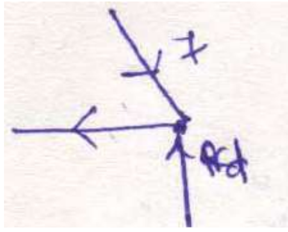
$\Rightarrow S_1 = 41.86KN$ (Tension)

Joint D



$\sum V = 0$

$S_7 \sin 60 = 77.5$

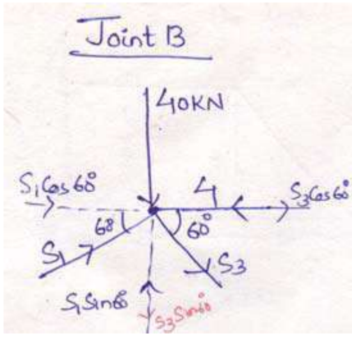
$\Rightarrow S_7 = 89.5KN$ (Compression)

$\sum H = 0$

$S_6 = S_7 \cos 60$

$\Rightarrow S_6 = 44.75KN$ (Tension)

Joint B



$\sum V = 0$

$S_1 \sin 60 = S_3 \cos 60 + 40$

$\Rightarrow S_3 = 37.532KN$ (Tension)

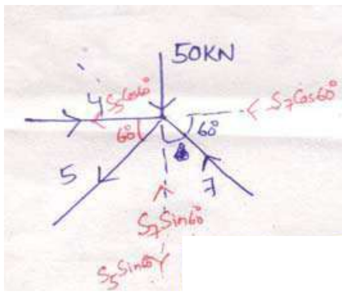
$\sum H = 0$

$S_4 = S_1 \cos 60 + S_3 \cos 60$

$\Rightarrow S_4 = 37.532 \cos 60 + 83.72 \cos 60$

$\Rightarrow S_4 = 60.626KN$ (Compression)

Joint C



$\sum V = 0$

$S_5 \sin 60 + 50 = S_7 \sin 60$

$\Rightarrow S_5 = 31.76KN$ (Tension)

Plane Truss (Method of Section)

In case of analysing a plane truss, using method of section, after determing the support reactions a section line is drawn passing through not more than three members in which forces are unknown, such that the entire frame is cut into two separate parts.

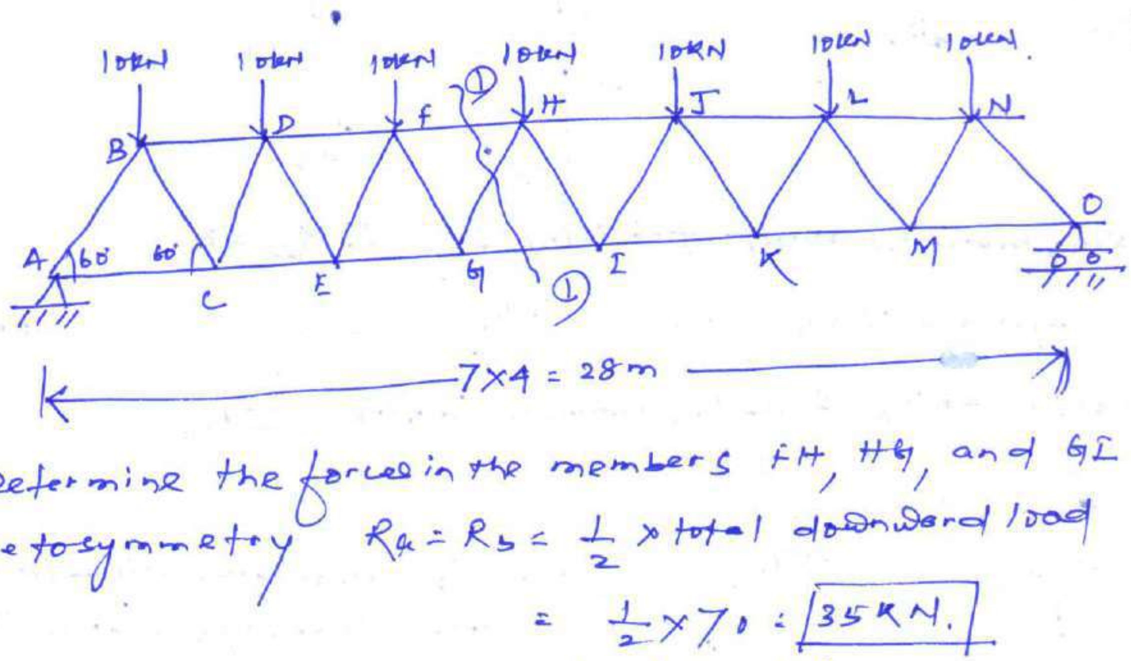
~~Ea~~ Each part should be in equilibrium under the action of loads, reactions and the forces in the members.

Method of section is preferred for the following cases:

(i) analysis of large truss in which forces in only few members are required

(ii) If method of joint fails to start or proceed with analysis for not getting a joint with only two unknown forces.

Example 1.

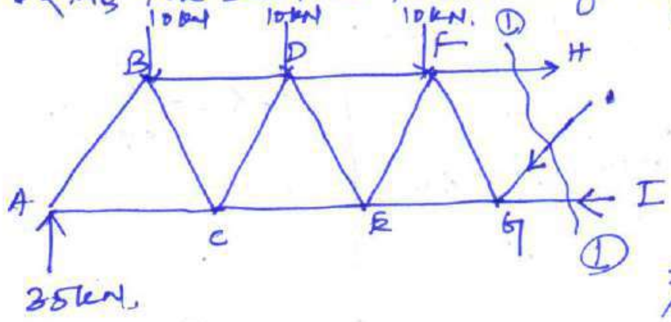


Determine the forces in the members FH, HG, and GI in the truss

Due to symmetry $R_A = R_O = \frac{1}{2} \times$ total downward load

$= \frac{1}{2} \times 70 = \boxed{35\,kN.}$

Taking the section to the left of the cut.



Taking moment about G

$\Sigma M_G = 0.$

$F_{FH} \times 4 \sin 60 + 35 \times 12$

$= 10 \times 2 + 10 \times 6 + 10 \times 10$

$\Rightarrow F_{FH} = \dfrac{(20+60+100) - 420}{4 \sin 60}$

$= -69.28\ kN.$

Negative sign indicates that direction should have opposite i.e it is compressive in nature.

Now Resolving all the forces vertically $\Sigma Y = 0$

$$10 + 10 + 10 + F_{GH} \sin 60 = 35$$

$$\Rightarrow F_{GH} = \frac{35 - 30}{\sin 60^\circ}$$

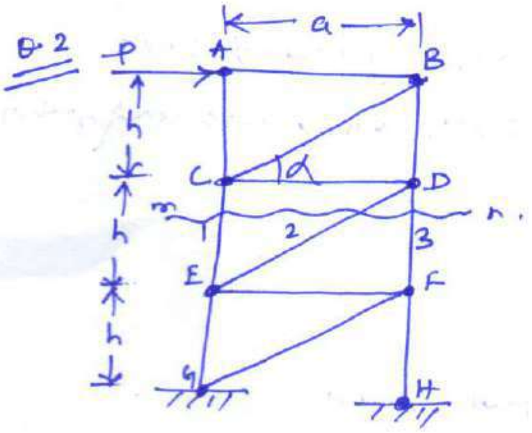
$$\Rightarrow \boxed{F_{GH} = 5.78\ kN.}\ \text{(compressive)}$$

Resolving all the forces horizontally $\Sigma X = 0$.

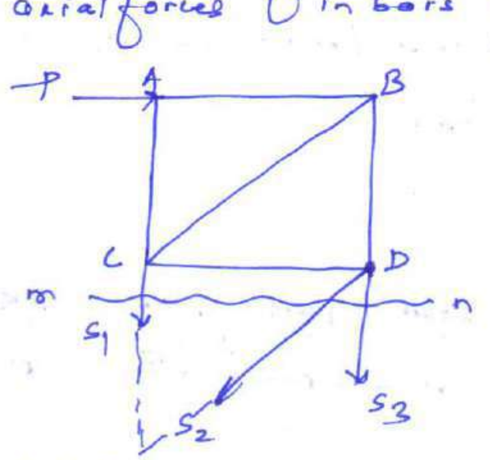
$$F_{FH} + F_{GH} \cos 60 = F_{GI}$$

$$\Rightarrow F_{GI} = 69.28 + 5.78 \cos 60^\circ = \boxed{72.17\ kN.}\ \text{(tension)}$$

Q.2



Using method of sections determine the axial forces in bars 1, 2 and 3.



Taking moment about ~~poi~~ joint D $\Sigma M_D = 0$.

$$S_1 \times a = P \times h \Rightarrow \boxed{S_1 = \frac{Ph}{a}} \quad \text{— (1)} \quad \text{(tension)}$$

Similarly taking E as the moment centre $\Sigma M_E = 0$

$$S_3 \times a + P \times 2h = 0$$

$$\Rightarrow \boxed{S_3 = \frac{-2Ph}{a}}$$

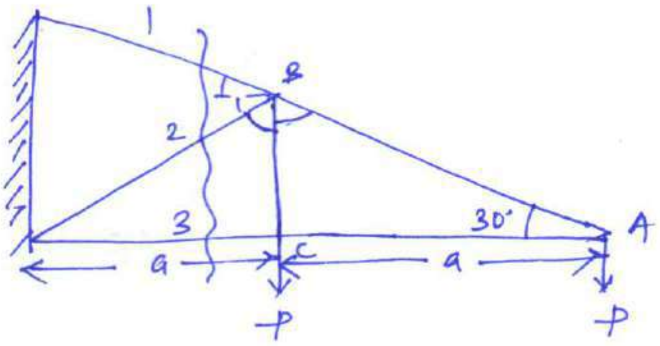
(−ve sign indicates direction of force will be opposite and it will be compressive in nature).

Resolving all the forces horizontally. $\Sigma X = 0$.

$$S_2 \cos\alpha = P$$

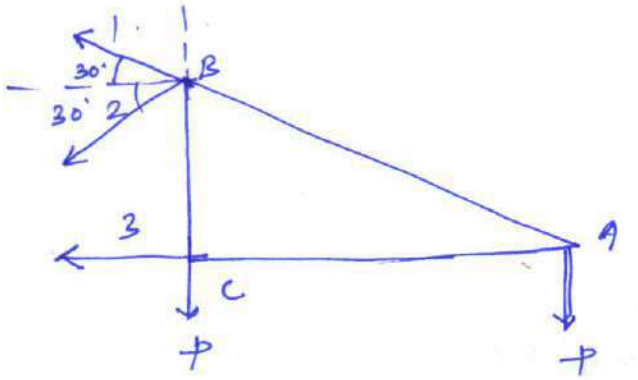
$$\Rightarrow S_2 = \frac{P}{\cos\alpha} = \boxed{\frac{P\sqrt{a^2+h^2}}{a}}\ \text{(Ans)}$$

$$\cos\alpha = \frac{a}{\sqrt{a^2+h^2}}$$



$\frac{BC}{AC} = \tan 30°$

$\Rightarrow BC = a \tan 30 = \boxed{0.578\,a}$



$\Sigma M_B = 0.$

$S_3 \times 0.578\,a + P \times a = 0$

$\Rightarrow S_3 = \frac{-Pa}{0.578\,a} = -1.73P$

(-ve sign indicates direction is opposite and it is compressive in nature)

Resolving vertically $\Sigma Y = 0$

$S_1 \sin 30 = 2P + S_2 \sin 30$

$\Rightarrow S_1 = \frac{2P + S_2/2}{\sin 30} = (4P + S_2) \quad — (2)$

Now resolving horizontally $\Sigma X = 0.$

$S_1 \cos 30 + S_2 \cos 30 = 1.73P$

$\Rightarrow (4P + S_2) \times \frac{\sqrt{3}}{2} + S_2 \frac{\sqrt{3}}{2} = 1.73P$

$\Rightarrow 2\sqrt{3}P + \frac{\sqrt{3}}{2} S_2 + \frac{\sqrt{3}}{2} S_2 = 1.73P$

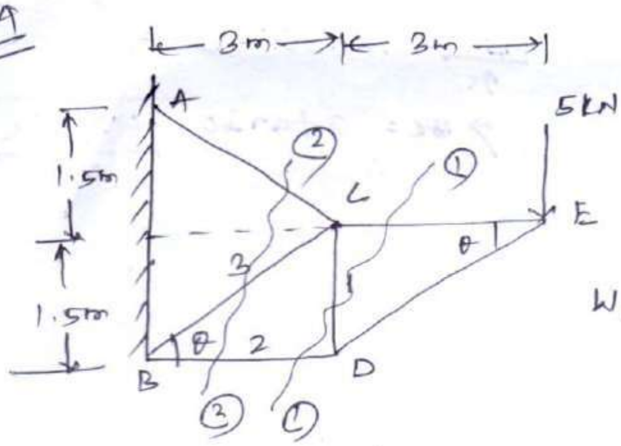
$\Rightarrow \frac{\sqrt{3}}{\cancel{2}} S_2 = 1.73P - 2\sqrt{3}P$

$= -1.73P$

$\Rightarrow S_2 = \frac{-1.73P}{\sqrt{3}} = \boxed{-P}$ (-ve sign indicates the direction is opposite and it is compressive)

Now $S_1 = 4P - P = \boxed{3P}$ (tension)

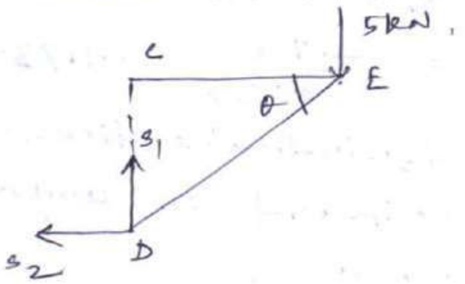
Q.4



Using method of sections, find axial forces in each bar 1, 2 and 3 of the plane truss.

We have $\tan\theta = \left(\frac{1.5}{3}\right) \Rightarrow \boxed{\theta = 26.56^\circ}$

considering section 1-1



Resolving vertically, $\Sigma Y = 0$

$\boxed{S_1 = 5kN}$ (tension)

Now taking moment about C

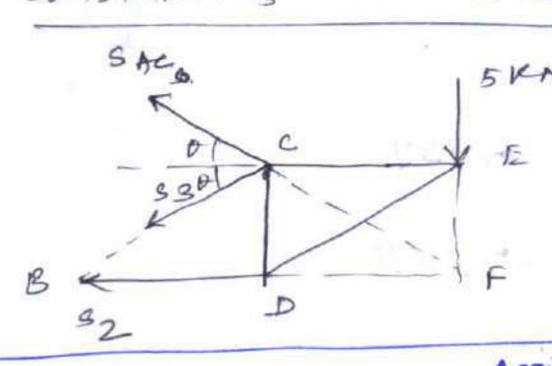
$S_2 \times 1.5 + 5 \times 3 = 0$

$\Rightarrow S_2 = -10kN$

-ve sign indicates direction should have been opposite

$\boxed{S_2 = 10kN}$ (compression)

Considering section 2-2

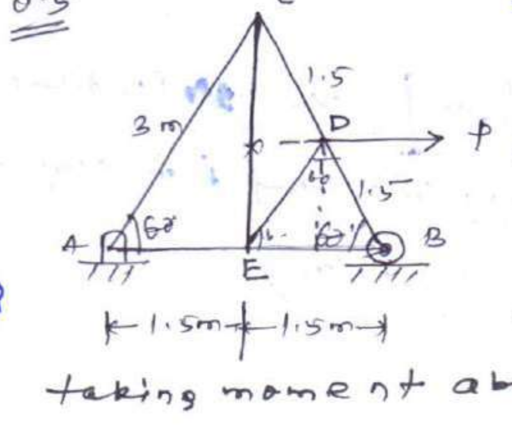


Taking moment about F

$\Sigma M_F = 0$

$\Rightarrow \boxed{S_3 = 0}$

Q.5



Assignment

Using method of joint and method of section find the axial force in the bar x.

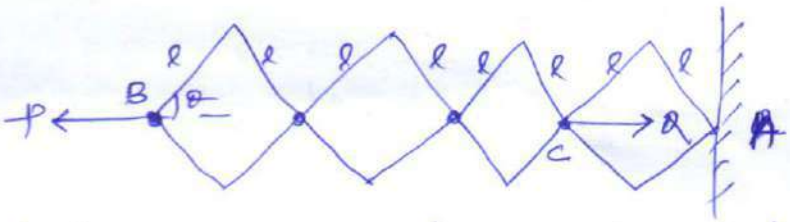
Method of Joint

considering the whole structure and taking moment about A $\Sigma M_A = 0$.

$R_B \times 3 = P \times 1.5 \sin 60$

$\Rightarrow \boxed{R_B = \frac{\sqrt{3}}{4} P.}$

Q.1 (6.3) Calculate the relation betn active forces P and Q for equilibrium of system of bars. The bars are so arranged that they form identical rhombuses.



Let l = length of each side of bar.

θ = angle made by each side of the rhombus

Distance of P from fixed point A = $6l\cos\theta$

" Q " $= 2l\cos\theta$

Let the virtual displacement of P is B-B'

$B\text{-}B' = dx_1 = \frac{d}{d\theta}(6l\cos\theta)\,d\theta = -6l\sin\theta\, d\theta$

Similarly the virtual displacement of Q is C-C'

$= dx_2 = -2l\sin\theta\, d\theta$

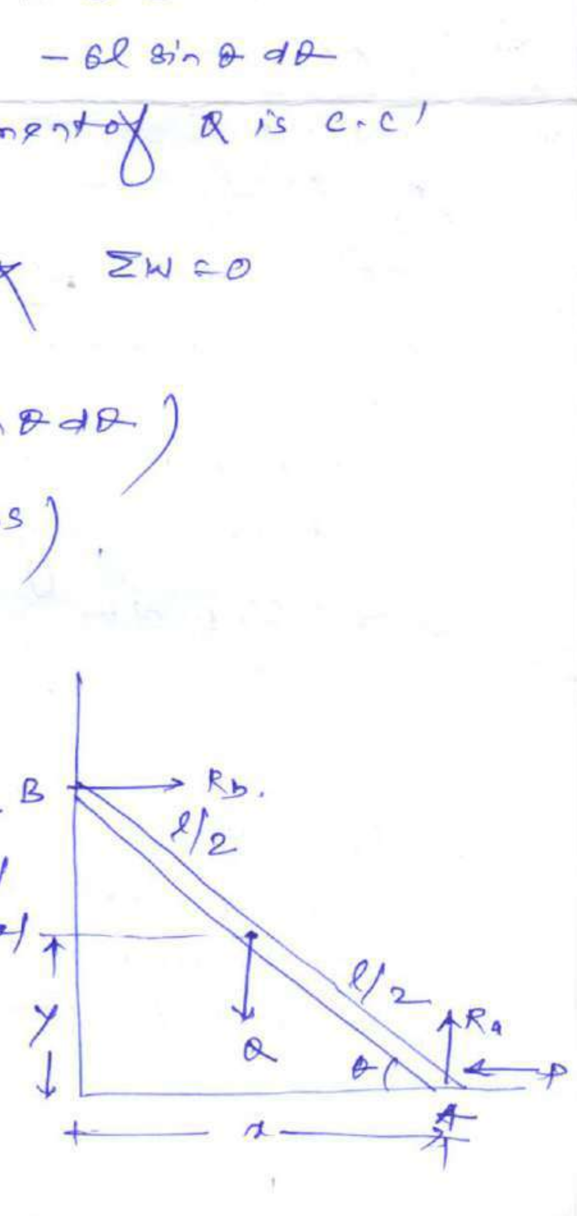
Applying principle of virtual work $\Sigma W = 0$

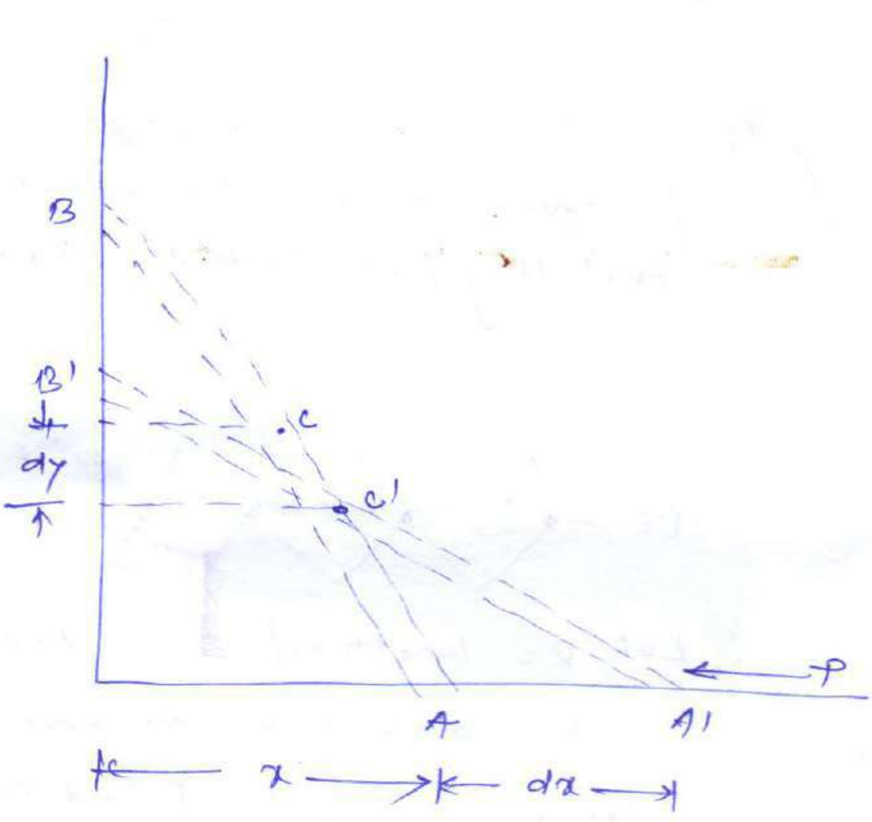
$P \cdot dx_1 = Q \cdot dx_2$

$\Rightarrow P\cdot(6l\sin\theta\, d\theta) = Q(2l\sin\theta\, d\theta)$

$\Rightarrow \boxed{P = \frac{Q}{3}}$ (Ans).

Q.2 A prismatic bar AB of length l and wt. Q stands in a vertical plane and is supported by smooth surfaces at A and B. Using principle of virtual work find the magnitude of horizontal force P applied at A if the bar is in equilibrium.





Let the horizontal distance of P from D is x

$x = l\cos\theta$

$AA' = dx = -l\sin\theta\, d\theta$

vertical distance of Q from D is y

$y = \frac{l}{2}\sin\theta$

$CC' = dy = \frac{l}{2}\cos\theta\, d\theta$

Normal reactions R_a and R_b have no work along the planes.

Applying principle of virtual work $\Sigma W = 0$

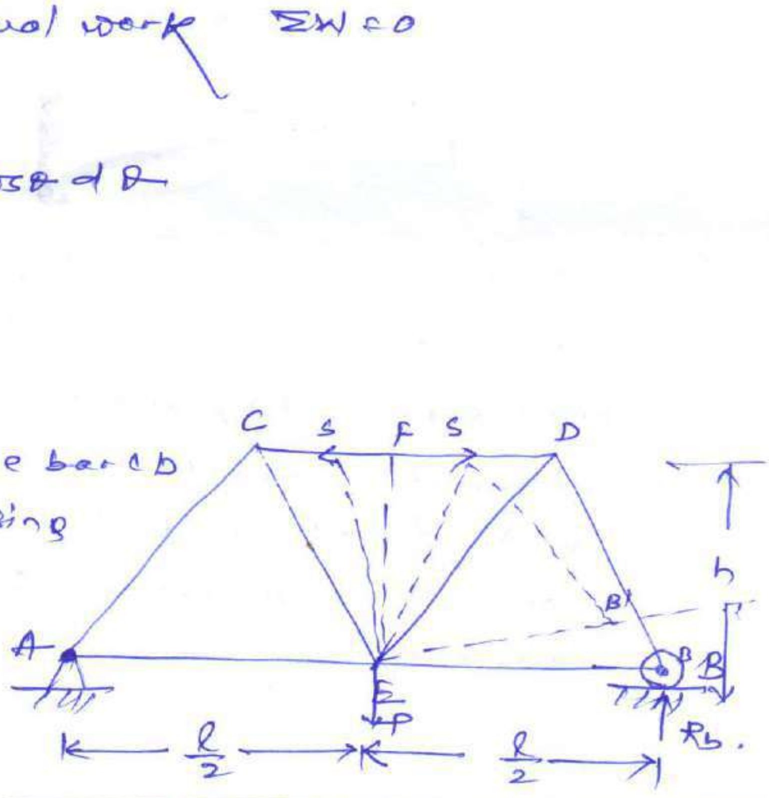
$P\,dx = Q\,dy$

$Pl\sin\theta\, d\theta = Q\frac{l}{2}\cos\theta\, d\theta$

$\Rightarrow \boxed{P = \frac{Q}{2}\cdot\cot\theta}$

<u>Q.3</u> (6.14)

find axial forces in the bar CD of the simple truss by using method of virtual work.



13/11/14 (2)

Let S be the compressive force in bar CD.

consider the part EBDF of the truss under the action of force R_b, P and S

Keeping E fixed and giving EB an angular displacement $d\alpha$

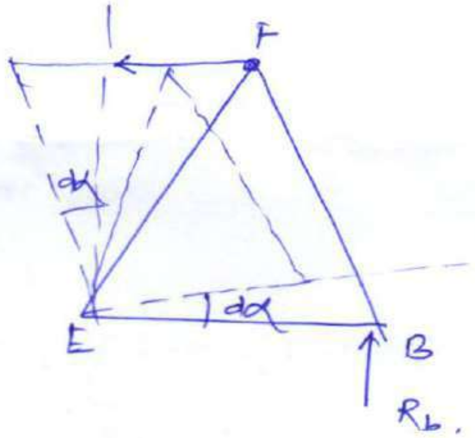
$\Sigma W = 0$.

$R_b \times BB' = S \times FF'$

$BB' = \frac{l}{2} d\alpha$

$FF' = h\, d\alpha$

$R_b \times \frac{l}{2} d\alpha = S \times h\, d\alpha$



$\Rightarrow \boxed{S = \frac{R_b l}{2h}}$ — (1)

Now considering whole frame as equilibrium body $\Sigma y = 0$.

$R_a + R_b = P$.

$R_b . l = P . \frac{l}{2} \Rightarrow \boxed{R_b = \frac{P}{2}}$ — (2)

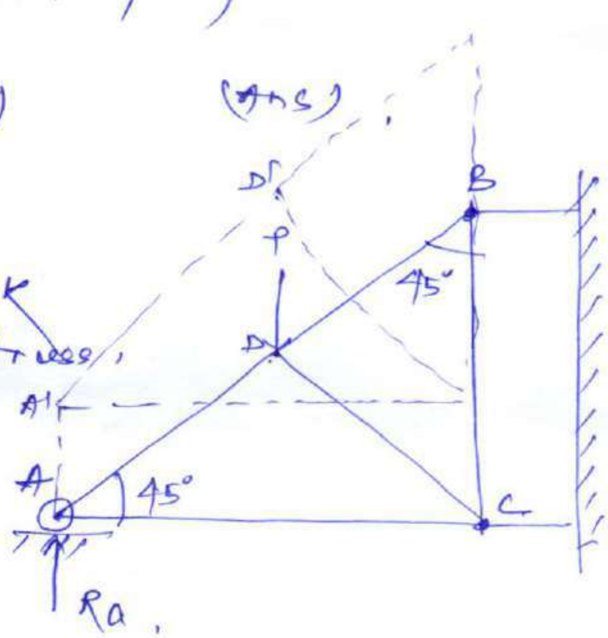
Substituting the value of R_b in eq. (1)

$\boxed{S = \frac{Pl}{4h}}$ — (3) (Ans)

Q.4 (6.15)

Using principle of virtual work find reactions R_a for the truss,

Let the truss is virtual displaced by an amount dy



$\Sigma W = 0$.

$R_a \times AA' = P \times DD'$

where $AA' = DD' = dy$

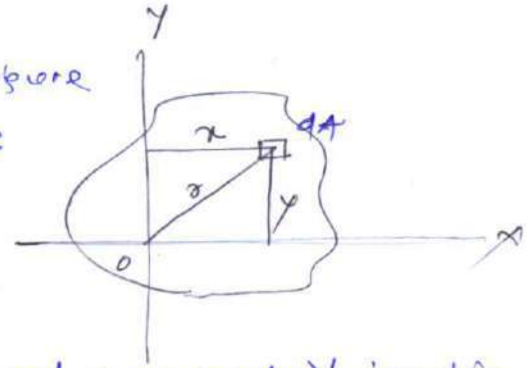
$\Rightarrow \boxed{R_a = P}$

modipada to bigbazar near jagnnath mandir righthand side

Moment of Inertia of Plane figures 02/12/14

The moment of inertia of any plane figure with respect to x and y axes in its plane are expressed as

$I_x = \int y^2 dA \quad I_y = \int x^2 dA$



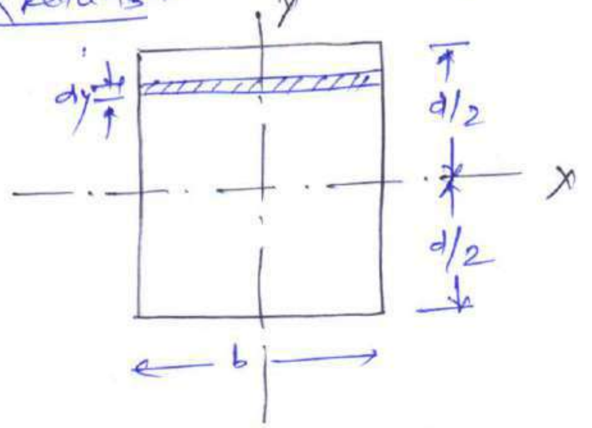
- Ixx and Iyy are also known as second moment of ~~inertia~~ area about the axes as it is distance is squared from corresponding axis.

unit

Unit of moment of inertia of area is expressed as m^4 or mm^4.

Moment of inertia of Plane figures:-

(i) Rectangle



considering a rectangle of width b and depth d.
Moment of inertia about Centroidal axis x-x parallel to the short side i.e b
Now considering an elementary strip of width dy

Moment of inertia of the elemental strip about centroidal axis xx is

$$I_{xx} = y^2 dA = y^2 b\,dy$$

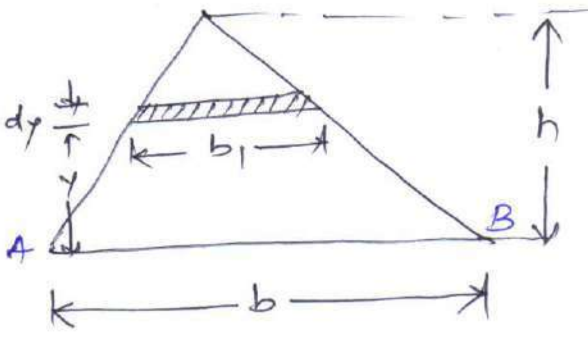
so moment of inertia of entire ~~figure~~ area

$$I_{xx} = \int_{-d/2}^{d/2} y^2 b\,dy = b\left|\frac{y^3}{3}\right|_{-\frac{d}{2}}^{d/2} = b\left[\frac{d^3}{24} + \frac{d^3}{24}\right]$$

$$\Rightarrow \boxed{I_{xx} = \frac{bd^3}{12}}$$

Similarly ~~moment of~~ $\boxed{I_{yy} = \frac{db^3}{12}}$

(ii) Triangle :- (Moment of inertia of a triangle about it's b



Consider a small elementary strip at a distance y from the base of thickness dy. Let dA is the area of strip

$dA = b_1 dy$

And $b_1 = \frac{(h-y)}{h} \times b$.

Moment of inertia of strip about base AB

$= y^2 dA = y^2 b_1 dy$

$= y^2 \frac{(h-y)}{h} . b\,dy$

$\therefore$ Moment of inertia of the triangle about AB

$$I_{AB} = \int_0^h \frac{y^2(h-y)\,b\,dy}{h} = \int_0^h \left(y^2 - \frac{y^3}{h}\right) b\,dy$$

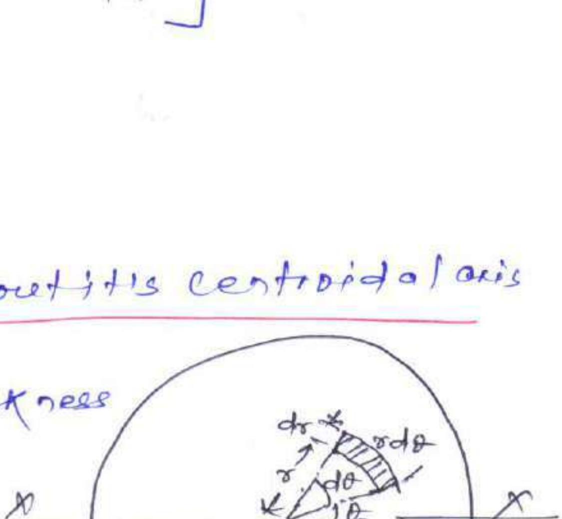
$$= b\left[\frac{y^3}{3} - \frac{y^4}{4h}\right]_0^h = b\left[\frac{h^3}{3} - \frac{h^4}{4h}\right]$$

$$= b\left[\frac{h^3}{3} - \frac{h^3}{4}\right] = \frac{bh^3}{12}$$

$$\Rightarrow \boxed{I_{AB} = \frac{bh^3}{12}}$$

(iii) Moment of inertia of a circle about it's centroidal axis

Considering an elementary strip of thickness dr, the side of strip $r\,d\theta$.



moment of inertia of strip about xx

$= y^2 dA$

$= (r\sin\theta)^2 \, r\,d\theta\,dr$

$= r^3 \sin^2\theta \, d\theta\,dr$

$\therefore$ Moment of inertia of circle about xx axis

$$I_{xx} = \int^R \int^{2\pi} r^3 \sin^2\theta \, d\theta\, dr$$

$$= \int_0^R \int_0^{2\pi} r^3 \left(\frac{1-\cos 2\theta}{2}\right) d\theta\, dr$$

$$= \int_0^R \frac{r^3}{2}\left[\theta - \frac{\sin 2\theta}{2}\right]_0^{2\pi} dr$$

$$= \int_0^R \frac{r^3}{2}\left(2\pi - \frac{\sin 4\pi}{2}\right) dr$$

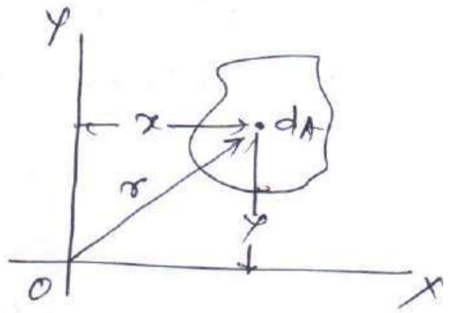
$$= \left[\frac{r^4}{8}\right]_0^R \left[2\pi - 0\right]$$

$$= \frac{R^4}{8} 2\pi = \frac{\pi R^4}{4}$$

$$\Rightarrow \boxed{I_{xx} = \frac{\pi R^4}{4} = \frac{\pi D^4}{64}} \qquad \left(\because R = \frac{D}{2}\right)$$

Polar moment of inertia:-

Moment of inertia about an axis perpendicular to the plane of area is called polar moment of inertia it may denoted as J or I_{zz}



$$\boxed{I_{zz} = \Sigma r^2 dA}$$

Radius of Gyration:-

Radius of gyration may be defined by a relation

$$\boxed{K = \sqrt{\frac{I}{A}}}$$

where K = radius of gyration

I = moment of inertia

A = cross-sectional area

so, we can have the following relations

$$\boxed{\begin{aligned} K_{xx} &= \sqrt{\frac{I_{xx}}{A}} \\ K_{yy} &= \sqrt{\frac{I_{yy}}{A}} \\ K_{AB} &= \sqrt{\frac{I_{AB}}{A}} \end{aligned}}$$

Theorems of Moment of inertia

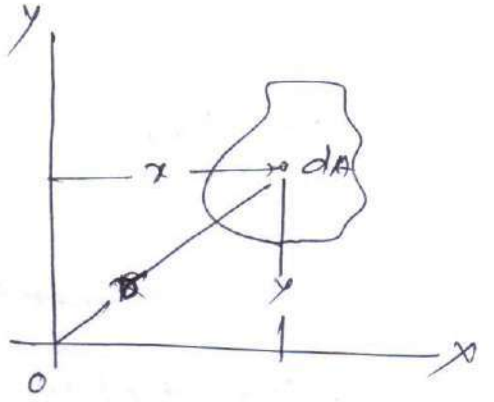
There are two theorems of moment of inertia

(a) Perpendicular axis theorem

(b) parallel axis theorem.

Perpendicular axis theorem:-

Moment of inertia of an area about an axis ⊥r to it's plane at any point O is equal to the sum of moments of inertia about any two mutually perpendicular axis through the same point O and lying in the plane of area.



$$I_{zz} = I_{xx} + I_{yy}$$

$$I_{zz} = \Sigma r^2 dA$$

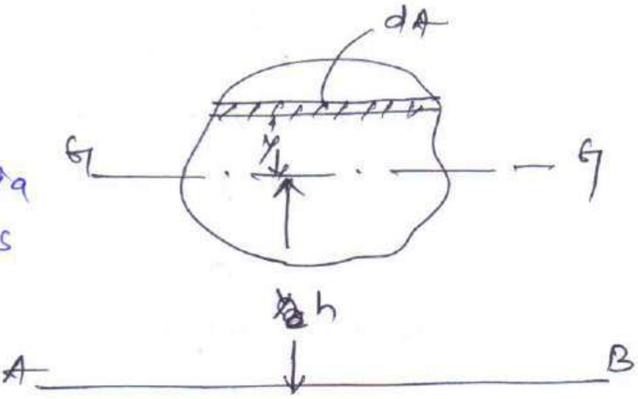
$$= \Sigma(x^2+y^2)\, dA$$

$$= \Sigma x^2 dA + \Sigma y^2 dA$$

$$\Rightarrow \boxed{I_{zz} = I_{xx} + I_{yy}}$$

Parallel axis theorem:-

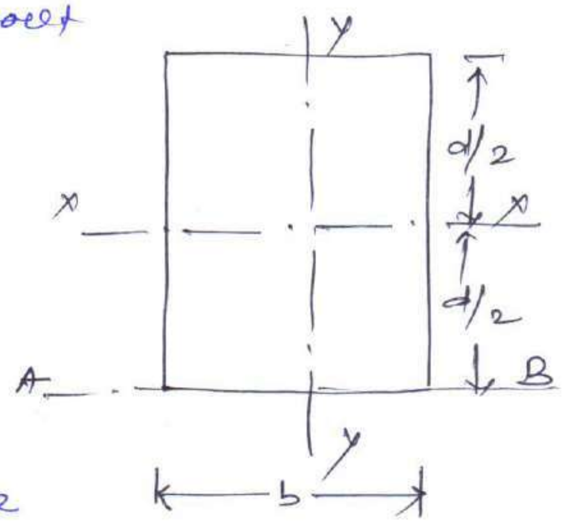
Moment of inertia about an axis in the plane of an area is equal to the sum of moment of inertia about a parallel centroidal axis and the product of area and square of the distance betn the two parallel axes.



$$\boxed{I_{AB} = I_{GG} + Ah^2}$$

Moment of inertia of standard sections:-

i) Moment of inertia of a rectangle about it's centroidal axis xx



$$I_{xx} = \frac{bd^3}{12}$$

Similarly moment of inertia about it's centroidal axis yy

$$I_{yy} = \frac{db^3}{12}$$

Now moment of inertia of rectangle about it's base AB can be obtained by applying parallel axis theorem

$$I_{AB} = I_{xx} + Ah^2$$

$$= \frac{bd^3}{12} + (bd)\left(\frac{d}{2}\right)^2$$

$$= \frac{bd^3}{12} + \frac{bd^3}{4}$$

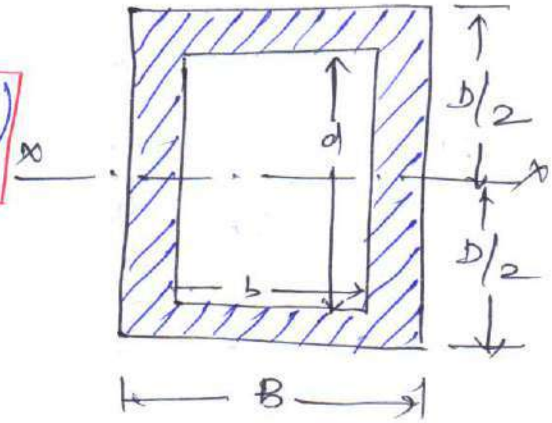
$$= \frac{3bd^3 + bd^3}{12} = \frac{bd^3}{3}$$

$$\Rightarrow \boxed{I_{AB} = \frac{bd^3}{3}}$$

(ii) Moment of inertia of a hollow rectangular section:-

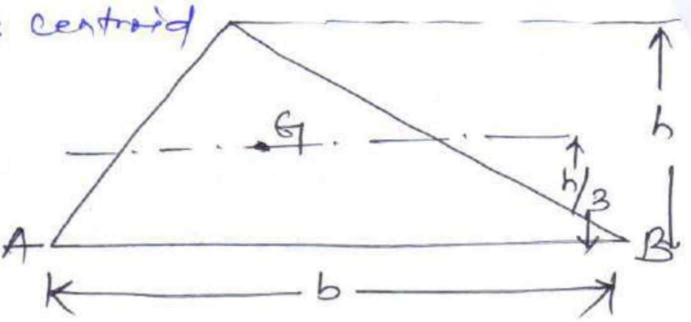
Moment of inertia of hollow rectangular section

$$\boxed{I_{xx} = \frac{BD^3}{12} - \frac{bd^3}{12} = \frac{1}{12}\left(BD^3 - bd^3\right)}$$



(iii) Moment of inertia of triangle about it's ~~base~~ centroidal

Moment of inertia of triangle about it's base = moment of inertia about its centroid $+ Ah^2$ (using parallel axis theorem)



$\Rightarrow I_{AB} = I_{xx} + Ah^2$

$$\Rightarrow \frac{bh^3}{12} = I_{xx} + \frac{1}{2} b \times h \times \left(\frac{h}{3}\right)^2$$

$$= I_{xx} + \frac{bh^3}{18}$$

$$\Rightarrow I_{xx} = \frac{bh^3}{12} - \frac{bh^3}{18} = \frac{3bh^3 - 2bh^3}{36} = \frac{bh^3}{36}$$

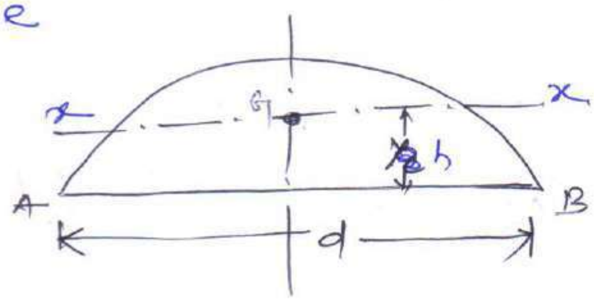
$$\Rightarrow \boxed{I_{xx} = \frac{bh^3}{36}}$$

(iv) Moment of inertia of semicircle

(a) about diametral axis

Moment of inertia of semicircle about $AB = \frac{1}{2} \frac{\pi d^4}{64}$

$$= \boxed{\frac{\pi d^4}{128}}$$



(b) about centroidal axis xx

$$\boxed{h = \frac{4R}{3\pi} = \frac{2d}{3\pi}}$$

area $A = \frac{1}{2} \frac{\pi d^2}{4} = \frac{\pi d^2}{8}$

Using parallel axis theorem

$I_{AB} = I_{xx} + Ah^2$

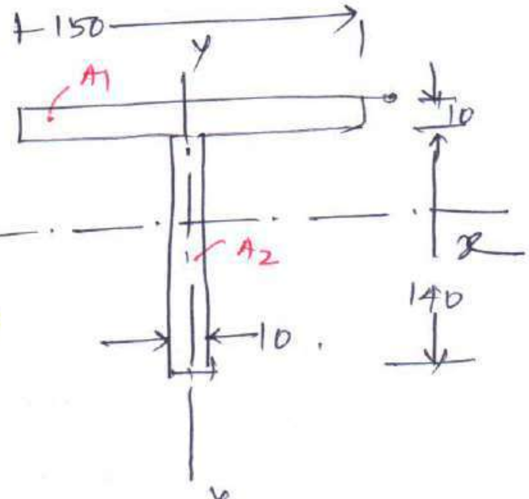
$$\Rightarrow \frac{\pi d^4}{128} = I_{xx} + \frac{\pi d^2}{8} \times \left(\frac{2d}{3\pi}\right)^2 =$$

$$\Rightarrow \frac{\pi d^4}{128} = I_{xx} + \frac{\pi d^2}{8} \times \frac{4d^2}{9\pi^2}$$

$$= I_{xx} + \frac{d^4}{18\pi}$$

$$\Rightarrow \boxed{I_{xx} = \left(\frac{\pi d^4}{128} - \frac{d^4}{18\pi}\right)}$$

Moment of inertia of composite figures:-



Q.1 Determine the moment of inertia of the composite section about an axis passing through the centroidal axis. Also determine MI about axis of symmetry and radius of gyration.

Soln Dividing the composite area into A_1 and A_2

$A_1 = 150 \times 10 = 1500 \text{ mm}^2$

$A_2 = 140 \times 10 = 1400 \text{ mm}^2$

Distance of centroid from base of the composite figure

$$\bar{y} = \frac{A_1 y_1 + A_2 y_2}{(A_1 + A_2)} = \frac{1500 \times 145 + 1400 \times 70}{2900}$$

$$= 108.79 \text{ mm}$$

Moment of inertia of the area about xx axis

$$I_{xx} = \left\{\frac{150 \times 10^3}{12} + 1500 \times (145 - 108.79)^2\right\} + \left\{\frac{10 \times 140^3}{12} + 1400 \times (108.79 - 70)^2\right\}$$

$$= (12500 + 1966746.15) + (2286666.667 + 2106529.74)$$

$$= 6372442.557 \text{ mm}^4$$

Similarly

$$I_{yy} = \frac{10 \times 150^3}{12} + \frac{140 \times 10^3}{12} = 2812500 + 11666.66667$$

$$= 2824166.667 \text{ mm}^4$$