$$\frac{Wa}{g} = (W-P)$$

$$\frac{(W-Q)a}{g} = P-(W-Q)$$

$$\frac{Wa+(W-Q)a}{g} = W-\cancel{P}+\cancel{P}-(W-Q) = Q$$

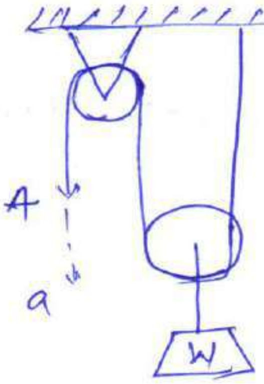
$$\Rightarrow \frac{Wa+Wa-Qa}{g} = Q$$

$$\Rightarrow 2Wa = Qg+Qa$$

$$\Rightarrow \boxed{Q = \frac{2Wa}{(g+a)}}$$

Q.1 A wt. W = 4450N is supported in a vertical plane by string and pulleys arranged shown in fig. If the free end A of the string is pulled vertically downward with constant acceleration $a = 18\ m/s^2$ find tension S in the string.

Differential equation of motion for the system is



$$2S - W = \frac{W}{g} \times \frac{a}{2}$$

$$\Rightarrow 2S = W + \frac{Wa}{2g}$$

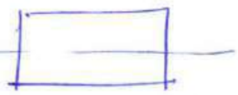
$$= \cancel{\frac{W}{2}\left(2+\frac{a}{2g}\right)}$$

$$= W\left(1+\frac{a}{2g}\right)$$

$$\Rightarrow S = \frac{W}{2}\left(1+\frac{a}{2g}\right)$$

$$= \frac{4450}{2}\left(1+\frac{18}{2\times 9.81}\right) = \boxed{4266.28\ N.}$$

Q.2 An elevator of gross wt W = 4450N starts to move upward direction with a constant acceleration and acquires a velocity v = 18m/s, after travelling a distance = 1.8m, find tensile force S in the cable during it's motion.

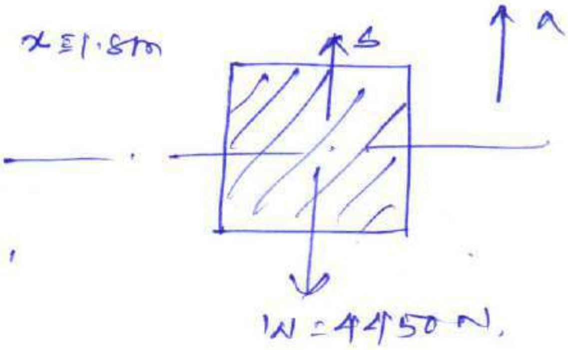


W = 4450N.

V = 18 m/s.

initial velocity u = 0

distance travelled x = 1.8 m,



$$S - W = \frac{W}{g}\cdot a$$

$$\Rightarrow S = W + \frac{W}{g}a = W\left(1 + \frac{a}{g}\right) \quad \text{---(1)}$$

Now applying equation of kinematics

$$v^2 - u^2 = 2as$$

$$\Rightarrow 18^2 - 0 = 2a \times 1.8$$

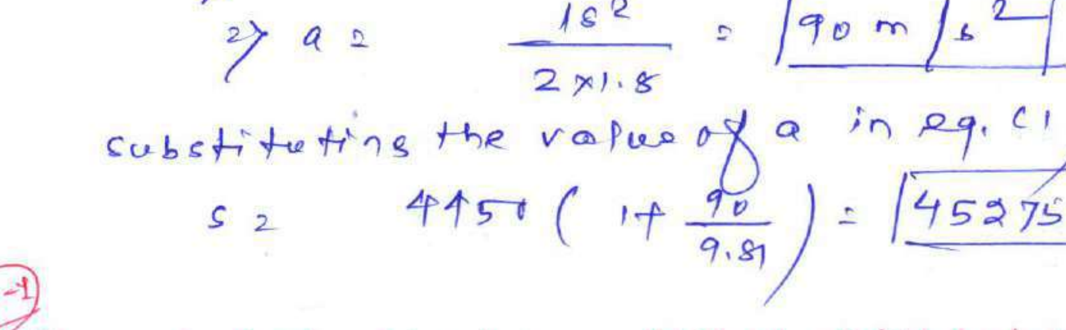
$$\Rightarrow a = \frac{18^2}{2 \times 1.8} = \boxed{90\ m/s^2}$$

substituting the value of a in eq. (1)

$$S = 4450\left(1 + \frac{90}{9.81}\right) = \boxed{45275.7\ N.}$$

(A-1)
Q.3 A train weighing 1870N without the locomotive starts to move with constant acceleration along a straight track and in first 60s acquires a velocity of 56 Kmph. Determine the tension S in draw bar both locomotive and train if the air resistance is 0.005 times the wt. of the train.

V = 56 Kmph = 15.56 m/s.



$S - F = \frac{W}{g} \cdot a$

$\Rightarrow S = 0.005W + \frac{Wa}{g}$ —(1)

from eq. of kinematics.

$v = u + at$

$\Rightarrow a = \left(\frac{15.56 - 0}{60}\right) = 0.26\ m/sec^2$

substituting the value of a in eq. (1)

$S = W\left(0.005 + \frac{a}{g}\right)$

$= 1870\left(0.005 + \frac{0.26}{9.81}\right) = \boxed{58.9\ kN.}$

A-2
Q. 1

A wt. W is attached to the end of a small flexible rope of dia. d = 6.25 mm. and is raised vertically by winding the rope on a reel. if the reel is turned uniformly at a rate of 2 rps. what will be the tension in rope.

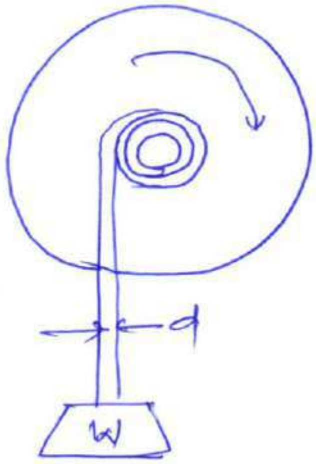
dia of rope d = 6.25 mm = 0.00625 m.

No of revolutions N = 2 rps.

let x = initial radius of reel.

t = time taken for N revolutions.

Net radius after t sec.



$R = [x + (Nt d)]$

Now mean velocity $v = R\omega$

$\omega = 2\pi N.$

$\therefore v = (x + Ntd)\, 2\pi N$

acceleration of rope $= a = \frac{dv}{dt}$

$a = \frac{d}{dt}\left[2\pi N x + 2\pi N^2 t d\right] = 2\pi N^2 d$

$S - W = \frac{W}{g} \cdot a \Rightarrow S = W + \frac{Wa}{g} = W\left(1 + \frac{a}{g}\right)$

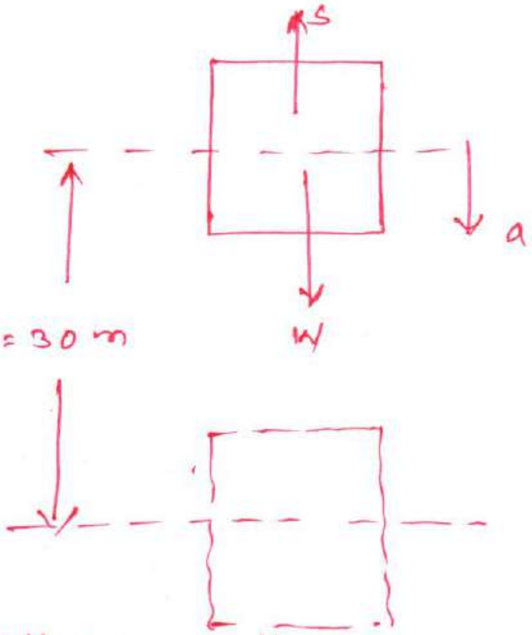
$\Rightarrow S = W\left(1 + \frac{2\pi N^2 d}{g}\right)$ ~~W(1+~~

$$\Rightarrow S = W\left(1 + \frac{2\pi \times 2^2 \times 0.00625}{9.81}\right)$$

$=$

Ass-3

Q.5 A mine case of wt $W = 8.9$ kN starts from rest and moves downward with constant acceleration travelling a distance $s = 30$ m in 10 sec. Find the tensile force in the cable.



Wt. of case $W = 8.9$ kN.

initial velocity $u = 0$.

distance travelled $s = 30$ m

time $t = 10$ sec.

$$s = \cancel{ut}^{0} + \frac{1}{2} a t^2$$

$$\Rightarrow 30 = \frac{1}{2} a \times 10^2$$

$$\Rightarrow t = \frac{60}{10^2} = \boxed{0.6\ m/sec^2}$$

Differential equation of rectilinear motion

$$W - S = \frac{W}{g} \cdot a$$

$$\Rightarrow S = W - \frac{W}{g} a = W\left(1 - \frac{a}{g}\right)$$

$$= 8.9\left(1 - \frac{0.6}{9.81}\right)$$

$$\Rightarrow \boxed{S = 8.35\ kN.}$$ (Ans)

D'Alembert's Principle

25/11/14 ①

Differential equation of motion (rectilinear) can be written as

$$X - m\ddot{x} = 0 \quad \text{—— (1)}$$

Where X = Resultant of all applied force in the direction of motion

m = mass of the particle

The above equation may be treated as equation of dynamic equilibrium. To express this equation, in addition to the real forces acting on the particle a fictitious force $m\ddot{x}$ is required to be considered. This force is equal to the product of mass of the particle and it's acceleration and directed in opposite direction, and is called the inertia force of the particle.

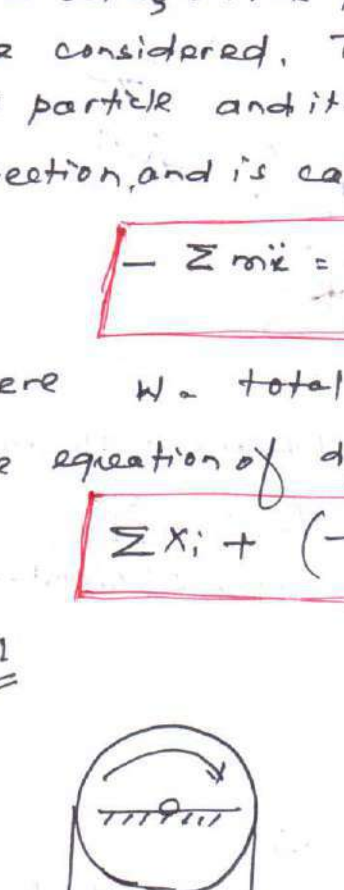
$$\boxed{-\Sigma m\ddot{x} = -\ddot{x}\,\Sigma m = -\frac{W}{g}\ddot{x}}$$

Where W = total weight of the body

so the equation of dynamic equilibrium can be expressed as:

$$\boxed{\Sigma X_i + \left(-\frac{W}{g}\ddot{x}\right) = 0} \quad \text{—— (2)}$$

Example 1



For the example shown considering the motion of pulley as shown by the arrow mark. where upward acceleration $\ddot{x}_2$ for W_2 and downward acceleration $\ddot{x}_1$ for W_1

- corresponding inertia forces and their direction are indicated by dotted line.
- By adding inertia forces to the real forces (such as W_1, W_2 and tension in strings S) we obtain, for each particle, a system of forces in equilibrium.

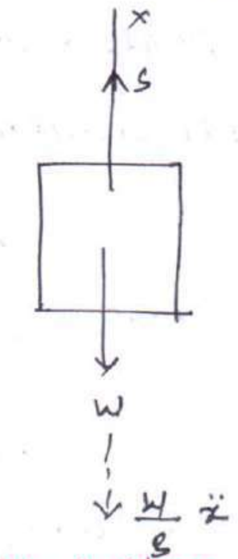
The equilibrium equation for the entire system without S

$$W_2 + m_2\ddot{x} = W_1 - m_1\ddot{x}$$

$$\Rightarrow (m_1+m_2)\ddot{x} = (W_1 - W_2) \Rightarrow \boxed{\ddot{x} = \frac{W_1-W_2}{(W_1+W_2)}\cdot g}$$

Example 2 A body is moving in upward direction by a rope.

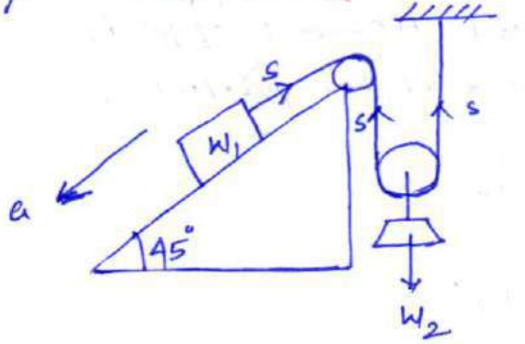
So the equation of dynamic equilibrium considering the real and inertia forces.

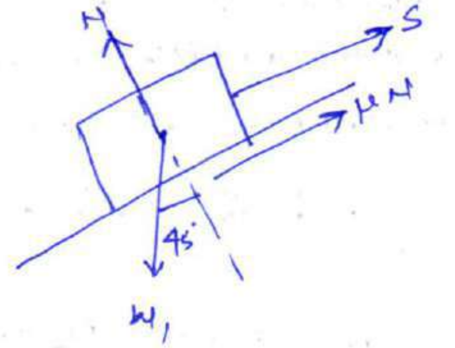


$S - W - \frac{W}{g} a = 0$, so tensile force in rope

$$\Rightarrow \boxed{S = W\left(1 + \frac{a}{g}\right)}$$

Q.1 Find tensions in the string during motion of the system if $W_1 = 900N$, $W_2 = 450N$. The μ betn the inclined plane and block $W_1 = 0.2$





When W_1 moves downward in the inclined plane with an acceleration a, then acceleration of $W_2 = \frac{a}{2}$

Considering dynamic equilibrium of W_1, from D'Alembert's principle

$$\left(W_1 \sin 45° - \mu N - S\right) - \frac{W_1}{g} a = 0$$

$$\Rightarrow \frac{W_1}{g} a = W_1 \sin 45° - \mu N - S$$

$$= W_1 \sin 45 - \mu W_1 \cos 45 - S$$

$$\Rightarrow a = \left(900 \times \frac{1}{\sqrt{2}} - 0.2 \times 900 \times \frac{1}{\sqrt{2}} - S\right) \frac{9.81}{900}$$

$$= (636.4 - 127.28 - S)\ 0.0109$$

$$\Rightarrow a = \cancel{5.549}\ 6.93676 - 1.387352 - 0.0109S \quad \text{— (1)}$$

Similarly ~~from~~ for weight W_2

$$2S - W_2 - \frac{W_2}{g}\frac{a}{2} = 0$$

$$\Rightarrow \cancel{\frac{W_2 a}{2g}} = W_2\left(1 + \frac{a}{2g}\right) = 2S$$

$$\Rightarrow \cancel{2}S = \frac{450}{2}\left(1 + \frac{a}{19.62}\right) = 225 + 11.46a \quad \text{— (2)}$$

substituting the value of S in eq. (1)

~~a = 5.549 − 2.4525 − 0.125a~~ ⇒ a =

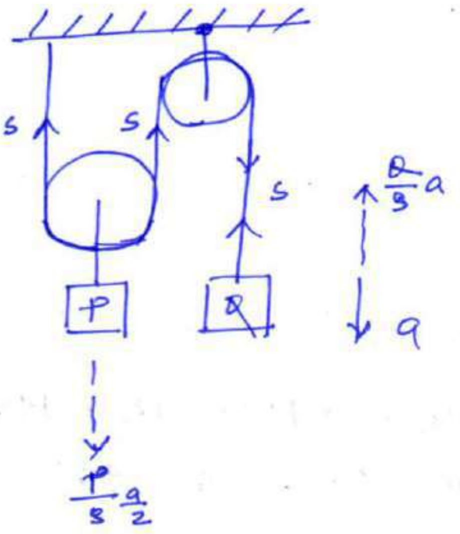
25/11/14

(2)

$$a = 6.93676 - 1.387352 - 0.0109\left(225 + 11.46a\right)$$
$$= \cancel{6.93}\ 5.549408 - 2.4525 - 0.124914a$$
$$= 3.096908 - 0.124914a$$
$$\Rightarrow \boxed{a = 2.75\ m/s^2}$$

Q.2 Two weights P and Q are connected by the arrangement shown in fig. Neglecting friction and inertia of pulley and cord find the acceleration a of wt. Q

Assume $P = 178\ N$, $Q = 133.5\ N$.



Applying D'Alembert's principle for Q

$$Q - S - \frac{Q}{g}a = 0$$
$$\Rightarrow S = Q\left(1 - \frac{a}{g}\right) \quad \text{--- (1)}$$
$$= 133.5\left(1 - \frac{a}{9.81}\right)$$

Applying D'Alembert's principle to P

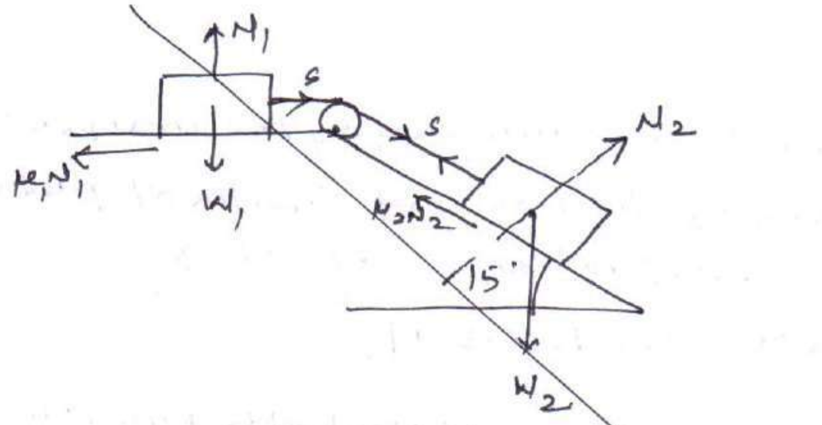
~~P - 2S = P~~

$$2S - P - \frac{Pa}{2g} = 0$$
$$\Rightarrow 2S = P\left(1 + \frac{a}{2g}\right)$$
$$\Rightarrow S = \frac{P}{2}\left(1 + \frac{a}{2g}\right) \quad \text{--- (2)}$$
$$= \frac{178}{2}\left(1 + \frac{a}{19.62}\right)$$

$$133.5\left(1 - \frac{a}{9.81}\right) = 89\left(1 + \frac{a}{19.62}\right)$$
$$\Rightarrow 133.5 - 13.608a = 89 + 4.536a$$
$$\Rightarrow 18.144a = 44.5$$
$$\Rightarrow \boxed{a = 2.45\ m/s^2} \quad \text{(Ans)}$$

Q.3 ~~Assuming the car in the fig. to have a velocity of 6 m/s find shortest distance s in which it can be stopped with constant decelaration without disturbing the block.~~ Data: $c = 0.6\ m$, $h = 0.9\ m$
$\mu = 0.5$

Q.3 Two blocks of wt $W_1 = 150\,N$ and $W_2 = 500\,N$ are connected by an inextensible string. Find the acceleration of the blocks and tension in the string. $\mu_1 = 0.1$, $\mu_2 = 0$.



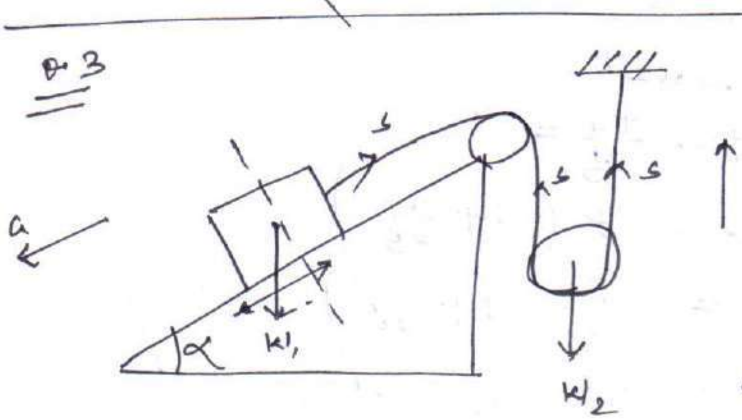
For block 1

$$S - \mu_1 N_1 = 0$$

$$\Rightarrow S = \mu_1 W_1 = 0.1 \times 150 = 15\,N.$$

for block 2

Q.3



$W_1 = 890\,N$ $W_2 = 445\,N$.
$\mu = 0.2$ $\alpha = 45°$
find S.

considering equilibrium of W_1 and applying D'Alembert's principle

$$W_1 \sin 45° - \mu N_1 - S - \frac{W_1}{g} a = 0$$

$$\Rightarrow S = W_1 \sin 45 - \mu N - \frac{W_1}{g} a$$

$$= \frac{890}{\sqrt{2}} - 0.2 \times 890 \times \frac{1}{\sqrt{2}} - \frac{890}{9.81} a$$

$$= 629.32 - 125.865 - 90.72a$$

$$\boxed{S = 503.455 - 90.72a} \quad -(1)$$

Applying D'Alembert's principle for W_2

$$2S - W_2 - \frac{W_2}{g}\frac{a}{2} = 0$$

$$\Rightarrow 2S = W_2\left(1 + \frac{a}{2g}\right)$$

$$\Rightarrow S = \frac{W_2}{2}\left(1 + \frac{a}{2g}\right) = \frac{445}{2}\left(1 + \frac{a}{19.62}\right) = 222.5 + 11.34a \quad -(2)$$

equating (1) and (2)

$503.455 - 90.72a = 222.5 + 11.34a$

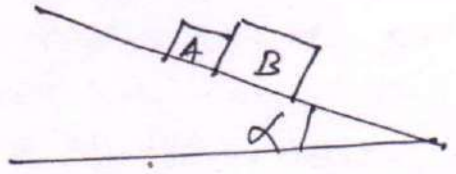
$\Rightarrow 102.0604\, a = 280.955$

$\Rightarrow \boxed{a = 2.75\ m/s^2}$

so $S = 222.5 + 11.34 \times 2.75$

$= \boxed{253.71\ N.}$

Q4

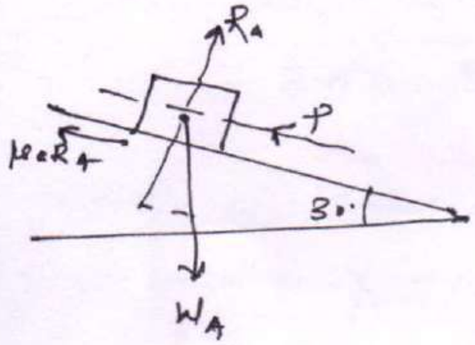


$W_A = 44.5N \quad W_B = 89N$

$\alpha = 30° \quad \mu_a = 0.15$

$\mu_B = 0.3$

find pressure P. betn blocks.



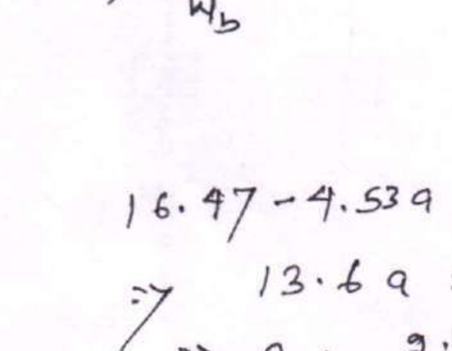
$W_a \sin 30 - P - \mu_a R_a - \frac{W_a}{g} a = 0$

$\Rightarrow P = W_a \sin 30 - \mu_a R_a - \frac{W_a}{g} a$

$= 44.5 \times \frac{1}{2} - 0.15 \times 44.5 \times \cos 30 - \frac{44.5}{9.81} a$

$= 22.25 - 5.78 - 4.53a$

$= 16.47 - 4.53a \quad —(1)$



$P + W_b \sin 30 - \mu_b R_b - \frac{W_b}{g} a = 0$

$\Rightarrow P = -\frac{W_b}{2} + 0.3 \times 89 \cos 30 + \frac{89}{9.81} a$

$= -\frac{89}{2} + 23.122 + 9.07a$

$= -21.378 + 9.07a \quad —(2)$

$16.47 - 4.53a = -21.378 + 9.07a$

$\Rightarrow 13.6a = 37.848$

$\Rightarrow a = 2.78\ m/s^2$

$P = 3.87\ N.$

27/11/2014 (1)

Momentum and Impulse

We have the differential equation of rectilinear motion of a particle

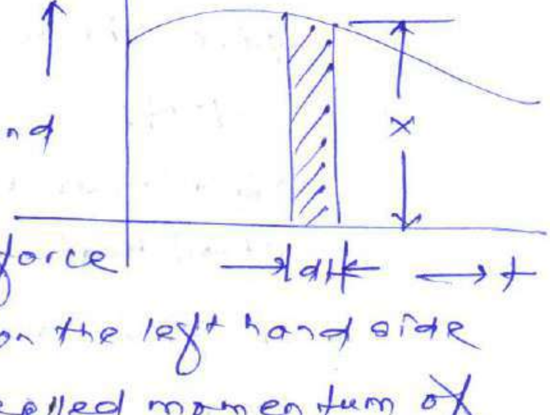
$$\frac{W}{g}\ddot{x} = X$$

Above equation may be written as

$$\frac{W}{g}\frac{d\dot{x}}{dt} = X$$

or $$\boxed{d\left(\frac{W}{g}\dot{x}\right) = X\,dt} \quad \text{--- (1)}$$

In the above equation we will assume force X as a function of time represented by a force time diagram.



The right hand side of eq.(1) is then represented by the area of shaded elemental strip of ht X and width dt. This quantity i.e $(X\,dt)$ is called impulse of the force X in time dt. The expression on the left hand side of the expression $\left(\frac{W}{g}\dot{x}\right)$ is called momentum of particle.

so the eq. (1) represents the differential change in momentum of a particle in time dt.

Integrating eq.(1) we have

$$\boxed{\frac{W}{g}\dot{x} + C = \int_0^t X\,dt} \quad \text{--- (2)}$$

where C is a constant of integration

Now assuming an initial moment, $t=0$, the particle has an initial velocity $\dot{x}_0$

so $$\boxed{C = -\frac{W}{g}\dot{x}_0} \quad \text{--- (3)}$$

So equation (2) becomes

$$\boxed{\frac{W}{g}\dot{x} - \frac{W}{g}\dot{x}_0 = \int_0^t X\,dt} \quad \text{--- (4)}$$

from equation (4f) it is clear that the total change of momentum of a particle during a finite interval of time is equal to the impulse of acting force.

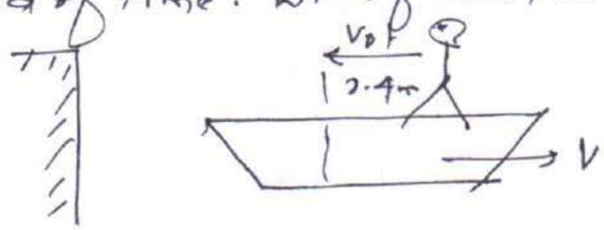
~~Example~~ in other words

$$\boxed{F \cdot dt = d(mv)}$$

where $m \times v$ = momentum

~~[illegible]~~

Q-1 A man of wt 712 N stands in a boat so that he is 4.5 m from a pier on the shore. He walks 2.4 m in the boat towards the pier and then stops. How far from the pier will he be at the end of time. Wt of boat is 890 N.



wt of man $W_1 = 712$ N

wt of boat $W_2 = 890$ N

Let v_0 is the initial velocity of man and t is time

then $v_0 t = x$

$\Rightarrow v_0 t = 2.4$ m

$\Rightarrow v_0 = \left(\frac{2.4}{t}\right)$ m/s.

let V = velocity of boat towards right

according to conservation of momentum

$$W_1 v_0 = (W_1 + W_2) V$$

$$\Rightarrow V = \frac{W_1 v_0}{(W_1 + W_2)}$$

distance covered by boat

$$s = V \cdot t = \frac{W_1 v_0}{(W_1 + W_2)} \cdot t$$

$$\Rightarrow s = \frac{712 \times 2.4}{\cancel{t}\,(712 + 890)} \cdot \cancel{t} = \boxed{1.067 \text{ m}}$$

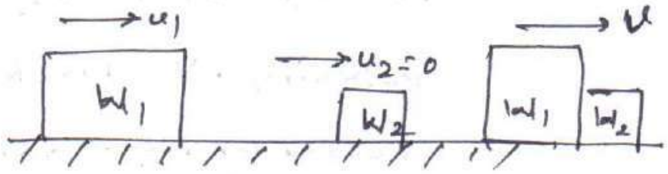
position of man from pier

$= 4.5 + s - x$

$= 4.5 + 1.667 - 2.4 = \boxed{3.167\ m}$ (Ans).

Q.2 A locomotive wt 534 kN has a velocity of 16 kmph and backs into a frieght car of wt 86 kN that is at rest on a track. after coupling at what velocity v the entire system continues to move. Neglect friction.

conservation of momentume



$W_1 u_1 + W_2 u_2 = (W_1 + W_2) V$

$\Rightarrow V = \dfrac{534 \times 4.45}{(534 + 86)} = \boxed{3.82\ m/s.}$

Q.3 A 667.5 man sits in a 333.75 N canoe and fire a rifle bullet horizontally. ~~directed over~~ find velocity v with which the canoe will move after the shot. the rifle has a muzzle velocity 660 m/s and wt. of bullet is 0.28 N.

Wt. of man W_1 = 667.5 N.

Wt. of canoe W_2 = 333.75 N.

Wt. of bullet W_3 = 0.28 N.

velocity of muzzle u = 660 m/s.

V = final velocity of canoe.

According to conservation of momentum

$\cancel{W_3 u}\ W_3 u = (W_1 + W_2) V$

$\Rightarrow V = \dfrac{0.28 \times 660}{(667.5 + 333.75)} = \boxed{0.182\ m/s.}$

Q.4 A wood block wt 22.25 N rests on a smooth horizontal surface. A revolver bullet weighing 0.14 N is shot horizontally into the side of block. If the block attains a velocity of 3 m/s what is muzzle velocity.

Wt. of wood block $W_1 = 22.25$ N.

Wt. of bullet $W_2 = 0.14$ N.

velocity of block $V = 3$ m/s.

velocity of muzzle $= u$

According to conservation of momentum

$$\cancel{W_1 V} = W_2 u = (W_1 + W_2) V$$

$$\Rightarrow u = \frac{(22.25 + 0.14)\,3}{0.14}$$

$$= \boxed{479.98 \text{ m/s.}}$$

Conservation of momentum

When the sum of impulses due to external force is zero the momentum of the system remain conserved

When $\Sigma \int_0^t X\, dt = 0$

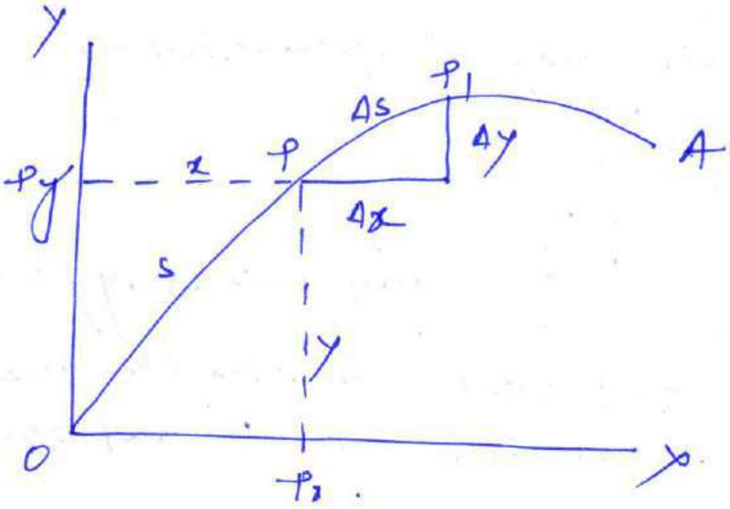
$$\boxed{\Sigma \left(\frac{W}{g}\right) \dot{x}_2 = \Sigma \left(\frac{W}{g}\right) \dot{x}_1}$$

$\therefore$ final momentum = initial momentum.

Curvilinear Translation

When a moving particle describes a curved path it is said to have curvilinear motion.

Displacement



consider a particle P in a plane on a curved path. To define the particle we need two coordinates x and y. As the particle moves, these coordinates ~~move~~ change with time and the displacement time equations are

$$x = f_1(t) \qquad y = f_2(t) \qquad \text{--- (1)}$$

The motion of particle can also be expressed as

$$y = f(x) \qquad s = f_1(t)$$

where $y = f(x)$ represents the equation of path OA and $s = f_1(t)$ gives displacement s measured along the path as a function of time.

velocity :-

Considering an infinitesimal time difference from t to $t+\Delta t$ during which the particle moves from P to P_1 along it's path.

then velocity of particle may be expressed as

$$\boxed{\bar{v}_{av} = \frac{\Delta \bar{s}}{\Delta t}}$$

$$\boxed{\begin{aligned}(v_{av})_x &= \frac{\Delta x}{\Delta t} \\ (v_{av})_y &= \frac{\Delta y}{\Delta t}\end{aligned}}$$

(average velocity along x and y coordinates)

It can also be expressed as

$$v_x = \frac{dx}{dt} = \dot{x}$$

$$v_y = \frac{dy}{dt} = \dot{y}$$

So the total velocity may be represented by

$$\boxed{v = \sqrt{\dot{x}^2 + \dot{y}^2}}$$

and $\cos(v, x) = \frac{\dot{x}}{v}$ and $\cos(v, y) = \frac{\dot{y}}{v}$

where (v, x) and (v, y) denotes the angles betn the direction of velocity vector $\vec{v}$ and the coordinate axes.

Acceleration :—

The acceleration particles may be described as

$$\boxed{\begin{aligned} a_x &= \frac{d\dot{x}}{dt} = \ddot{x} \\ a_y &= \frac{d\dot{y}}{dt} = \ddot{y} \end{aligned}}$$

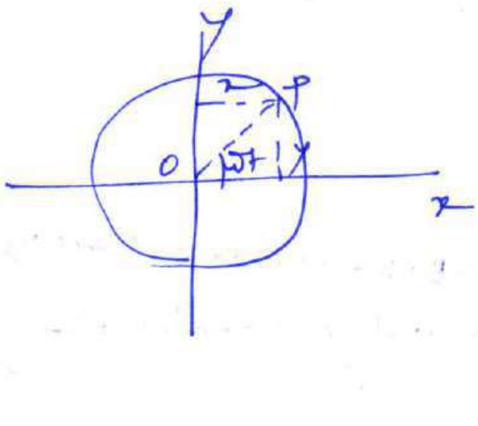
It is also known as instantaneous acceleration

Total acceleration $a = \sqrt{\ddot{x}^2 + \ddot{y}^2}$

Considering particular path for above case.

$x = r\cos\omega t \quad y = r\sin\omega t.$

$x^2 + y^2 = r^2$



$\dot{x} = -r\omega\sin\omega t \quad \dot{y} = r\omega\cos\omega t$

$v = \sqrt{\dot{x}^2 + \dot{y}^2}$

$\ddot{x} = -r\omega^2\cos\omega t \quad \ddot{y} = -r\omega^2\sin\omega t$

$a = \sqrt{\ddot{x}^2 + \ddot{y}^2}$