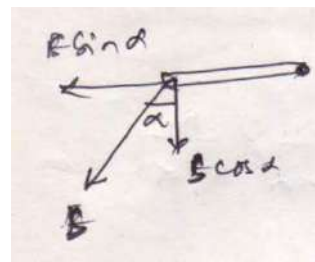
$$\text{Now } \alpha = \sin^{-1} \left(\frac{178}{380} \right) = 27.93^\circ$$

$$S \cos \alpha = F$$

$$\Rightarrow S = \frac{F}{\cos \alpha} = 6331.29 \text{ N}$$

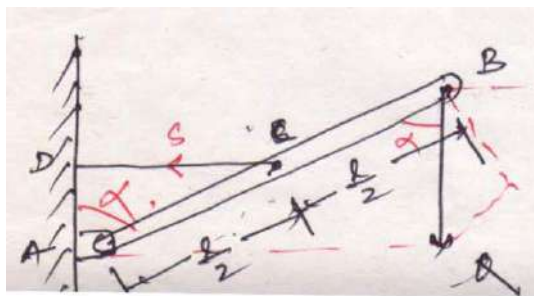
Now moment entered on crankshaft,

$$S \cos \alpha \times 0.178 = 995.7 \text{ N} = 1 \text{ kN}$$



Problem 4:

A rigid bar AB is supported in a vertical plane and carrying a load Q at its free end. Neglecting the weight of bar, find the magnitude of tensile force S in the horizontal string CD.



Taking moment about A,

$$\sum M_A = 0$$

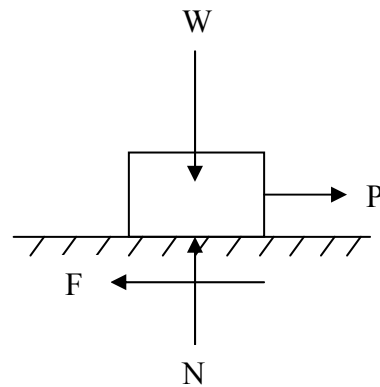
$$S \cdot \frac{l}{2} \cos \alpha = Q \cdot l \sin \alpha$$

$$\Rightarrow S = \frac{Q \cdot l \sin \alpha}{\frac{l}{2} \cos \alpha}$$

$$\Rightarrow S = 2Q \cdot \tan \alpha$$

Friction

- The force which opposes the movement or the tendency of movement is called **Frictional force or simply friction**. It is due to the resistance to motion offered by minutely projecting particles at the contact surfaces. However, there is a limit beyond which the magnitude of this force cannot increase.
- If the applied force is more than this limit, there will be movement of one body over the other. This limiting value of frictional force when the motion is impending, it is known as **Limiting Friction**.
- When the applied force is less than the limiting friction, the body remains at rest and such frictional force is called **Static Friction**, which will be having any value between zero and the limiting friction.
- If the value of applied force exceeds the limiting friction, the body starts moving over the other body and the frictional resistance experienced by the body while moving is known as **Dynamic Friction**. Dynamic friction is less than limiting friction.
- Dynamic friction is classified into following two types:
 - a) Sliding friction
 - b) Rolling friction
- Sliding friction is the friction experienced by a body when it slides over the other body.
- Rolling friction is the friction experienced by a body when it rolls over a surface.
- It is experimentally found that the magnitude of limiting friction bears a constant ratio to the normal reaction between two surfaces and this ratio is called **Coefficient of Friction**.



$$\text{Coefficient of friction} = \frac{F}{N}$$

where F is limiting friction and N is normal reaction between the contact surfaces.

Coefficient of friction is denoted by μ .

$$\text{Thus, } \mu = \frac{F}{N}$$

Laws of friction

1. The force of friction always acts in a direction opposite to that in which body tends to move.
2. Till the limiting value is reached, the magnitude of friction is exactly equal to the force which tends to move the body.
3. The magnitude of the limiting friction bears a constant ratio to the normal reaction between the two surfaces of contact and this ratio is called coefficient of friction.
4. The force of friction depends upon the roughness/smoothness of the surfaces.
5. The force of friction is independent of the area of contact between the two surfaces.
6. After the body starts moving, the dynamic friction comes into play, the magnitude of which is less than that of limiting friction and it bears a constant ratio with normal force. This ratio is called **coefficient of dynamic friction**.

Angle of friction

Consider the block shown in figure resting on a horizontal surface and subjected to horizontal pull P. Let F be the frictional force developed and N the normal reaction. Thus, at contact surface the reactions are F and N. They can be graphically combined to get the reaction R which acts at angle θ to normal reaction. This angle θ called the angle of friction is given by

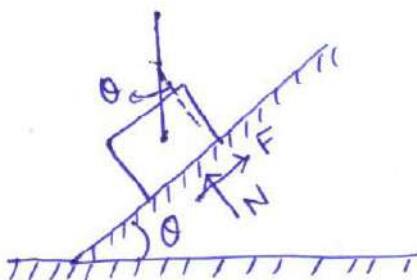
$$\tan \theta = \frac{F}{N}$$

As P increases, F increases and hence θ also increases. θ can reach the maximum value α when F reaches limiting value. At this stage,

$$\tan \alpha = \frac{F}{N} = \mu$$

This value of α is called Angle of Limiting Friction. Hence, the angle of limiting friction may be defined as the angle between the resultant reaction and the normal to the plane on which the motion of the body is impending.

Angle of repose



Consider the block of weight W resting on an inclined plane which makes an angle θ with the horizontal. When θ is small, the block will rest on the plane. If θ is gradually increased, a stage is reached at which the block start sliding down the plane. The angle θ for which the motion is impending, is called the angle of repose. Thus, the maximum inclination of the plane on which a body, free from external forces, can repose is called **Angle of Repose**.

Resolving vertically,
 $N = W \cdot \cos \theta$

Resolving horizontally,
 $F = W \cdot \sin \theta$

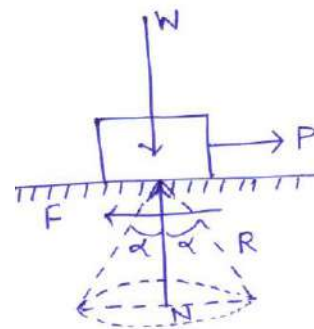
Thus, $\tan \theta = \frac{F}{N}$

If ϕ is the value of θ when the motion is impending, the frictional force will be limiting friction and hence,

$$\begin{aligned} \tan \phi &= \frac{F}{N} \\ &= \mu = \tan \alpha \\ \Rightarrow \phi &= \alpha \end{aligned}$$

Thus, the value of angle of repose is same as the value of limiting angle of repose.

Cone of friction

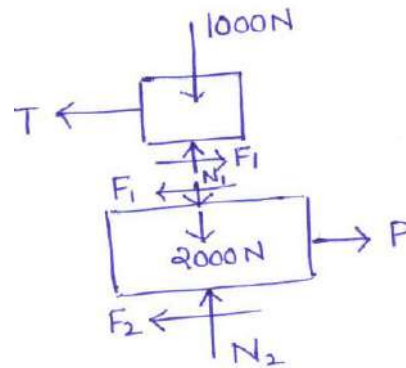
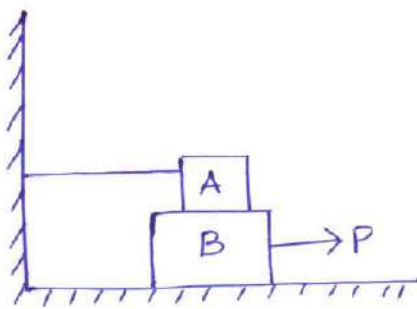


- When a body is having impending motion in the direction of force P , the frictional force will be limiting friction and the resultant reaction R will make limiting angle α with the normal.
- If the body is having impending motion in some other direction, the resultant reaction makes limiting frictional angle α with the normal to that direction. Thus, when the direction of force P is gradually changed through 360° , the resultant R generates a right circular cone with semi-central angle equal to α .

Problem 1: Block A weighing 1000N rests over block B which weighs 2000N as shown in figure. Block A is tied to wall with a horizontal string. If the coefficient of friction between blocks A and B is 0.25 and between B and floor is $\frac{1}{3}$, what should be the value of P to move the block (B), if

- (a) P is horizontal.
- (b) P acts at 30° upwards to horizontal.

Solution: (a)



Considering block A,

$$\sum V = 0$$

$$N_1 = 1000N$$

Since F_1 is limiting friction,

$$\frac{F_1}{N_1} = \mu = 0.25$$

$$F_1 = 0.25N_1 = 0.25 \times 1000 = 250N$$

$$\sum H = 0$$

$$F_1 - T = 0$$

$$T = F_1 = 250N$$

Considering equilibrium of block B,

$$\sum V = 0$$

$$N_2 - 2000 - N_1 = 0$$

$$N_2 = 2000 + N_1 = 2000 + 1000 = 3000N$$

$$\frac{F_2}{N_2} = \mu = \frac{1}{3}$$

$$F_2 = 0.3N_2 = 0.3 \times 3000 = 1000N$$

$$\sum H = 0$$

$$P = F_1 + F_2 = 250 + 1000 = 1250 \text{ N}$$

(b) When P is inclined:

$$\sum V = 0$$

$$N_2 - 2000 - N_1 + P \sin 30 = 0$$

$$\Rightarrow N_2 + 0.5P = 2000 + 1000$$

$$\Rightarrow N_2 = 3000 - 0.5P$$

From law of friction,

$$F_2 = \frac{1}{3} N_2 = \frac{1}{3} (3000 - 0.5P) = 1000 - \frac{0.5}{3} P$$

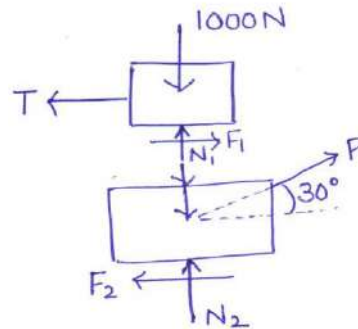
$$\sum H = 0$$

$$P \cos 30 = F_1 + F_2$$

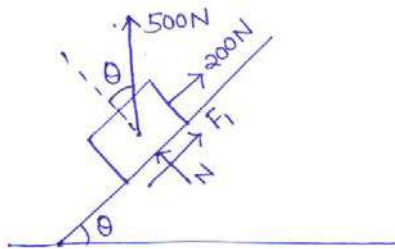
$$\Rightarrow P \cos 30 = 250 + \left(1000 - \frac{0.5}{3} P \right)$$

$$\Rightarrow P \left(\cos 30 + \frac{0.5}{3} \right) = 1250$$

$$\Rightarrow P = 1210.43 \text{ N}$$



Problem 2: A block weighing 500N just starts moving down a rough inclined plane when supported by a force of 200N acting parallel to the plane in upward direction. The same block is on the verge of moving up the plane when pulled by a force of 300N acting parallel to the plane. Find the inclination of the plane and coefficient of friction between the inclined plane and the block.



$$\sum V = 0$$

$$N = 500 \cos \theta$$

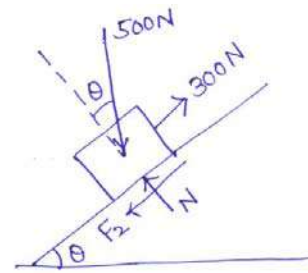
$$F_1 = \mu N = \mu \cdot 500 \cos \theta$$

$$\begin{aligned}\sum H &= 0 \\ 200 + F_1 &= 500 \cdot \sin \theta \\ \Rightarrow 200 + \mu \cdot 500 \cos \theta &= 500 \cdot \sin \theta\end{aligned}\tag{1}$$

$$\begin{aligned}\sum V &= 0 \\ N &= 500 \cdot \cos \theta \\ F_2 &= \mu N = \mu \cdot 500 \cdot \cos \theta\end{aligned}$$

$$\begin{aligned}\sum H &= 0 \\ 500 \sin \theta + F_2 &= 300 \\ \Rightarrow 500 \sin \theta + \mu \cdot 500 \cos \theta &= 300\end{aligned}\tag{2}$$

Adding Eqs. (1) and (2), we get



$$\begin{aligned}500 &= 1000 \cdot \sin \theta \\ \sin \theta &= 0.5 \\ \theta &= 30^\circ\end{aligned}$$

Substituting the value of θ in Eq. 2,

$$500 \sin 30 + \mu \cdot 500 \cos 30 = 300$$

$$\mu = \frac{50}{500 \cos 30} = 0.11547$$

Parallel forces on a plane

Like parallel forces: Coplanar parallel forces when act in the same direction.



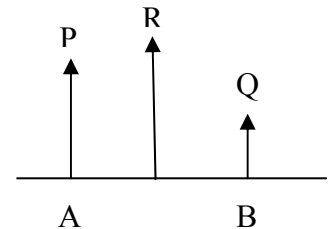
Unlike parallel forces: Coplanar parallel forces when act in different direction.



Resultant of like parallel forces:

Let P and Q are two like parallel forces act at points A and B.

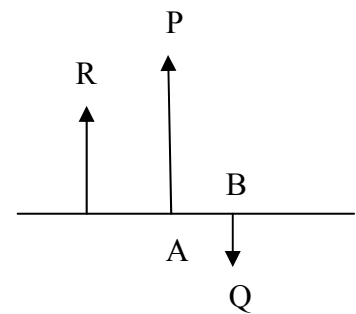
$$R = P + Q$$



Resultant of unlike parallel forces:

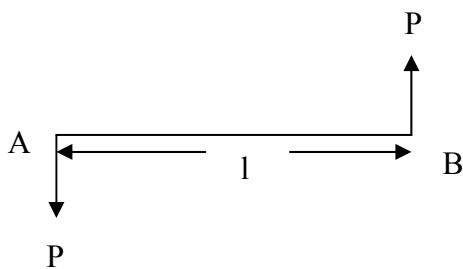
$$R = P - Q$$

R is in the direction of the force having greater magnitude.



Couple:

Two unlike equal parallel forces form a couple.

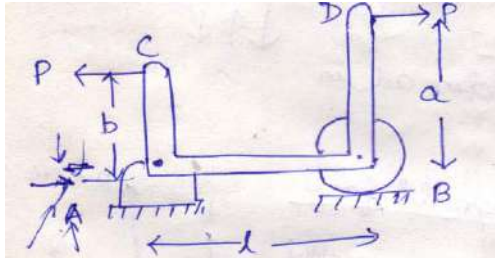


The rotational effect of a couple is measured by its moment.

$$\text{Moment} = P \times l$$

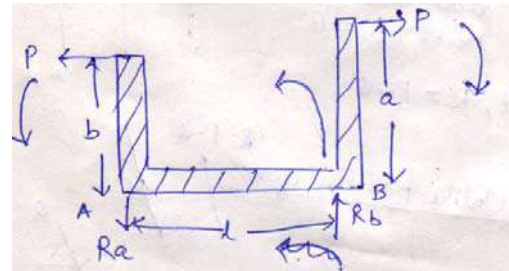
Sign convention: Anticlockwise couple (Positive)
Clockwise couple (Negative)

Problem 1 : A rigid bar CABD supported as shown in figure is acted upon by two equal horizontal forces P applied at C and D . Calculate the reactions that will be induced at the points of support. Assume $l = 1.2$ m, $a = 0.9$ m, $b = 0.6$ m.



$$\sum V = 0$$

$$R_a = R_b$$



Taking moment about A,

$$R_a = R_b$$

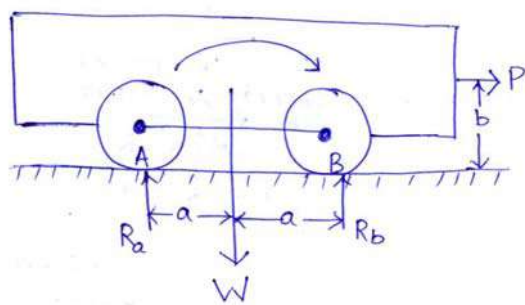
$$R_b \times l + P \times b = P \times a$$

$$\Rightarrow R_b = \frac{P(0.9 - 0.6)}{1.2}$$

$$\Rightarrow R_b = 0.25P(\uparrow)$$

$$\Rightarrow R_a = 0.25P(\downarrow)$$

Problem 2: Owing to weight W of the locomotive shown in figure, the reactions at the two points of support A and B will each be equal to $W/2$. When the locomotive is pulling the train and the drawbar pull P is just equal to the total friction at the points of contact A and B , determine the magnitudes of the vertical reactions R_a and R_b .



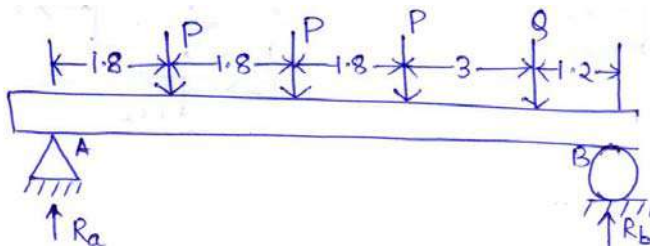
$$\sum V = 0$$

$$R_a + R_b = W$$

Taking moment about B,

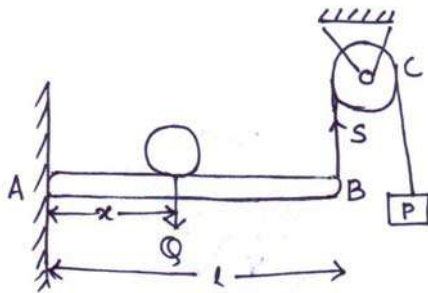
$$\begin{aligned}
\sum M_B &= 0 \\
R_a \times 2a + P \times b &= W \times a \\
\Rightarrow R_a &= \frac{W \cdot a - P \cdot b}{2a} \\
\therefore R_b &= W - R_a \\
\Rightarrow R_b &= W - \left(\frac{W \cdot a - P \cdot b}{2a} \right) \\
\Rightarrow R_b &= \frac{W \cdot a + P \cdot b}{2a}
\end{aligned}$$

Problem 3: The four wheels of a locomotive produce vertical forces on the horizontal girder AB. Determine the reactions R_a and R_b at the supports if the loads $P = 90$ KN each and $Q = 72$ KN (All dimensions are in m).

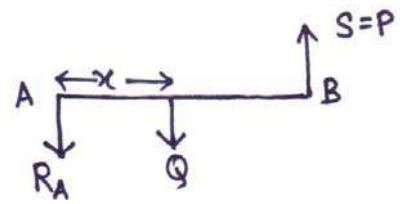
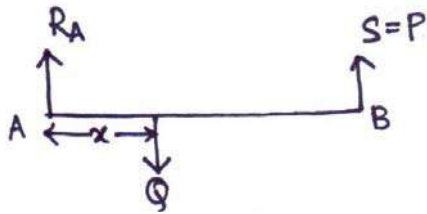


$$\begin{aligned}
\sum V &= 0 \\
R_a + R_b &= 3P + Q \\
\Rightarrow R_a + R_b &= 3 \times 90 + 72 \\
\Rightarrow R_a + R_b &= 342 \text{ KN} \\
\sum M_A &= 0 \\
R_b \times 9.6 &= 90 \times 1.8 + 90 \times 3.6 + 90 \times 5.4 + 72 \times 8.4 \\
\Rightarrow R_b &= 164.25 \text{ KN} \\
\therefore R_a &= 177.75 \text{ KN}
\end{aligned}$$

Problem 4: The beam AB in figure is hinged at A and supported at B by a vertical cord which passes over a frictionless pulley at C and carries at its end a load P. Determine the distance x from A at which a load Q must be placed on the beam if it is to remain in equilibrium in a horizontal position. Neglect the weight of the beam.



FBD

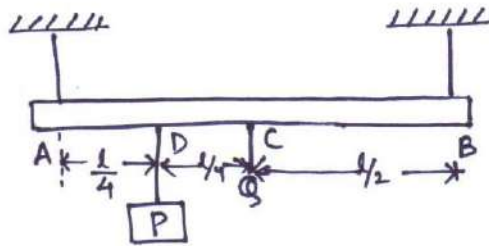


$$\sum M_A = 0$$

$$S \times l = Q \times x$$

$$\Rightarrow x = \frac{P \cdot l}{Q}$$

Problem 5: A prismatic bar AB of weight $Q = 44.5 \text{ N}$ is supported by two vertical wires at its ends and carries at D a load $P = 89 \text{ N}$ as shown in figure. Determine the forces S_a and S_b in the two wires.



$$Q = 44.5 \text{ N}$$

$$P = 89 \text{ N}$$

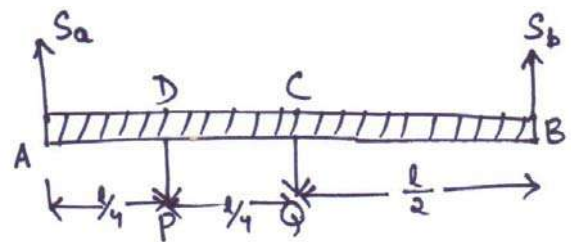
Resolving vertically,

$$\sum V = 0$$

$$S_a + S_b = P + Q$$

$$\Rightarrow S_a + S_b = 89 + 44.5$$

$$\Rightarrow S_a + S_b = 133.5 \text{ N}$$



$$\sum M_A = 0$$

$$S_b \times l = P \times \frac{l}{4} + Q \times \frac{l}{2}$$

$$\Rightarrow S_b = \frac{P}{4} + \frac{Q}{2}$$

$$\Rightarrow S_b = \frac{89}{4} + \frac{44.5}{2}$$

$$\Rightarrow S_b = 44.5$$

$$\therefore S_a = 133.5 - 44.5$$

$$\Rightarrow S_a = 89N$$

Centre of gravity

Centre of gravity: It is that point through which the resultant of the distributed gravity force passes regardless of the orientation of the body in space.

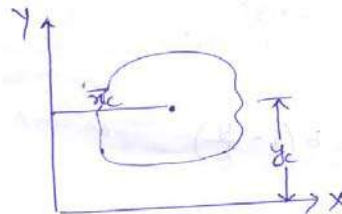
- As the point through which resultant of force of gravity (weight) of the body acts.

Centroid: Centroid of an area lies on the axis of symmetry if it exists.

Centre of gravity is applied to bodies with mass and weight and centroid is applied to plane areas.

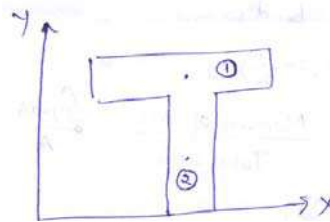
$$x_c = \sum A_i x_i$$

$$y_c = \sum A_i y_i$$



$$x_c = \frac{A_1 x_1 + A_2 x_2}{A_1 + A_2}$$

$$y_c = \frac{A_1 y_1 + A_2 y_2}{A_1 + A_2}$$

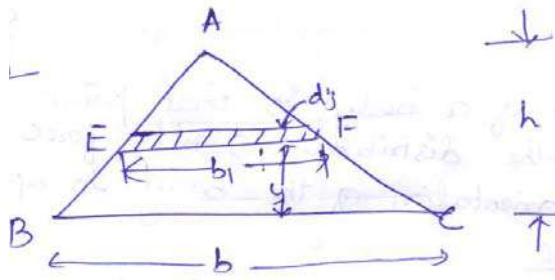


$$x_c = y_c = \frac{\text{Moment of area}}{\text{Total area}}$$

$$x_c = \frac{\int x.dA}{A}$$

$$y_c = \frac{\int y.dA}{A}$$

Problem 1: Consider the triangle ABC of base 'b' and height 'h'. Determine the distance of centroid from the base.



Let us consider an elemental strip of width 'b₁' and thickness 'dy'.

$$\triangle AEF \sim \triangle ABC$$

$$\therefore \frac{b_1}{b} = \frac{h-y}{h}$$

$$\Rightarrow b_1 = b \left(\frac{h-y}{h} \right)$$

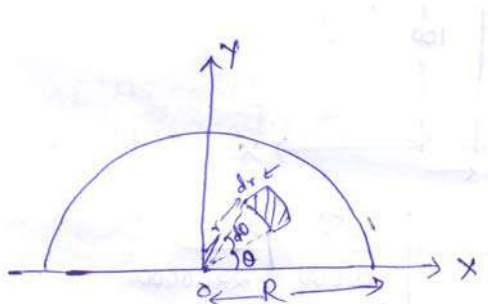
$$\Rightarrow b_1 = b \left(1 - \frac{y}{h} \right)$$

$$\begin{aligned} \text{Area of element EF (dA)} &= b_1 \times dy \\ &= b \left(1 - \frac{y}{h} \right) dy \end{aligned}$$

$$\begin{aligned} y_c &= \frac{\int y \cdot dA}{A} \\ &= \frac{\int_0^h y b \left(1 - \frac{y}{h} \right) dy}{\frac{1}{2} b \cdot h} \\ &= \frac{b \left[\frac{y^2}{2} - \frac{y^3}{3h} \right]_0^h}{\frac{1}{2} b \cdot h} \\ &= \frac{2}{h} \left[\frac{h^2}{2} - \frac{h^3}{3} \right] \\ &= \frac{2}{h} \times \frac{h^2}{6} \\ &= \frac{h}{3} \end{aligned}$$

Therefore, y_c is at a distance of h/3 from base.

Problem 2: Consider a semi-circle of radius R. Determine its distance from diametral axis.



Due to symmetry, centroid ' y_c ' must lie on Y-axis.

Consider an element at a distance ' r ' from centre ' o ' of the semicircle with radial width dr .

Area of element = $(r.d\theta) \times dr$

Moment of area about x = $\int y.dA$

$$= \int_0^\pi \int_0^R (r.d\theta).dr \times (r.\sin \theta)$$

$$= \int_0^\pi \int_0^R r^2 \sin \theta .dr.d\theta$$

$$= \int_0^\pi \int_0^R (r^2.dr) .\sin \theta .d\theta$$

$$= \int_0^\pi \left[\frac{r^3}{3} \right]_0^R .\sin \theta .d\theta$$

$$= \int_0^\pi \frac{R^3}{3} .\sin \theta .d\theta$$

$$= \frac{R^3}{3} [-\cos \theta]_0^\pi$$

$$= \frac{R^3}{3} [1+1]$$

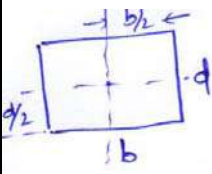
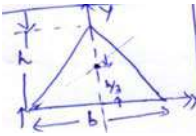
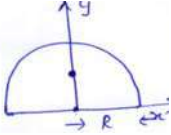
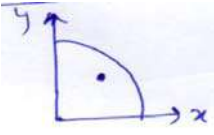
$$= \frac{2}{3} R^3$$

$$y_c = \frac{\text{Moment of area}}{\text{Total area}}$$

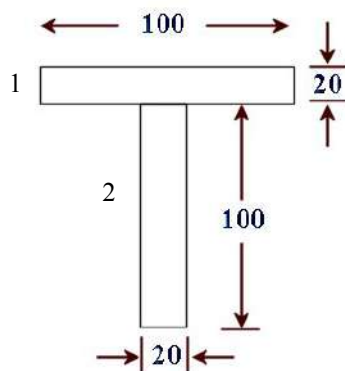
$$\begin{aligned}
 &= \frac{\frac{2}{3}R^3}{\pi R^2 / 2} \\
 &= \frac{4R}{3\pi}
 \end{aligned}$$

Therefore, the centroid of the semicircle is at a distance of $\frac{4R}{3\pi}$ from the diametric axis.

Centroids of different figures

Shape	Figure	$\bar{x}$	$\bar{y}$	Area
Rectangle		$\frac{b}{2}$	$\frac{d}{2}$	bd
Triangle		0	$\frac{h}{3}$	$\frac{bh}{2}$
Semicircle		0	$\frac{4R}{3\pi}$	$\frac{\pi r^2}{2}$
Quarter circle		$\frac{4R}{3\pi}$	$\frac{4R}{3\pi}$	$\frac{\pi r^2}{4}$

Problem 3: Find the centroid of the T-section as shown in figure from the bottom.

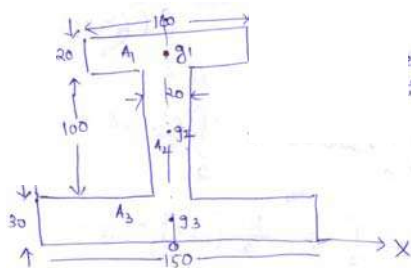


Area (A_i)	x_i	y_i	$A_i x_i$	$A_i y_i$
2000	0	110	10,000	22,0000
2000	0	50	10,000	10,0000
4000			20,000	32,0000

$$y_c = \frac{\sum A_i y_i}{A_i} = \frac{A_1 y_1 + A_2 y_2}{A_1 + A_2} = \frac{32,0000}{4000} = 80$$

Due to symmetry, the centroid lies on Y-axis and it is at distance of 80 mm from the bottom.

Problem 4: Locate the centroid of the I-section.



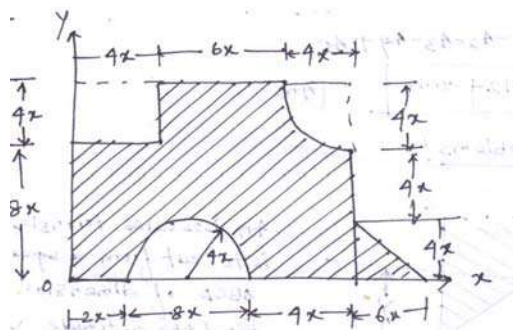
As the figure is symmetric, centroid lies on y-axis. Therefore, $\bar{x} = 0$

Area (A_i)	x_i	y_i	$A_i x_i$	$A_i y_i$
2000	0	140	0	280000
2000	0	80	0	160000
4500	0	15	0	67500

$$y_c = \frac{\sum A_i y_i}{A_i} = \frac{A_1 y_1 + A_2 y_2 + A_3 y_3}{A_1 + A_2 + A_3} = 59.71 \text{ mm}$$

Thus, the centroid is on the symmetric axis at a distance 59.71 mm from the bottom.

Problem 5: Determine the centroid of the composite figure about x-y coordinate. Take $x = 40$ mm.



$$A_1 = \text{Area of rectangle} = 12x \cdot 14x = 168x^2$$

$$A_2 = \text{Area of rectangle to be subtracted} = 4x \cdot 4x = 16x^2$$

$$A_3 = \text{Area of semicircle to be subtracted} = \frac{\pi R^2}{2} = \frac{\pi(4x)^2}{2} = 25.13x^2$$

$$A_4 = \text{Area of quatercircle to be subtracted} = \frac{\pi R^2}{4} = \frac{\pi(4x)^2}{4} = 12.56x^2$$

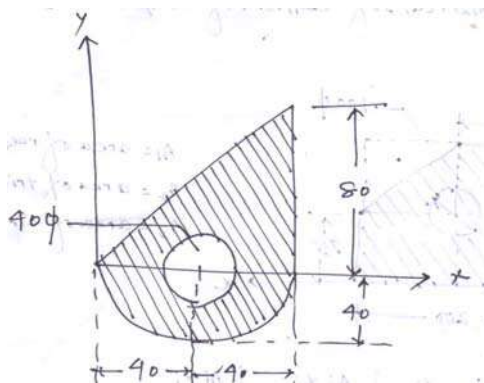
$$A_5 = \text{Area of triangle} = \frac{1}{2} \times 6x \times 4x = 12x^2$$

Area (A_i)	x_i	y_i	$A_i x_i$	$A_i y_i$
$A_1 = 268800$	$7x = 280$	$6x = 240$	75264000	64512000
$A_2 = 25600$	$2x = 80$	$10x = 400$	2048000	10240000
$A_3 = 40208$	$6x = 240$	$\frac{4 \times 4x}{3\pi} = 67.906$	9649920	2730364.448
$A_4 = 20096$	$10x + \left(4x - \frac{4 \times 4x}{3\pi}\right) = 492.09$	$8x + \left(4x - \frac{4 \times 4x}{3\pi}\right) = 412.093$	9889040.64	8281420.926
$A_5 = 19200$	$14x + \frac{6x}{3} = 16x = 640$	$\frac{4x}{3} = 53.33$	12288000	1023936

$$x_c = \frac{A_1 x_1 - A_2 x_2 - A_3 x_3 - A_4 x_4 + A_5 x_5}{A_1 - A_2 - A_3 - A_4 + A_5} = 326.404 \text{ mm}$$

$$y_c = \frac{A_1 y_1 - A_2 y_2 - A_3 y_3 - A_4 y_4 + A_5 y_5}{A_1 - A_2 - A_3 - A_4 + A_5} = 219.124 \text{ mm}$$

Problem 6: Determine the centroid of the following figure.



$$A_1 = \text{Area of triangle} = \frac{1}{2} \times 80 \times 80 = 3200 \text{ m}^2$$

$$A_2 = \text{Area of semicircle} = \frac{\pi d^2}{8} - \frac{\pi R^2}{2} = 2513.274 \text{ m}^2$$

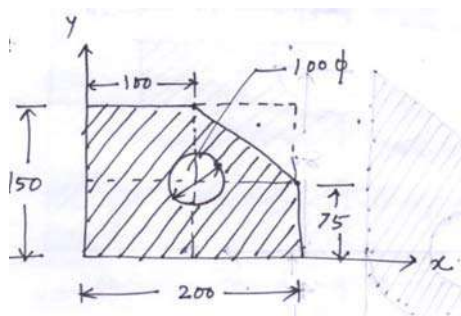
$$A_3 = \text{Area of semicircle} = \frac{\pi D^2}{2} = 1256.64 \text{ m}^2$$

Area (A_i)	x_i	y_i	$A_i x_i$	$A_i y_i$
3200	$2 \times (80/3) = 53.33$	$80/3 = 26.67$	170656	85344
2513.274	40	$\frac{-4 \times 40}{3\pi} = -16.97$	100530.96	-42650.259
1256.64	40	0	50265.6	0

$$x_c = \frac{A_1 x_1 + A_2 x_2 - A_3 x_3}{A_1 + A_2 + A_3} = 49.57 \text{ mm}$$

$$y_c = \frac{A_1 y_1 + A_2 y_2 - A_3 y_3}{A_1 + A_2 - A_3} = 9.58 \text{ mm}$$

Problem 7: Determine the centroid of the following figure.



A_1 = Area of the rectangle

A_2 = Area of triangle

A_3 = Area of circle

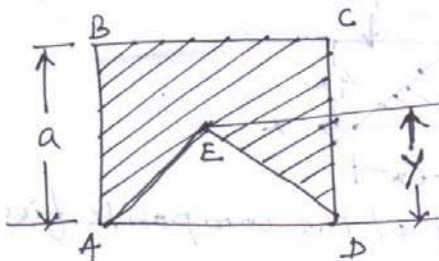
Area (A_i)	x_i	y_i	$A_i x_i$	$A_i y_i$
30,000	100	75	3000000	2250000
3750	$100 + 200/3 = 166.67$	$75 + 150/3 = 125$	625012.5	468750
7853.98	100	75	785398	589048.5

$$x_c = \frac{\sum A_i x_i}{\sum A_i} = \frac{A_1 x_1 - A_2 x_2 - A_3 x_3}{A_1 - A_2 - A_3} = 86.4 \text{ mm}$$

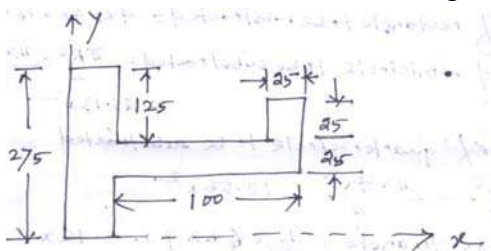
$$y_c = \frac{\sum A_i y_i}{\sum A_i} = \frac{A_1 y_1 - A_2 y_2 - A_3 y_3}{A_1 - A_2 - A_3} = 64.8 \text{ mm}$$

Numerical Problems (Assignment)

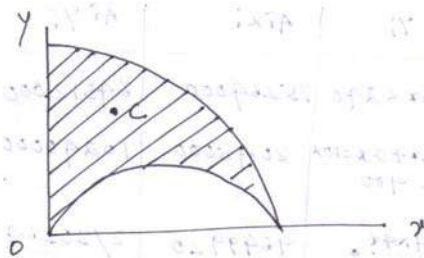
1. An isosceles triangle ADE is to cut from a square ABCD of dimension 'a'. Find the altitude 'y' of the triangle so that vertex E will be centroid of remaining shaded area.



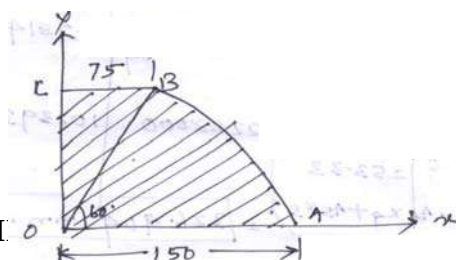
2. Find the centroid of the following figure.



3. Locate the centroid C of the shaded area obtained by cutting a semi-circle of diameter 'a' from the quadrant of a circle of radius 'a'.



4. Locate the centroid of the composite figure.



Truss/ Frame: A pin jointed frame is a structure made of slender (cross-sectional dimensions quite small compared to length) members pin connected at ends and capable of taking load at joints.

Such frames are used as roof trusses to support sloping roofs and as bridge trusses to support deck.

Plane frame: A frame in which all members lie in a single plane is called plane frame. They are designed to resist the forces acting in the plane of frame. Roof trusses and bridge trusses are the example of plane frames.

Space frame: If all the members of frame do not lie in a single plane, they are called as space frame. Tripod, transmission towers are the examples of space frames.

Perfect frame: A pin jointed frame which has got just sufficient number of members to resist the loads without undergoing appreciable deformation in shape is called a perfect frame. Triangular frame is the simplest perfect frame and it has 03 joints and 03 members.

It may be observed that to increase one joint in a perfect frame, two more members are required. Hence, the following expression may be written as the relationship between number of joint j , and the number of members m in a perfect frame.

$$m = 2j - 3$$

- (a) When $LHS = RHS$, Perfect frame.
- (b) When $LHS < RHS$, Deficient frame.
- (c) When $LHS > RHS$, Redundant frame.

Assumptions

The following assumptions are made in the analysis of pin jointed trusses:

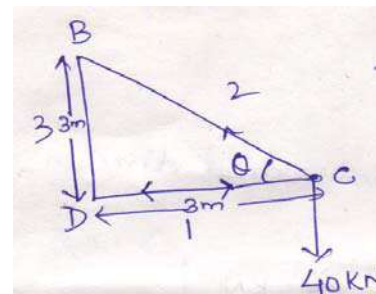
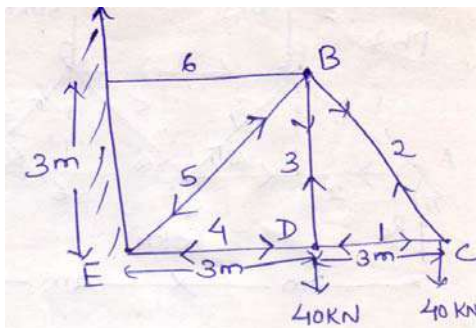
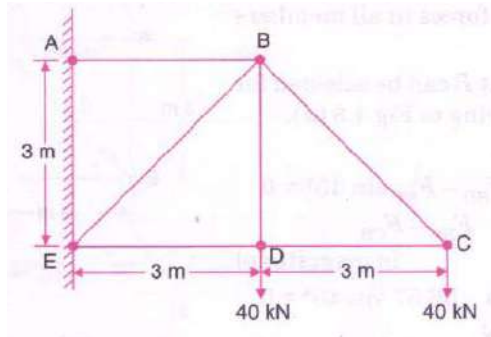
- 1. The ends of the members are pin jointed (hinged).
- 2. The loads act only at the joints.
- 3. Self weight of the members is negligible.

Methods of analysis

- 1. Method of joint
- 2. Method of section

Problems on method of joints

Problem 1: Find the forces in all the members of the truss shown in figure.



$$\tan \theta = 1$$

$$\Rightarrow \theta = 45^\circ$$

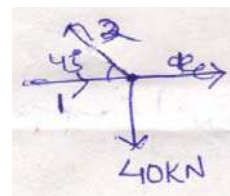
Joint C

$$S_1 = S_2 \cos 45$$

$$\Rightarrow S_1 = 40 \text{ kN (Compression)}$$

$$S_2 \sin 45 = 40$$

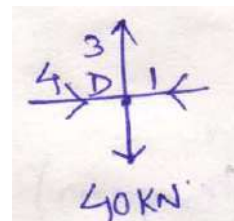
$$\Rightarrow S_2 = 56.56 \text{ kN (Tension)}$$



Joint D

$$S_3 = 40 \text{ kN (Tension)}$$

$$S_1 = S_4 = 40 \text{ kN (Compression)}$$

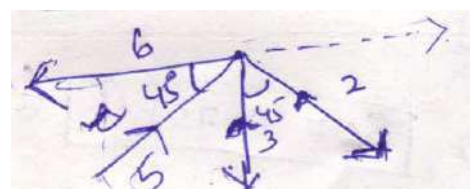


Joint B

Resolving vertically,

$$\sum V = 0$$

$$S_5 \sin 45 = S_3 + S_2 \sin 45$$



$$\Rightarrow S_5 = 113.137 \text{ kN (Compression)}$$

Resolving horizontally,

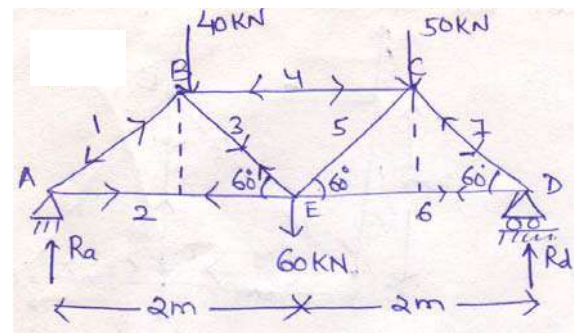
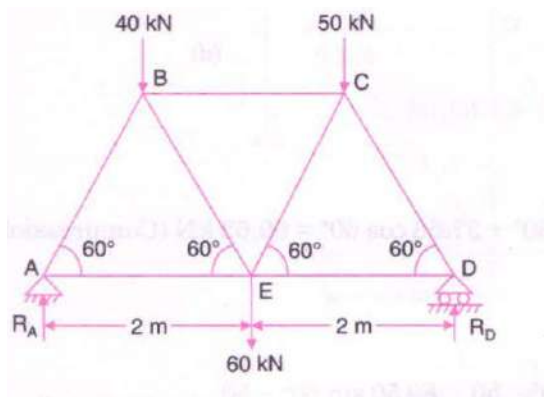
$$\sum H = 0$$

$$S_6 = S_5 \cos 45 + S_2 \cos 45$$

$$\Rightarrow S_6 = 113.137 \cos 45 + 56.56 \cos 45$$

$$\Rightarrow S_6 = 120 \text{ kN (Tension)}$$

Problem 2: Determine the forces in all the members of the truss shown in figure and indicate the magnitude and nature of the forces on the diagram of the truss. All inclined members are at 60° to horizontal and length of each member is 2m.



Taking moment at point A,

$$\sum M_A = 0$$

$$R_d \times 4 = 40 \times 1 + 60 \times 2 + 50 \times 3$$

$$\Rightarrow R_d = 77.5 \text{ kN}$$

Now resolving all the forces in vertical direction,

$$\sum V = 0$$

$$R_a + R_d = 40 + 60 + 50$$

$$\Rightarrow R_a = 72.5 \text{ kN}$$

Joint A

$$\sum V = 0$$

$$\Rightarrow R_a = S_1 \sin 60$$

$$\Rightarrow S_1 = 83.72 \text{ kN (Compression)}$$

$$\sum H = 0$$

$$\Rightarrow S_2 = S_1 \cos 60$$

