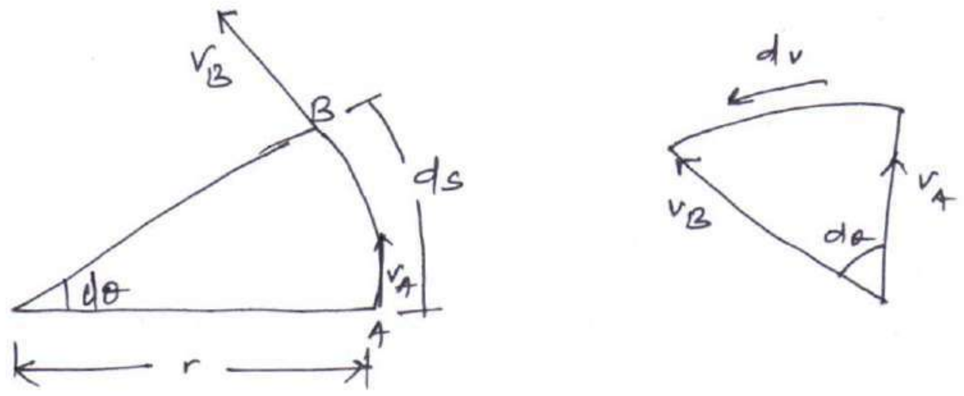
D'Alembert's Principle in Curvilinear Motion

Acceleration during circular motion



V_A = tangential velocity at A
= tangential velocity at B
$= V_B = V$

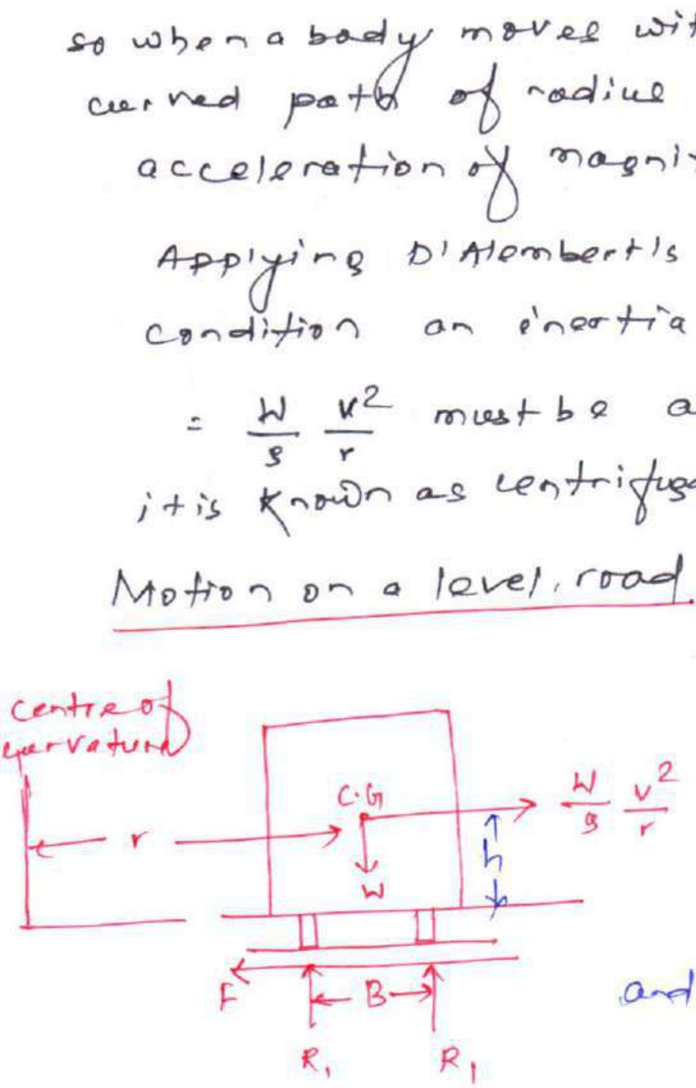
Now $\quad dv = v\,d\theta = v\,\dfrac{ds}{r} = \dfrac{v}{r}\,ds$

acceleration $= \dfrac{dv}{dt} = \boxed{\dfrac{v^2}{r}}$

so when a body moves with uniform velocity v along a curved path of radius r, it has a radial inward acceleration of magnitude $\dfrac{v^2}{r}$

Applying D'Alembert's principle to get equilibrium condition an inertia force of magnitude $\dfrac{W}{g}a$ $= \dfrac{W}{g}\dfrac{v^2}{r}$ must be applied in outward direction it is known as centrifugal force.

Motion on a level road



Consider a body is moving with uniform velocity on a curvilinear curve of radius r. Let the road is flat.

Let W = wt. of the body
and inertia force is given by

$$\frac{W}{g}a = \frac{W}{g}\frac{v^2}{r}$$

Condition for skidding :-

Let W = wt. of vehicle

R_1, R_2 = reactions at wheel

F = frictional force.

$\frac{W}{g} \cdot \frac{v^2}{r}$ = inertia force

Skidding takes place when the frictional force reaches limiting value i.e

$F = \mu W$

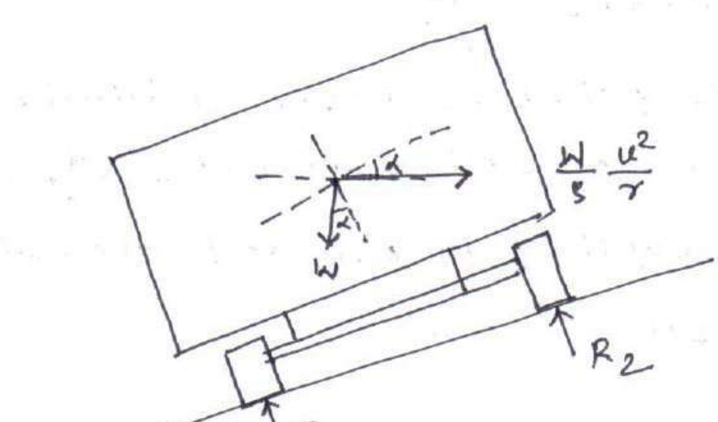
Then max permissible speed to avoid skidding

$$\boxed{v = \sqrt{\frac{gr}{2} \frac{B}{h}}}$$

The distance betn inner and outer wheel is equal to the gauge of railway track and expressed as G.

so $$\boxed{v = \sqrt{\frac{gr}{2} \frac{G}{h}}}$$

Designed speed and angle of Braking



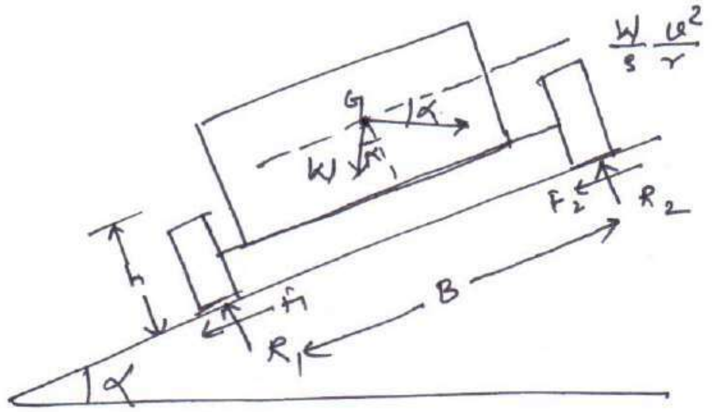
Σ of all the forces in the inclined plane

$$\frac{W}{g} \frac{v^2}{r} \cos\alpha - W \sin\alpha = 0$$

$$\Rightarrow \boxed{\tan\alpha = \frac{v^2}{gr}}$$

Relation betn the angle of braking and designed speed is $\boxed{\tan\alpha = \frac{v^2}{gr}}$

condition for skidding and overturning:-



(a) condition for skidding

$$\boxed{v = \sqrt{\tan(\alpha + \phi) \times g r}}$$

where α = angle of inclination

$\tan\phi = \mu$

g = ~~coeffic~~ gravitational acceleration

r = radius of curve

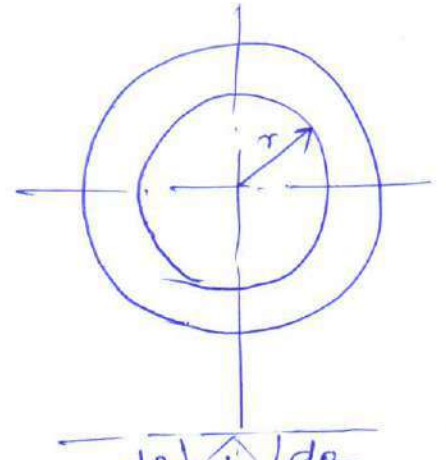
~~then the~~ vehicle will skid if the velocity is more than this value.

(b) condition for overturning:

Limiting speed from consideration of overturning

$$\boxed{v = \sqrt{g r \frac{G + (2he/G)}{2h - e}}}$$

Q.1 A circular ring has a mean radius r = 500 mm and is made of steel for which w = 77.12 kN/m³ and for which ultimate strength in tension is 413.85 MPa. Find the uniform speed of rotation about its geometrical axis perpendicular to the plane of the ring at which it will burst?

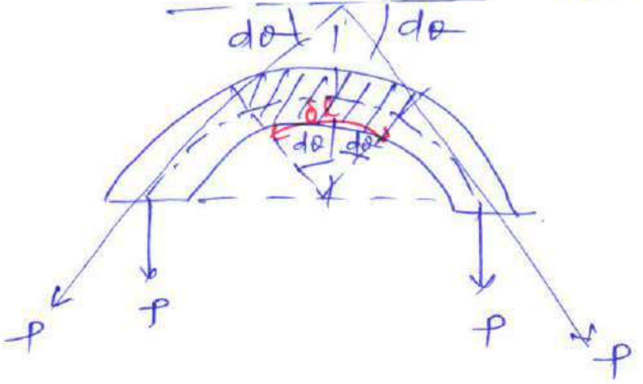


mean radius $r = 500\ mm = 0.5\ m$,

density of the wheel $w = 77.12\ kN/m^3$

σ_t = ultimate strength = $413.85 \times 10^6\ Pa$.

Now considering an infinitesimal small elementary ring extruded at an angle of $2d\theta$



centrifugal force acting

$$\delta F_c = \frac{dW}{g} \cdot \frac{v^2}{r}$$

Let P = tension on the ring

A = cross-sectional area of ring.

dW = wt. of the element

$= w \times$ volume

$= w \times A \times \delta l$

$= w \times A \times r\,2d\theta$

Now centrifugal force

$$\frac{w}{g}(A\,dl) \times \frac{v^2}{r} = \frac{w}{g} \times A\,\cancel{r}\,2d\theta \times \frac{v^2}{\cancel{r}} = \frac{2wA\,d\theta\,v^2}{g}$$

Balancing forces along the radius $= 2P\sin d\theta$

$$= \frac{2wA\,d\theta\,v^2}{g} \quad \text{--- (1)}$$

as $d\theta$ is very small $\sin d\theta \simeq d\theta$

Eq. (1) may be written as

$$\cancel{2}P\,\cancel{d\theta} = \frac{\cancel{2}wA\,\cancel{d\theta}\cdot v^2}{g}$$

$$\Rightarrow \boxed{P = \frac{wAv^2}{g}} \quad \text{--- (2)}$$

Tensile stress on the ring $\sigma_t = \frac{P}{A} = \frac{wv^2}{g}$

Now substituting the values

$$413.85 \times 10^6 = \frac{77.12 \times 10^3 \times v^2}{9.81} \Rightarrow v = 229.45\ m/s.$$

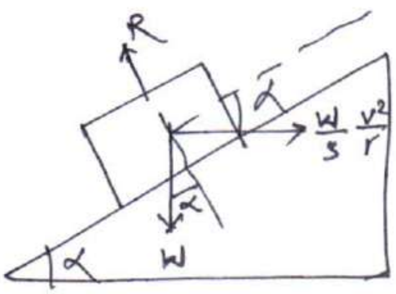
Now $v = \frac{\pi D N}{60} \Rightarrow N = \frac{60 \times 229.45}{\pi \times 1} = \boxed{4382\ rpm}$

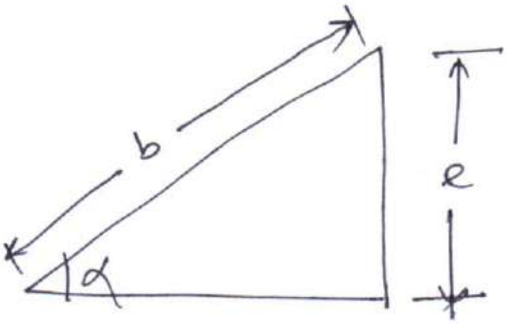
D' Alembert's Principle in Curvilinear Motion

Equation of motion of a particle may be written as

$$\left.\begin{aligned} X - m\ddot{x} = 0 \\ Y - m\ddot{y} = 0 \end{aligned}\right\} \quad \text{— (1)}$$

Q.2 Find the proper super elevation 'e' for a 7.2 m highway curve of radius r = 600m in order that a car travelling with a speed of 80 Kmph will have no tendency to skid sidewise.





$b = 7.2\,m \qquad r = 600\,m \qquad v = 80\,Kmph = 22.23\ m/s.$

Resolving along the inclined plane

$$W\sin\alpha = \frac{W}{g}\cdot\frac{v^2}{r}\cos\alpha$$

$$\Rightarrow \boxed{\tan\alpha = \frac{v^2}{rg}}$$

from the geometry $\sin\alpha = \frac{e}{b}$, since α is very small

let $\sin\alpha \simeq \tan\alpha$

$$\frac{v^2}{rg} = \frac{e}{b} \Rightarrow e = \frac{bv^2}{rg} = \frac{7.2 \times 22.23^2}{600 \times 9.81}$$

$$= 0.604\,m \quad \text{(Ans)}$$

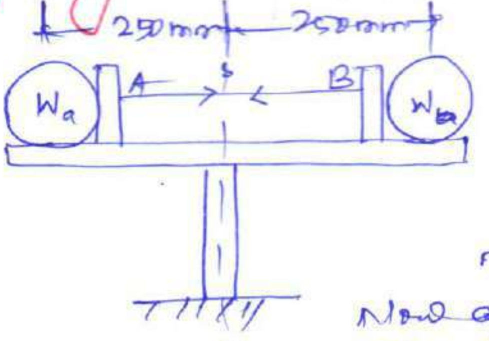
Q.3 A racing car travels around a circular track of 300m radius with a speed of 384 kmph. What angle α should the floor of the track make with horizontal in order to safeguard against skidding.

velocity v = 384 kmph $\quad r = 300m$
= 106.67 m/s.

We have angle of banking $\tan\alpha = \frac{v^2}{rg}$

$\Rightarrow \alpha = \tan^{-1}\left(\frac{106.67^2}{300\times 9.81}\right) = \boxed{75.5^\circ}$ (Ans)

Q.4 Two balls of wt $W_a = 44.5N$ and $W_b = 66.75N$ are connected by an elastic string and supported on a turntable as shown. When the turntable is at rest, the tension in the string is $S = 222.5N$ and the balls exert this same force on each of the stops A and B. What forces will they exert on the stops when the turntable is rotating uniformly about the vertical axis CD at 60 rpm?



We have;

$W_a = 44.5N \quad W_b = 66.75N$

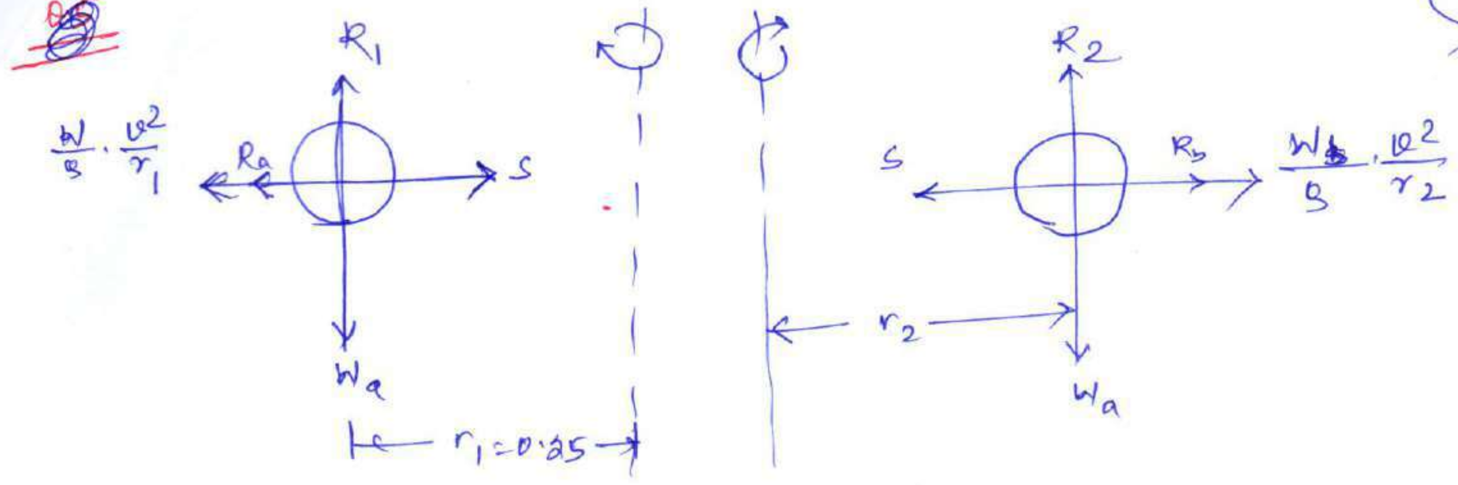
$S = 222.5N$

$N = 60 rpm$,

radius of rotation $r_1, r_2 = 0.25m$

Now angular velocity

$\omega = \frac{2\pi N}{60} = \frac{2\pi\times 60}{60} = 2\pi$ rad/s.



~~Consider~~ Considering the left hand side ball

$R_a + \frac{W_a}{g} \cdot r_1 \omega^2 = S$

$\Rightarrow R_a = 222.5 - \frac{44.5}{9.81} \times 0.25 \times (2\pi)^2$

$= \boxed{[illegible]}\ \boxed{177.72\ N.}$

Considering the ball on right hand side

$R_b + \frac{W_b}{g} \times r_2 \times \omega^2 = S$

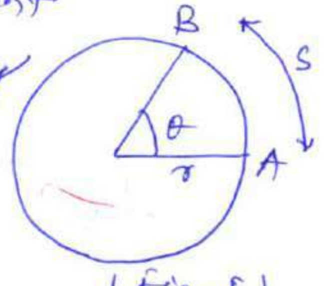
$\Rightarrow R_b = 222.5 - \frac{66.75}{9.81} \times 0.25 \times (2\pi)^2$

$= \boxed{155.34\ N.}$

Rotation of Rigid Bodies:-

Angular motion:-

The rate of change of angular displacement with time is called angular velocity and denoted by ω.



(fig-1)

$$\boxed{\omega = \frac{d\theta}{dt}} \quad —(1)$$

- The rate of change of angular velocity with time is called angular acceleration and denoted by α

$$\boxed{\alpha = \frac{d\omega}{dt} = \frac{d^2\theta}{dt^2}} \quad —(2)$$

Angular acceleration may also be expressed as:

$$\alpha = \frac{d\omega}{dt} = \frac{d\omega}{d\theta} \cdot \frac{d\theta}{dt}$$

$$\Rightarrow \boxed{\alpha = \omega \cdot \frac{d\omega}{d\theta}} \quad —(3) \quad \left(\because \frac{d\theta}{dt} = \omega\right)$$

Relationship between angular motion and linear motion

from fig-1 $\quad s = r\theta$

tangential velocity (linear) of the particle p.

$$\boxed{v = \frac{ds}{dt} = r \cdot \frac{d\theta}{dt}} \quad —(4)$$

linear acceleration $\boxed{a_t = \frac{dv}{dt} = r\frac{d^2\theta}{dt^2}} \quad —(5)$

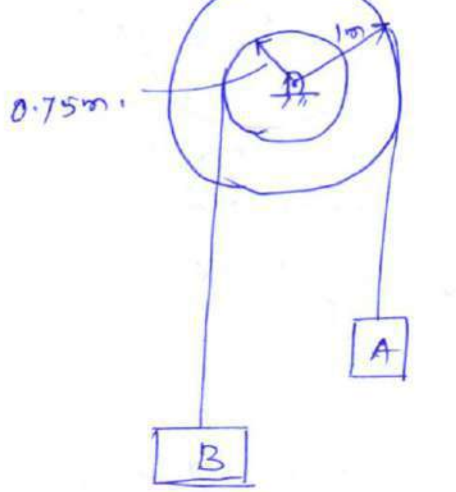
If $\frac{v^2}{r}$ = radial acceleration

Then $\boxed{a_n = \frac{v^2}{r} = r\omega^2}$ —(6) (where a_n = radial acceleration)

uniform angular velocity (ω)

$$\boxed{\omega = \frac{2\pi N}{60} \text{ rad/sec}} \quad —(7)$$

Q: The step pulley starts from rest and accelerates at 2 rad/s^2. How much time is required for block A to move 20m. Find also the velocity of A and B at that time.



When A moves by 20m, the angular displacement of pulley θ is given by

$$r\theta = s$$

$$\Rightarrow 1 \times \theta = 20$$

$$\Rightarrow \boxed{\theta = 20 \text{ rad}}$$

$\alpha = 2 \text{ rad/s}^2$ and $\omega_0 = 0$

from kinematic relation

$$\theta = \omega_0 t + \frac{1}{2}\alpha t^2$$

$$\Rightarrow 20 = 0 \times t + \frac{1}{\cancel{2}} \times \cancel{2} t^2$$

$$\Rightarrow \boxed{t = 4.472 \text{ sec.}}$$

velocity of pulley at this time

$$\omega = \omega_0 + \alpha t$$

$$= 0 + 2 \times 4.472$$

$$= \boxed{8.944 \text{ rad/s}}$$

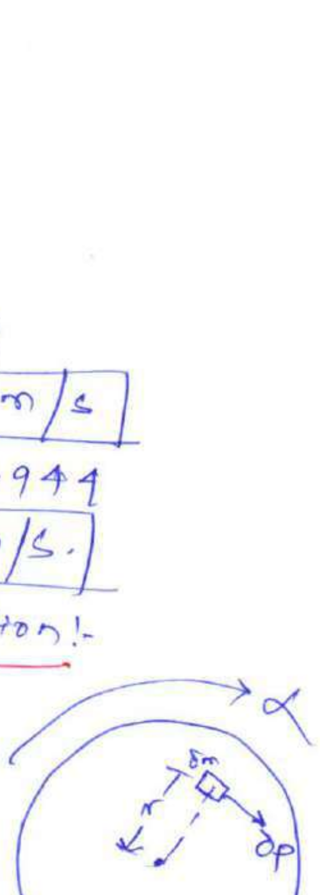
velocity of block A $v_A = 1 \times 8.944$

$$= \boxed{8.944 \text{ m/s}}$$

velocity of block B $v_B = 0.75 \times 8.944$

$$= \boxed{6.708 \text{ m/s.}}$$

Kinematics of rigid body for rotation:-



consider a wheel rotating about it's axis in clockwise direction with an acceleration α. Let δm be mass of an element at a distance r from the axis of rotation. δp be the resulting force on this element

$\delta p = \delta m \times a$ (a = tangential acceleration)

but $a = r \times \alpha$ (α = angular acceleration)

$\therefore \boxed{\delta p = \delta m \, r \, \alpha}$

Rotational moment $\delta M_t = \delta p \times r$

$= \delta m \, r^2 \alpha$

$M_t = \Sigma \, \delta M_t = \Sigma \, \delta m \, r^2 \alpha$

$= \alpha \, \Sigma \, \delta m \, r^2$

$= \alpha I$

$\Rightarrow \boxed{M_t = \alpha I}$ (I = mass moment of inertia)

Product of mass moment of inertia and angular velocity of rotating body is called angular momentum

so $\boxed{\text{Angular momentum} = I \omega}$

Kinetic energy of rotating bodies

$\boxed{K.E = \frac{1}{2} I \omega^2}$

Q.2 A flywheel weighing 50 kN and having radius of gyration 1m losses its speed from 400 rpm to 280 rpm in 2 min. Calculate
(a) retarding torque, (b) change in KE during the period, (c) change in angular momentum.

we have $\omega_0 = 400 \text{rpm} = \frac{2\pi \times 400}{60} = 41.89 \text{ rad/s}$

$\omega = 280 \text{rpm} = \frac{2\pi \times 280}{60} = 29.32 \text{ rad/s}.$

$t = 2 \text{min} = 120 \text{sec},$

$\omega = \omega_0 + \alpha t$

$\Rightarrow \alpha = \frac{\omega - \omega_0}{t} = \boxed{-.1047 \text{ rad/s}^2}$