Radius of gyration $K = \sqrt{\frac{I}{A}}$

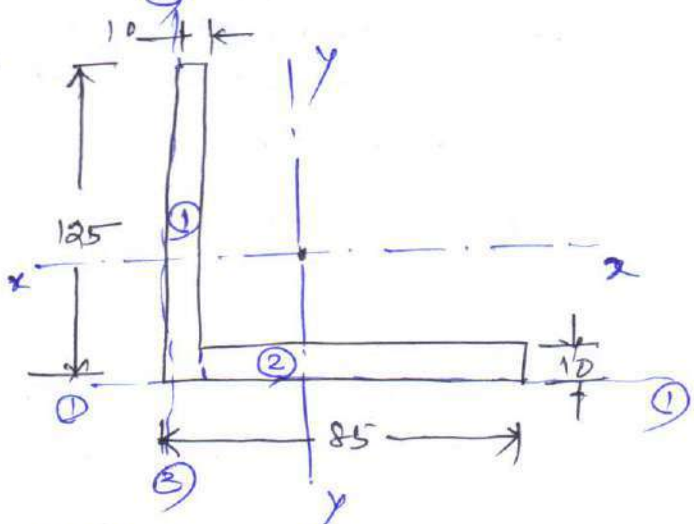
so $K_{xx} = \sqrt{\frac{I_{xx}}{A}}$

$= \sqrt{\frac{6372442.5}{2900}} = 46.87$ mm

Similarly $k_{yy} = \sqrt{\frac{I_{yy}}{A}} = \sqrt{\frac{2824166.667}{2900}}$

$= 31.206$ mm (Ans)

Q.2 Determine the MI of L-section about its centroidal axes parallel to the legs. Also find the polar moment of inertia.



We have $A_1 = 125 \times 10 = 1250$ mm^2

$A_2 = 75 \times 10 = 750$ mm^2

Total area $A_1 + A_2 = 2000$ mm^2

Distance of centroid from 1-1 axis

$\bar{y} = \frac{A_1 y_1 + A_2 y_2}{A_1 + A_2}$

$= \frac{1250 \times 62.5 + 750 \times 5}{2000} = 40.9375$ mm

Distance of centroidal axis yy from 2-2 axis

$\bar{x} = \frac{A_1 x_1 + A_2 x_2}{A_1 + A_2}$

$= \frac{1250 \times 5 + 750 \times \left(\frac{75}{2} + 10\right)}{2000}$

$= \frac{1250 \times 5 + 750 \times 47.5}{2000} = 20.93$ mm

Moment of inertia about xx axis

$I_{xx} = \left\{\frac{10 \times 125^3}{12} + 1250 \times (62.5 - 40.9375)^2\right\}$

$+ \left\{\frac{75 \times 10^3}{12} + 750 \times (40.9375 - 5)^2\right\}$

$= (1627604.167 + 581176.7578) + (6250 + 968627.9297)$

$= \boxed{3183658.854 \text{ mm}^4}$

Similarly MI about yy centroidal axis

$$I_{yy} = \left\{\frac{125\times10^3}{12} + 1250\times(20.93-5)^2\right\} + \left\{\frac{10\times75^3}{12} + 750\times(47.5-20.93)^2\right\}$$

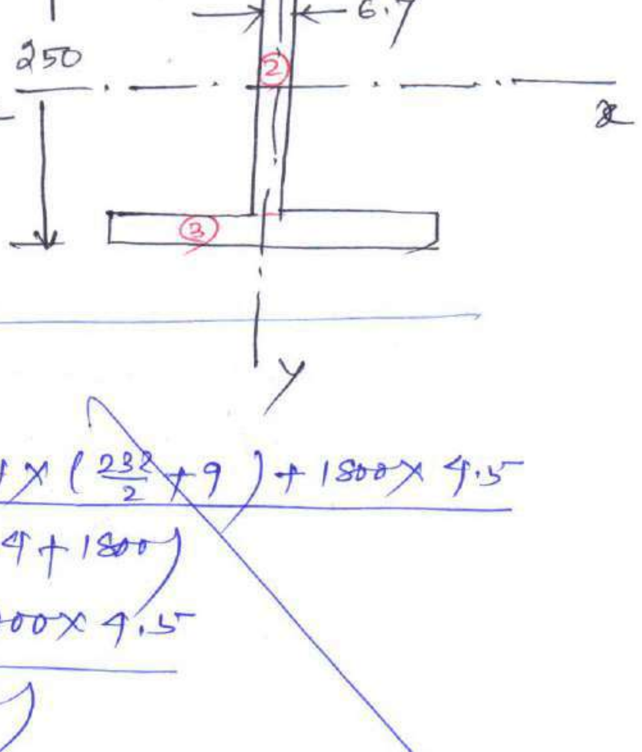
$$= (10416.66667+317206.125)+(351562.5+529473.675)$$

$$= \boxed{1208658.967\ mm^4}$$

Polar moment of inertia $I_{zz} = I_{xx} + I_{yy}$

$$= \boxed{4392317.821\ mm^4} \quad (Ans)$$

Q.3 Determine the MI of the symmetrical I section about its centroidal axes x-x and y-y. Also determine the polar moment of inertia of the section.



We have from the figure

$A_1 = 200\times9 = 1800\ mm^2$

$A_2 = 232\times6.7 = 1554.4\ mm^2$

$A_3 = 200\times9 = 1800\ mm^2$

~~Position of centroidal axis xx from base~~

$$\bar{y} = \frac{A_1y_1+A_2y_2+A_3y_3}{(A_1+A_2+A_3)}$$

$$= \frac{1800\times(4.5+232+9)+1554.4\times(\frac{232}{2}+9)+1800\times4.5}{(1800+1554.4+1800)}$$

$$= \frac{1800\times245.5+1554.4\times125+1800\times4.5}{(1800+1554.4+1800)}$$

$$= 125\ mm.$$

$$\bar{x} = \frac{A_1x_1+A_2x_2+A_3x_3}{(A_1+A_2+A_3)}$$

$$= \frac{1800\times100+1554.4\times96.65+1800\times100}{(1800+1554.4+1800)} = 98.98$$

MI about xx axis

$$I_{xx} = \left\{ \frac{200\times 9^3}{12} + 1800\times(125-4.5)^2 \right\} + \left\{ \frac{6.7\times 232^3}{12} + 1554.4\times(\right.$$

$$+ \left\{ \frac{200\times 9^3}{12} + 1800\times(125-4.5)^2 \right\}$$

$$= (12150 + 26136450) + (6972002.133 + 0)$$

$$+ (12150 + 26136450)$$

$$= 26148600 + 6972002.133 + 26148600$$

$$= \boxed{59269202.13 \text{ mm}^4}$$

MI about yy axis

$$I_{yy} = \frac{9\times 200^3}{12} + \frac{232\times 6.7^3}{12} + \frac{9\times 200^3}{12}$$

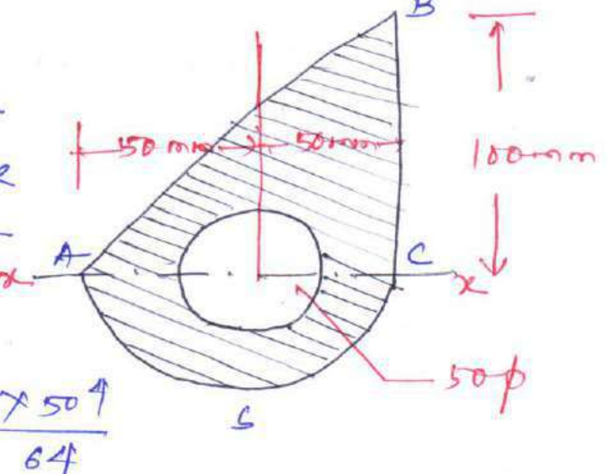
$$= 6000000 + 5814.751 + 6000000$$

$$= \boxed{12005814.75 \text{ mm}^4}$$

Polar moment of inertia $I_{zz} = I_{xx} + I_{yy}$

$$= \boxed{71275016.88 \text{ mm}^4}$$

<u>Q.4</u> Calculate the moment of inertia of the shaded area about xx axis.



MI of the shaded section about xx = MI of triangle ABC about xx + MI of semicircle ACS about xx − MI of circle

$$= \frac{100\times 100^3}{12} + \frac{\pi\times 100^4}{128} - \frac{\pi\times 50^4}{64}$$

$$= 8333333.333 + 2454369.261 - 306796.1576$$

$$= 10480906.44 \text{ mm}^4$$

$$= \boxed{1.048\times 10^7 \text{ mm}^4}$$

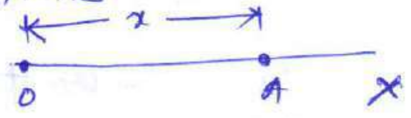
-: Rectilinear Translation :-

In statics, it was considered that the rigid bodies are at rest. In dynamics, it's considered that they are in motion. Dynamics is commonly divided into two branches. Kinematics and kinetics.

- In, kinematics we are concerned with space time relationship of a given motion of a body and not at all with the forces that cause the motion.
- In kinetics we are concerned with finding the kind of motion that a given body or system of bodies will have under the action of given forces or with what forces must be applied to produce a desired motion.

Displacement

From the fig. displacement of a particle can be defined by its x-coordinate, measured from the fixed reference point O.



- When the particle is to the right of fixed point O, this displacement can be considered positive and when it's towards the ~~rig~~ left hand side it's considered as negative.

General displacement time equation

$$x = f(t) \qquad -(1)$$

where $f(t)$ = function of time.

for example $\boxed{x = c + bt}$

In the above equation c, represents the initial displacement at $t = 0$, while the constant b shows the rate at which displacement increases. It is called uniform rectilinear motion.

Second example is $\boxed{x = \frac{1}{2} gt^2}$

where x is proportional to the square of time.

Velocity

Acceleration

Example The rectilinear motion of a particle is defined by the displacement - time equation $x = x_0 - v_0 t + \frac{1}{2} a t^2$ construct displacement - time and velocity diagram for this motion and find the displacement and velocity at time $t_2 = 2s$. $x_0 = 750$ mm, $v_0 = 500$ mm/s
$a = 0.125$ m/s^2

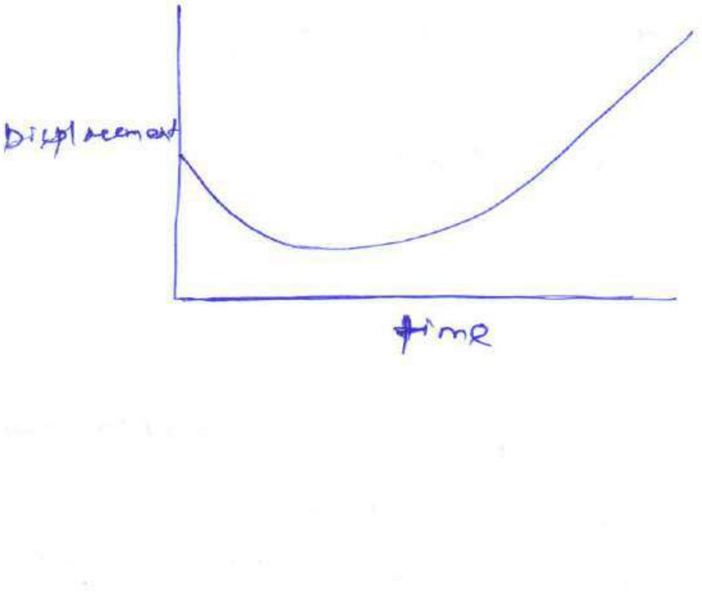
The equation of motion is

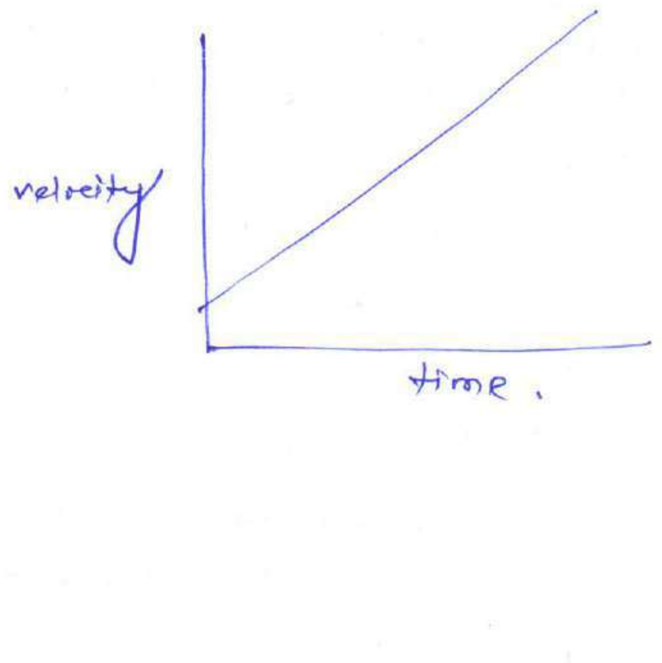
$$x = x_0 - v_0 t + \frac{1}{2} a t^2 \quad \text{— (1)}$$

$$v = \frac{dx}{dt} = -v_0 + a t \quad \text{— (2)}$$

substituting x_0, v_0 and a in equation (1)

~~x = 750 - 500~~





Q-1 A bullet leaves the muzzle of a gun with velocity $v = 750\ m/s$. Assuming constant acceleration from breech to muzzle find time t occupied by the bullet in travelling through gun barrel which is 750 mm long.

initial velocity of bullet $u = 0$
final velocity of bullet $v = 750\ m/s$.
total distance $s = 0.75\ m$.
$t = ?$

We have $v^2 - u^2 = 2as$,

$$\Rightarrow v^2 = 2as \Rightarrow a = \frac{v^2}{2s} = \frac{750^2}{2 \times 0.75}$$
$$= 375000\ m/sec^2$$

Again $v = u + at$

$$\Rightarrow 750 = 375000 \times t$$
$$\Rightarrow t = \frac{750}{375000} = \boxed{0.002\ sec.}$$

Q-2 A stone is dropped into well and falls vertically with constant acceleration $g = 9.81\ m/sec^2$. The sound of impact of stone on the bottom of well is heared after 6.5 sec. If velocity of sound is 336 m/s. How deep is the well?

$V = 336\ m/sec$
let s = depth of well
t_1 = time taken by the stone into the well
t_2 = time taken by the sound to be heared.
total time $t = (t_1 + t_2) = 6.5\ sec.$

Now $s = ut + \frac{1}{2} gt^2$

$$\Rightarrow s = 0 + \frac{1}{2} gt^2$$
$$\Rightarrow t_1 = \sqrt{\frac{2s}{g}}$$

When the sound travels with uniform velocity
$s = Vt_2$ or $t_2 = \frac{s}{V}$

$\sqrt{\frac{2s}{g}} + \frac{s}{V} = 6.5$

$\Rightarrow \frac{2s}{g} = \left(6.5 - \frac{s}{336}\right)^2$

$\Rightarrow 2s = 9.81\left(6.5 - \frac{s}{336}\right)^2$

$= 9.81\left(\frac{2184 - s}{336}\right)^2$

$= 0.0291\,(2184 - s)^2$

$= 0.0291\,(4769856 + s^2 - 4368s)$

$= 138802.809 + 0.0291s^2 - 127.1088s$

$\Rightarrow 0.0291s^2 - 129.1088s + 138802.809 = 0$

$\Rightarrow s =$

$0.2038s = 42.25 + 0.0000885s^2 - 0.0386s$

~~s = 174~~

$0.0000885s^2 - 0.1658s + 42.25 = 0$

$\boxed{s = 17.31\text{ m.}}$

A2 A rope AB is attached at B to a small block of negligible dimensions and passes over a pulley C so that it's free end A hangs 1.5 m above ground when the block rests on the floor. The end A of the rope is moved horizontally in a st. line by a man walking with a uniform velocity $v_0 = 3$ m/s. plot the velocity-time diagram
(b) find the time t required for the block to reach the pulley if h = 4.5 m, pully dimension are negligible.

A3 A particle starts from rest and moves along a st. line with constant acceleration a. If it acquires a velocity $v = 3$ m/s. after having travelled a distance $s = 7.5$ m. find magnitude of acceleration.

Principles of Dynamics:

Newton's law of motion!

First law! Everybody continues in its state of rest or of uniform motion in a straight line except in so far as it may be compelled by force to change that state.

Second Law !-

The acceleration of a given particle is proportional to the force applied to it and takes place in the direction of the straight line in which the force acts.

Third law To every action there is always an equal and contrary reaction or the mutual action's of any two bodies are always equal and oppositely directed.

General Equation of Motion of a Particle!

$$\boxed{ma = f}$$

Differential equation of Rectilinear motion!-

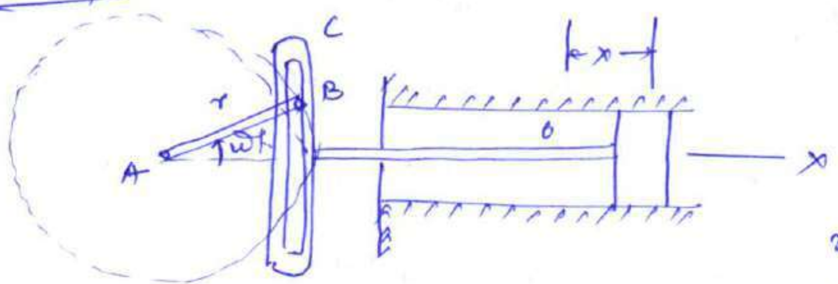
Differential form of equation for rectilinear motion can be expressed as

$$\boxed{\frac{W}{g}\ddot{x} = X}$$

where $\ddot{x}$ = acceleration

X = Resultant acting force.

Example



For the engine shown in fig., the combined wt. of piston and piston rod $W = 450N$, crank radius $r = 250mm$ and uniform speed of rotation $n = 120$ rpm. Determine the magnitude of resultant force acting in piston (a) at extreme position and (b) at the middle position piston has a simple harmonic motion representing displacement-time equation

$$x = r\cos\omega t \quad \text{---(1)}$$

$$\omega = \frac{2\pi n}{60} = \frac{2\pi\times120}{60} = 4\pi \text{ rad/s.}$$

$$\dot{x} = -r\omega\sin\omega t$$

$$\ddot{x} = -r\omega^2\cos\omega t \quad \text{---(2)}$$

Differential equation of motion

$$\frac{W}{g}\ddot{x} = X$$

$$\Rightarrow -\frac{W}{g}r\omega^2\cos\omega t = X$$

$$\Rightarrow X = -\frac{450}{9.81}\times0.25\,(4\pi)^2\cos(4\pi t)$$

for extreme position

$$\cos\omega t = -1$$

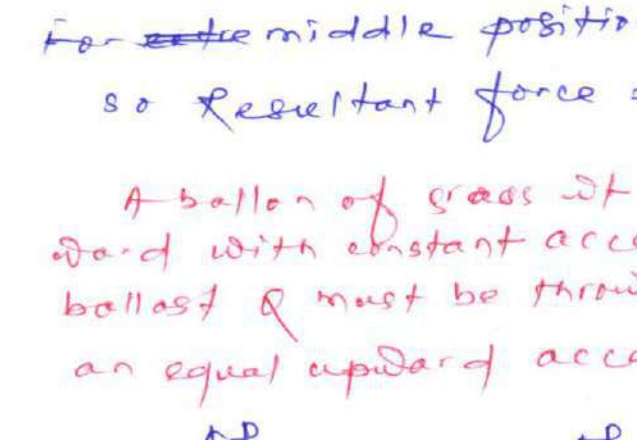
so $\boxed{X = 1810N.}$

For ~~extre~~ middle position $\cos\omega t = 0$.

so Resultant force = 0.

E-2 A ballon of gross wt W is falling vertically downward with constant acceleration a. What amount of ballast Q must be thrown out in order to give balloon an equal upward acceleration a.

P = buoyant force.



(i) considering 1st case when ballon is falling,

$$\frac{W}{g}a = W - P \quad \text{---(1)}$$

(ii) $$\frac{W-Q}{g}a = P-(W-Q) \quad \text{---(2)}$$

Eq(1) + Eq(2)

$$\frac{Q}{g}\cdot a = W+W-Q = 2W+Q$$

$$\Rightarrow Q\left(\frac{a}{g}-1\right) = 2W$$

$$\Rightarrow Q = \frac{2Wg}{(a-g)}$$

$$\frac{Wa}{g} = (W-P)$$

$$\frac{(W-Q)a}{g} = P-(W-Q)$$

$$\frac{Wa+(W-Q)a}{g} = W-\cancel{P}+\cancel{P}-(W-Q) = Q$$

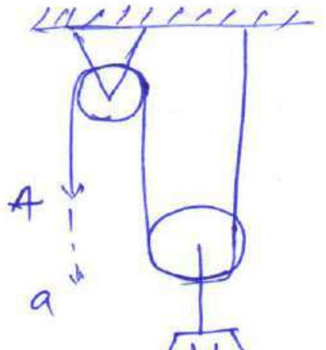
$$\Rightarrow \frac{Wa+Wa-Qa}{g} = Q$$

$$\Rightarrow 2Wa = Qg+Qa$$

$$\Rightarrow \boxed{Q = \frac{2Wa}{(g+a)}}$$

Q.1 A wt. $W = 4450N$ is supported in a vertical plane by string and pulleys arranged shown in fig. If the free end A of ~~oth~~ the string is pulled vertically downward with constant acceleration $a = 18\,m/s^2$ find tension S in the string.

Differential equation of motion for the system is



$$2S - W = \frac{W}{g}\times\frac{a}{2}$$

$$\Rightarrow 2S = W + \frac{Wa}{2g}$$

$$= \cancel{\frac{W}{2}\left(2+\frac{a}{2g}\right)}$$

$$= W\left(1+\frac{a}{2g}\right)$$

$$\Rightarrow S = \frac{W}{2}\left(1+\frac{a}{2g}\right)$$

$$= \frac{4450}{2}\left(1+\frac{18}{2\times 9.81}\right) = \boxed{4266.28\ N.}$$