

LINEAR WAVESHAPING

A linear network is a network made up of linear elements only. A linear network can be described by linear differential equations. The principle of superposition and the principle of homogeneity hold good for linear networks. In pulse circuitry, there are a number of waveforms, which appear very frequently. The most important of these are sinusoidal, step, pulse, square wave, ramp, and exponential waveforms. The response of RC , RL , and RLC circuits to these signals is described in this chapter. Out of these signals, the sinusoidal signal has a unique characteristic that it preserves its shape when it is transmitted through a linear network, i.e. under steady state, the output will be a precise reproduction of the input sinusoidal signal. There will only be a change in the amplitude of the signal and there may be a phase shift between the input and the output waveforms. The influence of the circuit on the signal may then be completely specified by the ratio of the output to the input amplitude and by the phase angle between the output and the input. No other periodic waveform preserves its shape precisely when transmitted through a linear network, and in many cases the output signal may bear very little resemblance to the input signal.

The process whereby the form of a non-sinusoidal signal is altered by transmission through a linear network is called linear wave shaping.

THE HIGH-PASS RC CIRCUIT

Figure 1.30 shows a high-pass RC circuit. At zero frequency the reactance of the capacitor is infinity and so it blocks the input and hence the output is zero. Hence, this capacitor is called the *blocking capacitor* and this circuit, also called the *capacitive coupling circuit*, is used to provide dc isolation between the input and the output. As the frequency increases, the reactance of the capacitor decreases and hence the output and gain increase. At very high frequencies, the capacitive reactance is very small so a very small voltage appears across C and, so the output is almost equal to the input and the gain is equal to 1. Since this circuit attenuates low-frequency signals and allows transmission of high-frequency signals with little or no attenuation, it is called a high-pass circuit.

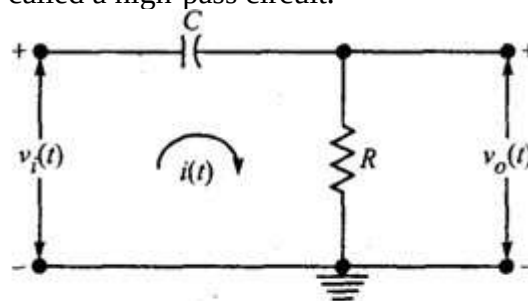


Figure The high-pass RC circuit.

Sinusoidal Input

Figure (a) shows the Laplace transformed high-pass RC circuit. The gain versus frequency curve of a high-pass circuit excited by a sinusoidal input is shown in Figure (b). For a sinusoidal input v_i , the output signal v_o increases in amplitude with increasing frequency. The frequency at which the gain is $1/\sqrt{2}$ of its maximum value is called the lower cut-off or lower 3-dB frequency. For a high-pass circuit, there is no upper cut-off frequency because all high frequency signals are transmitted with zero attenuation. Therefore, $f_2 - f_1$. Hence bandwidth $B.W = f_2 - f_1 = \infty$

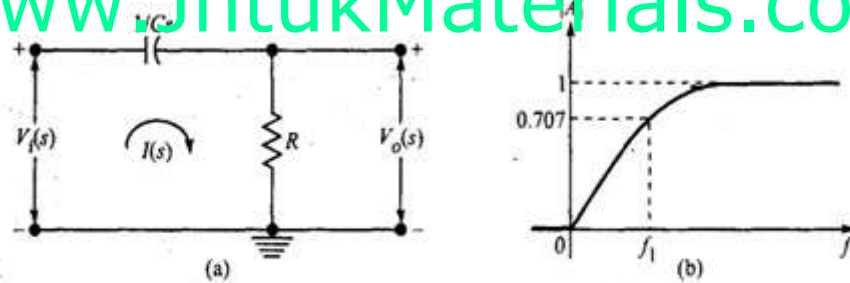


Figure (a) Laplace transformed high-pass circuit and (b) gain versus frequency plot.

Expression for the lower cut-off frequency

For the high-pass RC circuit shown in Figure (a), the magnitude of the steady-state gain A , and the angle θ by which the output leads the input are given by

$$A = \frac{V_o(s)}{V_i(s)} = \frac{R}{R + \frac{1}{Cs}} = \frac{1}{1 + \frac{1}{RCs}}$$

Putting $s = j\omega$, $A = \frac{1}{1 - j\frac{1}{\omega RC}} = \frac{1}{1 - j\frac{1}{2\pi f RC}}$

$$\therefore |A| = \frac{1}{\sqrt{1 + \left(\frac{1}{2\pi f RC}\right)^2}} \quad \text{and} \quad \theta = -\tan^{-1} \frac{1}{2\pi f RC}$$

At the lower cut-off frequency f_1 , $|A| = 1/\sqrt{2}$

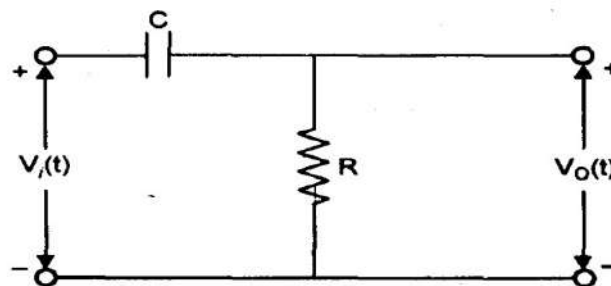
$$\therefore \frac{1}{\sqrt{1 + \left(\frac{1}{2\pi f_1 RC}\right)^2}} = \frac{1}{\sqrt{2}}$$

Squaring and equating the denominators,

$$\frac{1}{2\pi f_1 RC} = 1 \quad \text{i.e.} \quad f_1 = \frac{1}{2\pi RC}$$

This is the expression for the lower cut-off frequency of a high-pass circuit.

High pass RC circuit response to Step input Voltage:



RC high-pass filter

A step waveform is defined by the following expression:

$$v(t) = \begin{cases} 0 & \text{for } t < 0 \\ V & \text{for } t \geq 0 \end{cases}$$

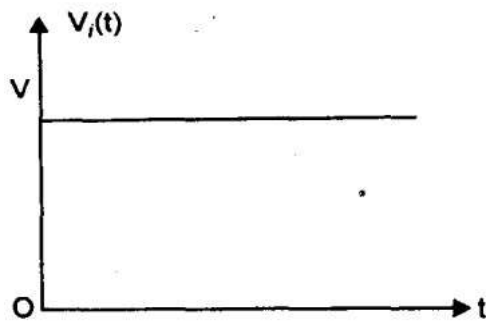


Fig:a The step waveform

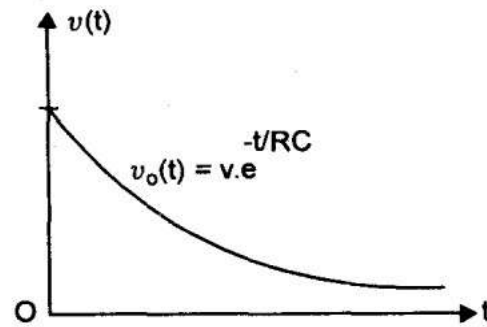


Fig:b Output waveform

The generalized transient expression has been derived at,

$$v_o(t) = v_o(\infty) + [v_o(0) - v_o(\infty)]e^{-t/RC} \quad \text{--- (1)}$$

The expression for $v(t)$ can be found if we know the initial condition $v(0)$ final condition $v(\infty)$, and the time constant ' RC ' of the circuit.

$$\begin{aligned} v_o(t) &= 0 + (V - 0)e^{-t/RC} \\ v_o(t) &= V.e^{-t/RC} \end{aligned} \quad \text{--- (2)}$$

The output waveform for the step input to an RC high pass filter is an exponentially falling waveform as shown in Fig. (b). This response reaches almost zero after a time ' t ' greater than ' $5RC$ '.

High pass RC circuit response to pulse input:

A positive pulse is mathematically represented as the combination of a positive step followed by a delayed negative step i.e., $v_i = V u(t) - V u(t - t_p)$ where, t_p is the duration of the pulse as shown in Fig.

To understand the response of a high-pass circuit to this pulse input, let us trace the sequence of events following the application of the input signal.

At $t = 0$, v_i abruptly rises to V . As a capacitor is connected between the input and output, the output also changes abruptly by the same amount. As the input remains constant, the output decays exponentially to V_1 at $t = t_p$. Therefore,

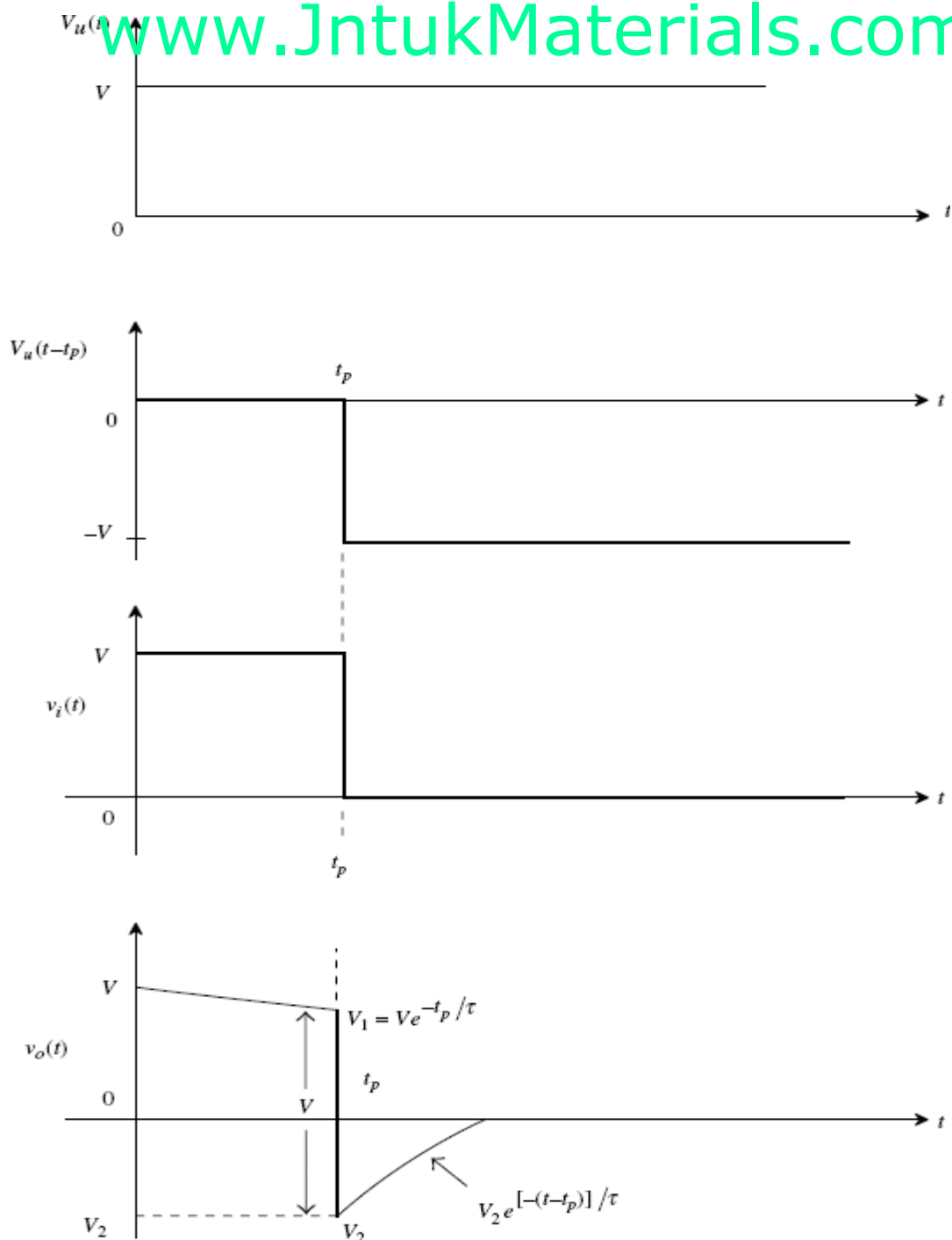
$$V_1 = V e^{-t_p/\tau}$$

At $t = t_p$, the input abruptly falls by V , v_o also falls by the same amount. In other words, $v_o = V_1 - V$. Since V_1 is less than V ; v_o is negative and its value is V_2 and this decays to zero exponentially. For $t > t_p$,

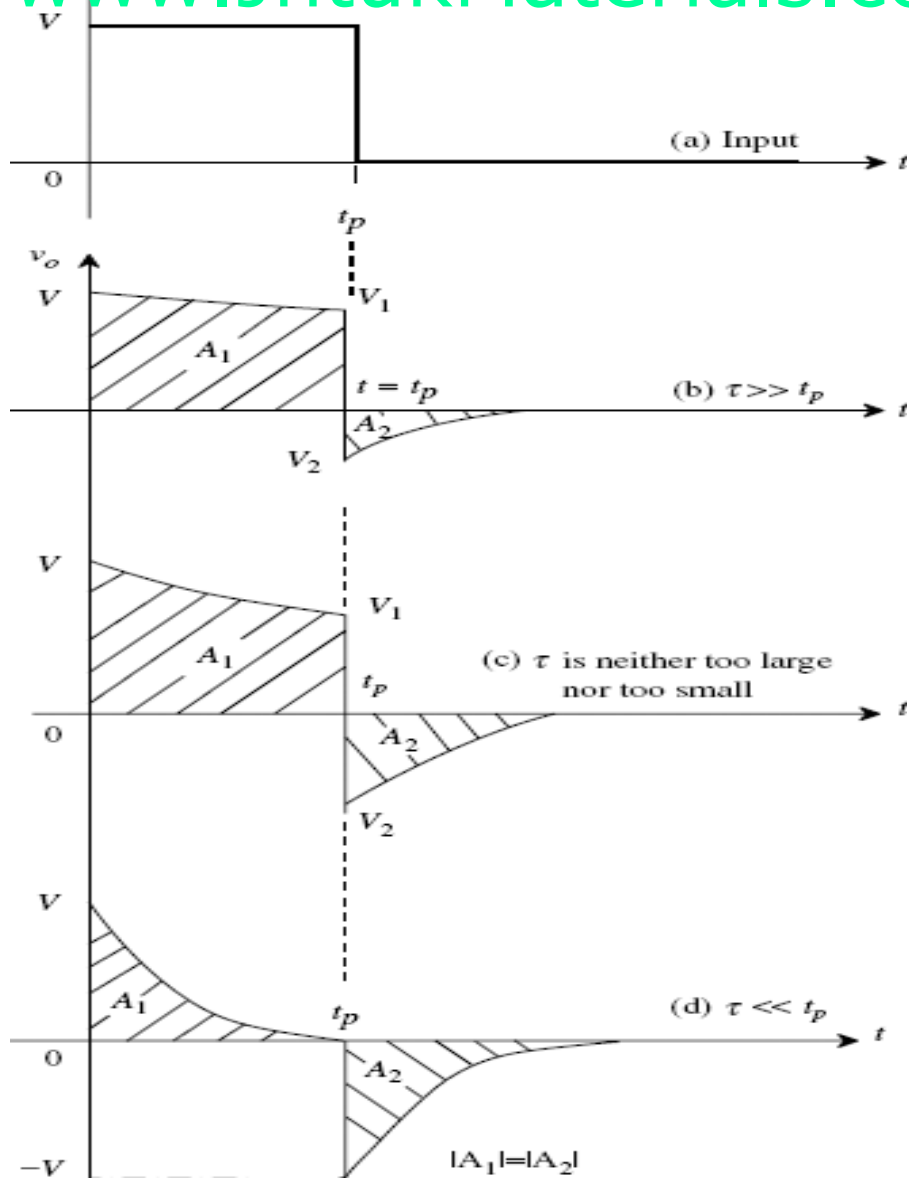
$$v_o = V_2 e^{[-(t-t_p)]/\tau} = (V_1 - V) e^{[-(t-t_p)]/\tau}$$

then

$$v_o = V(e^{-t_p/\tau} - 1) e^{[-(t-t_p)]/\tau}$$



The response of high-pass circuits with different values of τ to pulse input is plotted in Fig below. As is evident from the preceding discussion, when a pulse is passed through a high-pass circuit, it gets distorted. Only when the time constant τ is very large, the shape of the pulse at the output is preserved, as can be seen from Fig (b). However, as shown in Fig (c), when the time constant τ is neither too small nor too large, there is a tilt (also called a sag) at the top of the pulse and an under-shoot at the end of the pulse. If $\tau \ll t_p$, as in Fig (d), the output comprises a positive spike at the beginning of the pulse and a negative spike at the end of the pulse. In other words, a high-pass circuit converts a pulse into spikes by employing a small time constant; this process is called peaking.



If the distortion is to be negligible, τ has to be significantly larger than the duration of the pulse. In general, there is an undershoot at the end of the pulse. The larger the tilt (for small τ), the larger the undershoot and the smaller the time taken for this undershoot to decay to zero. The area above the reference level (A_1) is the same as the area below the reference level (A_2). Let us verify this using below Fig

Area A_1 : For $0 < t < t_p$:

$$v_o = V e^{-t/\tau}$$

$$A_1 = \int_0^{t_p} V e^{-t/\tau} dt = [-V \tau e^{-t/\tau}]_0^{t_p}$$

$$A_1 = [-V \tau e^{-t_p/\tau} + V \tau] = V \tau (1 - e^{-t_p/\tau})$$

Similarly,

$$A_2 = \int_{t_p}^{\infty} V(e^{-t_p/\tau} - 1)e^{-(t-t_p)/\tau} dt = \int_{t_p}^{\infty} [Ve^{-t/\tau} - Ve^{-(t-t_p)/\tau}] dt$$

$$= \left[\frac{Ve^{-t/\tau}}{-1/\tau} \right]_{t_p}^{\infty} - \left[V \frac{1}{-1/\tau} e^{-(t-t_p)/\tau} \right]_{t_p}^{\infty}$$

$$A_2 = [V\tau e^{-t_p/\tau} - V\tau] = -V\tau(1 - e^{-t_p/\tau})$$

So that,

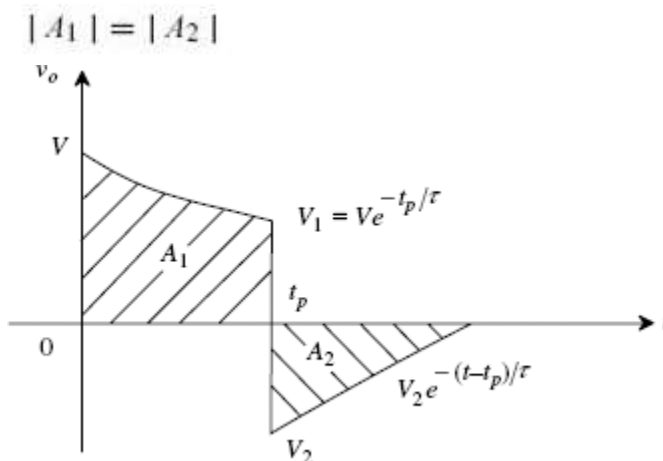


Fig: The calculation of A1 and A2

High pass RC circuit response to Square input:

A waveform that has constant amplitude, say, V' for a time T_1 and has another constant amplitude, V'' for a time T_2 , and which is repetitive with a time $T = (T_1 + T_2)$, is called a square wave. In a symmetric square wave, $T_1 = T_2 = T/2$. Figure 1.10 shows typical input – output waveforms of the high - pass circuit when a square wave is applied as the input signal.

As the capacitor blocks the DC, the DC component in the output is zero. Thus, as expected, even if the signal at the input is referenced to an arbitrary dc level, the output is always referenced to the zero level. It can be proved that whatever the dc component associated with a periodic input waveform, the dc level of the steady - state output signal for the high - pass circuit is always zero as shown in Fig. 1.10. To verify this statement, we write the KVL equation for the high – pass circuit:

$$v_i = \frac{q}{C} + v_o$$

where, q is the charge on the capacitor. Differentiating with respect to t :

$$\frac{dv_i}{dt} = \frac{1}{C} \frac{dq}{dt} + \frac{dv_o}{dt}$$

But $i = \frac{dv_i}{dt}$

Substituting this condition in above Eq, then

$$\frac{dv_i}{dt} = \frac{i}{C} + \frac{dv_o}{dt}$$

Since $v_o = iR$, $i = v_o/R$ and $RC = \tau$. Therefore,

$$\therefore \frac{dv_i}{dt} = \frac{v_o}{\tau} + \frac{dv_o}{dt}$$

Multiplying by dt and integrating over the time period T we get:

$$\int_0^T dv_i = [v_i]_0^T = v_i(T) - v_i(0)$$

$$\int_0^T \frac{v_o}{\tau} dt = \frac{1}{\tau} \int_0^T v_o dt$$

$$\int_0^T dv_o = [v_o]_0^T = v_o(T) - v_o(0)$$

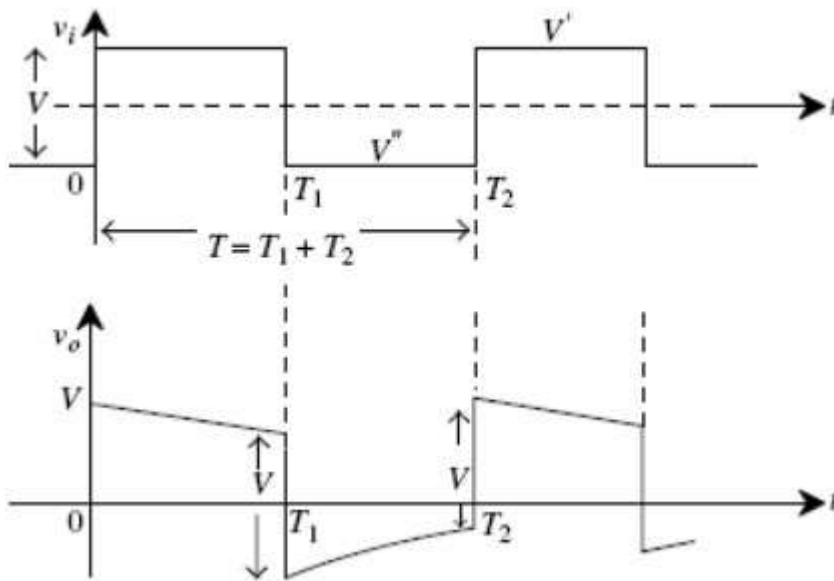


Fig: A typical steady - state output of a high - pass circuit with a square wave as input

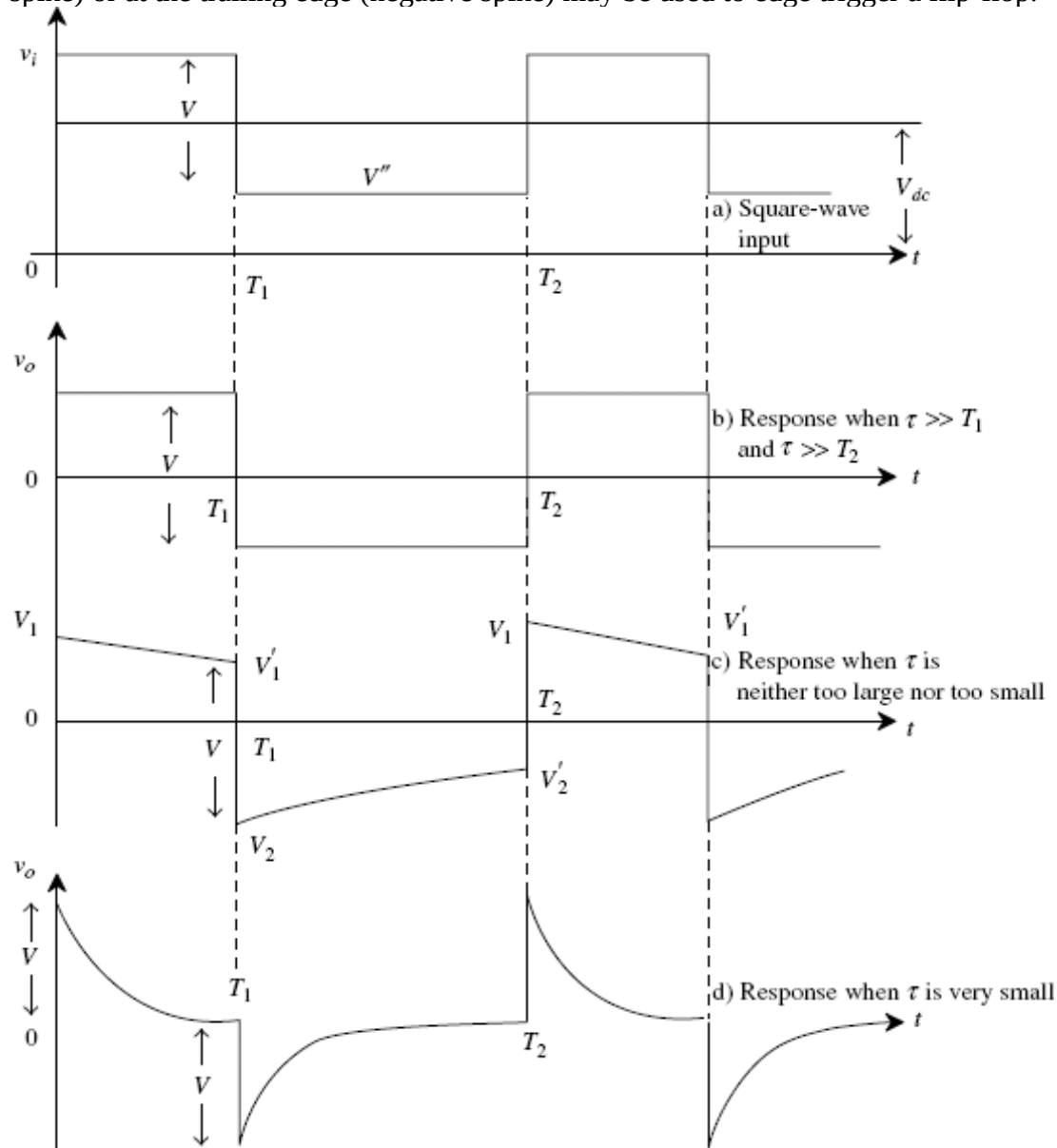
$$v_i(T) - v_i(0) = \frac{1}{\tau} \int_0^T v_o dt + [v_o(T) - v_o(0)]$$

Under steady state conditions, the output and the input waveforms are repetitive with a time Period T . Therefore, $v_i(T) = v_o(T)$ and $v_i(0) = v_o(0)$. Hence, from the above Eq.

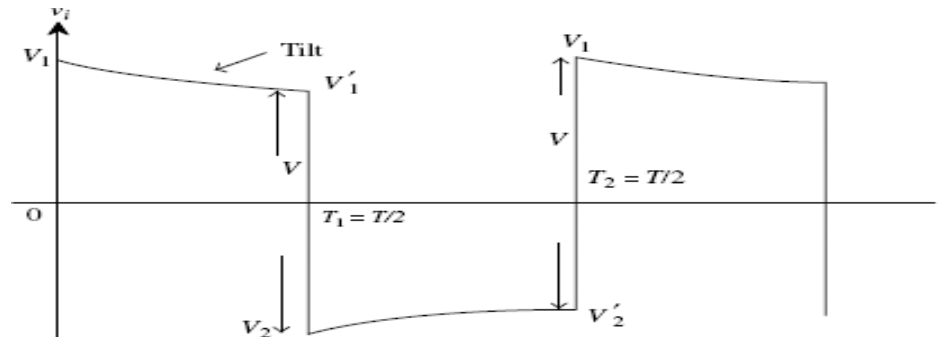
$$\int_0^T v_o dt = 0$$

As the area under the output waveform over one cycle represents the DC component in the output, from above Eq. it is evident that the DC component in the steady-state is always zero. Now let us consider the response of the high-pass RC circuit for a square-wave input for different values of the time constant τ , as shown in below Fig.

As is evident from the waveform in Fig.(b), there is no appreciable distortion in the output if τ is large. The output is almost the same as the input except for the fact that there is no DC component in the output. As τ decreases, as in Fig. (c), there is a tilt in the positive duration (amplitude decreases from V_1 to V_1' during the period 0 to T_1) and there is also a tilt in the negative duration (amplitude increases from V_2 to V_2' during the period T_1 to T_2). A further decrease in the value of τ [see Fig. (d)] gives rise to positive and negative spikes. There is absolutely no resemblance between the signals at the input and the output. However, this condition is imposed on high-pass circuits to derive spikes. In case a pulse is required to trigger another circuit, we see that the pulses obtained either at the rising edge (positive spike) or at the trailing edge (negative spike) may be used to edge trigger a flip-flop.



Let us consider the typical response of the high-pass circuit for a square-wave input shown in Fig below



From the above fig

$$V'_1 = V_1 e^{-T_1/\tau} \quad \text{and} \quad V'_1 - V_2 = V$$

$$V'_2 = V_2 e^{-T_2/\tau} \quad \text{and} \quad V_1 - V'_2 = V$$

..... (a)

For a symmetric square wave $T_1 = T_2 = T/2$. And, because of symmetry:

$$V_1 = -V_2 \quad \text{and} \quad V'_1 = -V'_2$$

.....(b)

From Eq (a) $V'_1 - V_2 = V$

But $V'_1 = V_1 e^{-T_1/\tau}$

Therefore,

$$V_1 e^{-T_1/\tau} - V_2 = V$$

.....(c)

From Eq (b) $V_1 = -V_2$

Substituting in Eq. (c):

$$V_1 e^{-T_1/\tau} + V_1 = V \quad = \quad V_1 (1 + e^{-T_1/\tau}) = V$$

Thus

$$V_1 = \frac{V}{1 + e^{-T_1/\tau}}$$

For a symmetric square wave, as $T_1 = T_2 = T/2$, then from above Eq. (2.39) we can written as:

$$V_1 = \frac{V}{1 + e^{-T/2\tau}}$$

$$V_1' = V_1 e^{-T/2\tau} = \frac{V}{(1 + e^{-T/2\tau})} \times e^{-T/2\tau}$$

There is a tilt in the output waveform. The percentage tilt, P , is defined as:

$$P = \frac{V_1 - V_1'}{\frac{V}{2}} \times 100\%$$

$$P = \frac{\frac{V}{1 + e^{-T/2\tau}} - \frac{V e^{-T/2\tau}}{1 + e^{-T/2\tau}}}{\frac{V}{2}} \times 100\% = \frac{V}{1 + e^{-T/2\tau}} \frac{[1 - e^{-T/2\tau}]}{\frac{V}{2}} \times 100\%$$

$$P = \frac{(1 - e^{-T/2\tau})}{(1 + e^{-T/2\tau})} \times 200\%$$

If $T/2\tau \ll 1$,

$$e^{-T/2\tau} = 1 - \frac{T}{2\tau}$$

Therefore

$$P = \frac{1 - \left(1 - \frac{T}{2\tau}\right)}{1 + \left(1 - \frac{T}{2\tau}\right)} \times 200\% = \frac{\frac{T}{2\tau}}{2 - \frac{T}{2\tau}} \times 200\% \cong \frac{T}{2\tau} \times 100\% \text{ since } \frac{T}{2\tau} \ll 1$$

Thus, for a symmetrical square wave:

$$P = \frac{T}{2\tau} \times 100\%,$$

Above Eq tells us that the smaller the value of τ when compared to the half-period of the square wave ($T/2$), the larger is the value of P . In other words, distortion is large with small τ and is small with large τ . The lower half-power frequency, $f_1 = 1/2\pi\tau$.

Therefore,

$$\frac{1}{2\tau} = \pi f_1$$

$$\text{So } P = \pi f_1 T \times 100\%$$

$$P = \frac{\pi f_1}{f} \times 100\% \quad \text{since } T = \frac{1}{f}$$

High pass RC circuit response to ramp input Voltage:

A waveform which is defined as: $V_i(t) = \begin{cases} 0 & \text{for } t < 0 \\ \alpha t & \text{for } t > 0 \end{cases}$

Then the output is $V_o(t) = i(t) R$

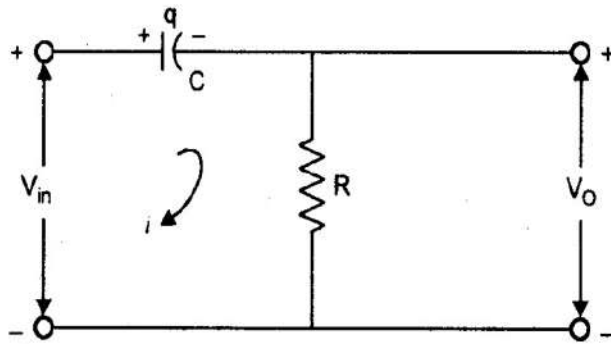


Fig. High pass RC circuit

$$\frac{dv_i(t)}{dt} = \frac{v_o(t)}{RC} + \frac{dv_o(t)}{dt}$$

Which becomes

$$\alpha = \frac{v_o(t)}{RC} + \frac{dv_o(t)}{dt}$$

This equation has the solution for $V(t)$ at $t = 0$. Taking Laplace Transform on both sides of the above equation becomes

$$\begin{aligned} \frac{\alpha}{s} &= \frac{v_o(s)}{RC} + sV_o(s) \\ &= V_o(s) \left[s + \frac{1}{RC} \right] \end{aligned}$$

$$v_o(s) = \frac{\alpha}{s \left[s + \frac{1}{RC} \right]}$$

By taking Inverse Laplace Transform,

$$v_o(t) = \alpha RC \left[1 - e^{-t/RC} \right]$$

For times ' t ' $\ll RC$,

$$v_o(t) = \alpha t \left[1 - \frac{t}{2RC} + \dots \right]$$

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The output signal falls away slightly from the input. As a measure of the departure from linearity let us define the transmission error e_t , as the difference between input and output divided by the input. The error at a time $t = T$, is then

$$e_t = \frac{v_i(\tau) - v_o(\tau)}{v_i(\tau)}$$

$$e_t = \frac{T}{2RC} = \pi f_1 T$$

where $f_1 = 1/2\pi RC$ is again the low frequency 3-dB point.

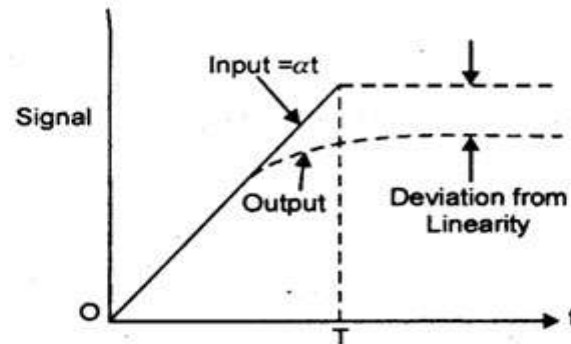
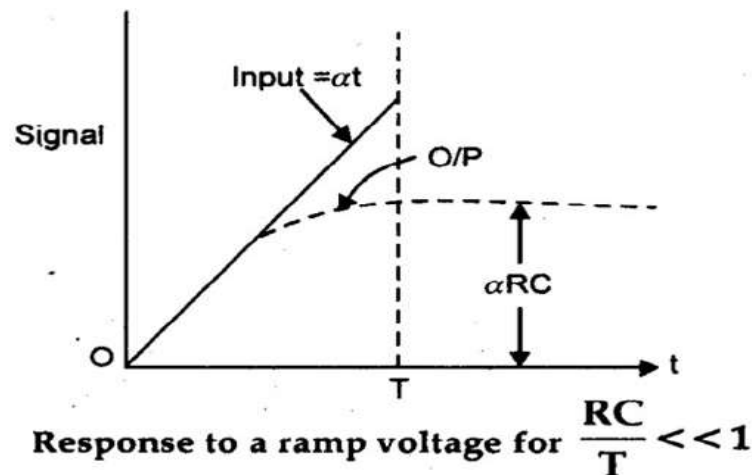


Fig. Response of a high-pass RC circuit to a ramp voltage for $\frac{RC}{T} \gg 1$

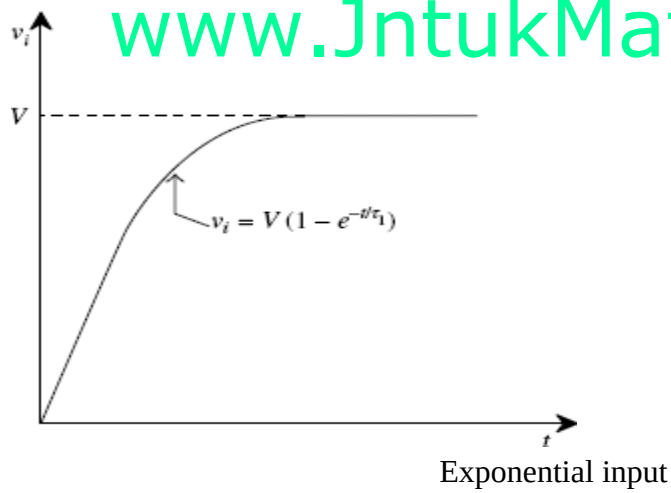


Response to a ramp voltage for $\frac{RC}{T} \ll 1$

Response of the High-pass RC Circuit to Exponential Input:

When a pulse is applied as input to an amplifier, it may while appearing at the actual input terminals of the amplifier, have a finite rise time. The result is that the input to the amplifier is

no longer a pulse with sharp rising edge, but an exponential. We would now like to know the response of the high-pass circuit to this exponential input. If the input to the high-pass circuit in Fig below is an exponential of the form:



We know

$$\frac{dv_i}{dt} = \frac{v_o}{\tau} + \frac{dv_o}{dt} \quad \dots\dots\dots(d)$$

As $v_i = V(1 - e^{-t/\tau_1})$,

$$\frac{dv_i}{dt} = \frac{V}{\tau_1} e^{-t/\tau_1}$$

Substituting above Eq. in Eq. (d): then

$$\frac{V}{\tau_1} e^{-t/\tau_1} = \frac{v_o}{\tau} + \frac{dv_o}{dt}$$

Taking Laplace transform on both sides:

$$\frac{\frac{V}{\tau_1}}{\left(s + \frac{1}{\tau_1}\right)} = \frac{v_o(s)}{\tau} + s v_o(s)$$

where, τ is the time constant of the high-pass circuit.

$$\frac{\frac{V}{\tau_1}}{\left(s + \frac{1}{\tau_1}\right)} = v_o(s) \left(s + \frac{1}{\tau}\right)$$

Therefore

$$v_o(s) = \frac{\frac{V}{\tau_1}}{\left(s + \frac{1}{\tau_1}\right) \left(s + \frac{1}{\tau}\right)}$$

Case 1: $\tau = \tau_1$

Applying partial fractions, above Eq. can be written as:

$$v_o(s) = \frac{A}{\left(s + \frac{1}{\tau_1}\right)} + \frac{B}{\left(s + \frac{1}{\tau}\right)} = \frac{\frac{V}{\tau_1}}{\left(s + \frac{1}{\tau_1}\right)\left(s + \frac{1}{\tau}\right)}$$

Therefore

$$\frac{V}{\tau_1} = A\left(s + \frac{1}{\tau}\right) + B\left(s + \frac{1}{\tau_1}\right)$$

Put $s = -1/\tau_1$ in above Eq.

$$\frac{V}{\tau_1} = A\left(\frac{-1}{\tau_1} + \frac{1}{\tau}\right) \quad \text{or} \quad A = \frac{\frac{V}{\tau_1}}{\left(\frac{1}{\tau} - \frac{1}{\tau_1}\right)} = \frac{V}{\left(\frac{\tau_1}{\tau} - 1\right)}$$

Now put $s = -1/\tau$, then

$$\frac{V}{\tau_1} = B\left(\frac{1}{\tau_1} - \frac{1}{\tau}\right)$$

Therefore

$$B = \frac{-V}{\left(\frac{\tau_1}{\tau} - 1\right)}$$

Substituting the values of A and B in $V_o(s)$:

$$v_o(s) = \frac{V}{\left(\frac{\tau_1}{\tau} - 1\right)\left(s + \frac{1}{\tau_1}\right)} - \frac{V}{\left(\frac{\tau_1}{\tau} - 1\right)\left(s + \frac{1}{\tau}\right)} = \frac{V}{\left(\frac{\tau_1}{\tau} - 1\right)} \left[\frac{1}{\left(s + \frac{1}{\tau_1}\right)} - \frac{1}{\left(s + \frac{1}{\tau}\right)} \right]$$

Taking inverse Laplace transform:

$$v_o(t) = \frac{V}{\left(\frac{\tau_1}{\tau} - 1\right)} (e^{-t/\tau_1} - e^{-t/\tau})$$

This is the expression for the output voltage where $\tau \neq \tau_1$.

Let $t/\tau_1 = x$ and $\tau/\tau_1 = n$. For $n \neq 1$, i.e., $\tau \neq \tau_1$, we have from above Eq:

$$v_o(t) = \frac{V}{\left(\frac{1}{n} - 1\right)} (e^{-x} - e^{-x/n}) \quad \text{since } \frac{t}{\tau_1} \times \frac{\tau_1}{\tau} = \frac{x}{n} = \frac{t}{\tau}$$

Therefore

$$v_o(t) = \frac{Vn}{(1-n)} (e^{-x} - e^{-x/n}) = \frac{Vn}{(n-1)} (e^{-x/n} - e^{-x})$$

If $\tau \gg \tau_1$, the second term in the above Eq. is small when compared to the first. Thus,

$$v_o(t) \cong \frac{Vn}{(n-1)} e^{-x/n} = \frac{Vn}{(n-1)} e^{-t/\tau}$$

Case 2: $\tau = \tau_1$, that is, $n = 1$.

$$v_o(s) = \frac{\frac{V}{\tau}}{\left(s + \frac{1}{\tau}\right)\left(s + \frac{1}{\tau}\right)} = \frac{\frac{V}{\tau}}{\left(s + \frac{1}{\tau}\right)^2}$$

Taking Laplace inverse:

$$v_o(t) = \frac{V}{\tau} t e^{-t/\tau}$$

As $t/\tau = x = t/\tau_1$ and $\tau/\tau_1 = n = 1$:

$$v_o(t) = V x e^{-x}$$

The response of the circuit is plotted for different values of n in Fig. below

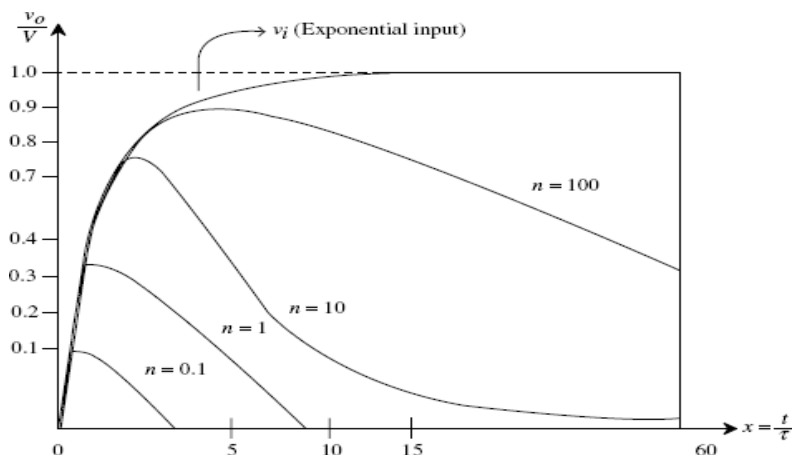


Fig: Response of a high-pass circuit to an exponential input

$$V \frac{n}{n-1} \left[\left(\frac{n-1}{n} \right) \left(\frac{1}{\tau} \right) e^{-x/n} - e^{-x} \left(\frac{n-1}{\tau} \right) \right] = 0$$

$$V \frac{n}{n-1} \left[\left(\frac{e^{-x}}{\tau} - \frac{e^{-x/n}}{n\tau} \right) \right] = 0 \quad e^{-x} = \frac{e^{-x/n}}{n}$$

$$n = e^{x \left(1 - \frac{1}{n} \right)} = e^{\left(\frac{x(n-1)}{n} \right)}$$

$$\ln n = \frac{x(n-1)}{n}$$

$$x = \frac{n}{n-1} \ln n$$

Since $x = t/\tau$, from above Eq., the time taken to rise to the peak t_p is given by:

$$t_p = \tau \frac{n}{n-1} \ln n$$

From above Eq.

$$-x = \frac{-n}{n-1} \ln n = \ln \left[n^{n/(1-n)} \right]$$

To obtain the maximum value of the output, substitute this value of $-x$ from above Eq. in the expression for $v_o(t)$ in below Eq.

$$v_o(t) = \frac{Vn}{(1-n)} (e^{-x} - e^{-x/n}) = \frac{Vn}{(n-1)} (e^{-x/n} - e^{-x})$$

$$v_o(\max) = \frac{Vn}{n-1} \exp \left[\frac{1}{n} \ln n^{n/(1-n)} - \ln n^{n/(1-n)} \right]$$

$$= V \frac{n}{n-1} \exp \left[\ln n^{1/(1-n)} - \ln n^{n/(1-n)} \right]$$

$$= V \frac{n}{n-1} \left[n^{1/(1-n)} - n^{n/(1-n)} \right] = \frac{V}{n-1} \left[n^{1+[1/(1-n)]} - n^{1+[n/(1-n)]} \right]$$

$$= \frac{V}{n-1} \left[n^{(2-n)/(1-n)} - n^{1/(1-n)} \right] = \frac{V}{n-1} \left[(n-1)^{1/(1-n)} \right] = V n^{1/(1-n)} \quad \text{for } n \neq 1$$

$$\frac{v_o(\max)}{V} = n^{1/(1-n)} \quad \text{for } n \neq 1$$

From the waveforms in above Figure and the subsequent mathematical relations derived, it is seen that, if an exponential signal is applied as an input to a high-pass circuit, the output is a pulse whose duration depends on $n (= \tau/\tau_1)$, where τ_1 is the time constant of the previous circuit that has generated the exponential signal and τ is the time constant of the high-pass circuit under consideration. The smaller the value of n , the smaller the duration of this output pulse and also the smaller its amplitude. As n increases, the duration as well as the amplitude of this output pulse increases. Hence, depending on our requirement, we adjust the value of n .

DIFFERENTIATORS

Sometimes, a square wave may need to be converted into sharp positive and negative spikes (pulses of short duration). By eliminating the positive spikes, we can generate a train of negative spikes and vice-versa. The pulses so generated may be used to trigger a multivibrator. In such cases, a differentiator is used. If in a circuit, the output is a differential of the input signal, then the circuit is called a differentiator.

A High-pass RC Circuit as a Differentiator

If the time constant of the high-pass RC circuit, shown in Fig. 1.1(a), is much smaller than the time period of the input signal, then the circuit behaves as a differentiator. If T is to be large when compared to τ , then the frequency must be small. At low frequencies, XC is very large when compared to R . Therefore, the voltage drop across R is very small when compared to the drop across C .

$$v_i = \frac{1}{C} \int i dt + iR$$

But $iR = v_o$ is small. Therefore,

$$v_i = \frac{1}{C} \int i dt \quad \text{or} \quad v_i = \frac{1}{\tau} \int v_o dt \quad (\text{since } i = V_o/R)$$

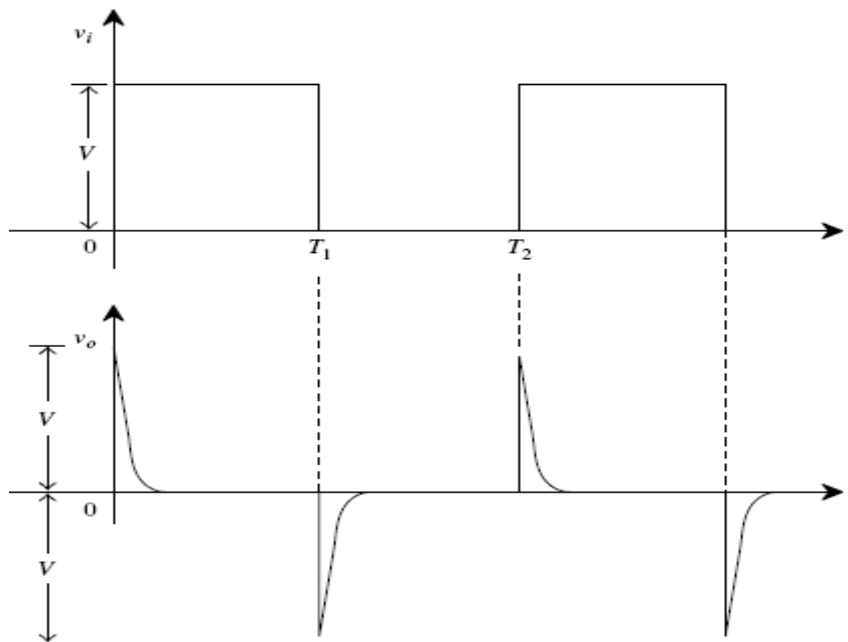


FIGURE: The output of a differentiator

Differentiating:

$$\frac{dv_i}{dt} = \frac{v_o}{\tau}$$

$$v_o = \tau \frac{dv_i}{dt}$$

Therefore

$$v_o \propto \frac{dv_i}{dt}$$

Thus, from above Eq. it can be seen that the output is proportional to the differential of the input signal.

THE LOW-PASS RC CIRCUIT

Figure 1.1 shows a low-pass RC circuit. A low-pass circuit is a circuit, which transmits only low-frequency signals and attenuates or stops high-frequency signals.

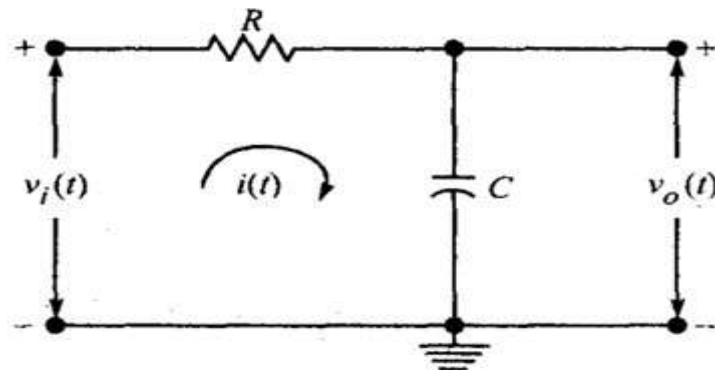


Fig The low-pass RC circuit.

At zero frequency, the reactance of the capacitor is infinity (i.e. the capacitor acts as an open circuit) so the entire input appears at the output, i.e. the input is transmitted to the output with zero attenuation. So the output is the same as the input, i.e. the gain is unity. As the frequency increases the capacitive reactance ($X_c = 1/2\pi fC$) decreases and so the output decreases.

At very high frequencies the capacitor virtually acts as a short-circuit and the output falls to zero.

Sinusoidal Input

The Laplace transformed low-pass RC circuit is shown in Figure (a). The gain versus frequency curve of a low-pass circuit excited by a sinusoidal input is shown in Figure (b). This curve is obtained by keeping the amplitude of the input sinusoidal signal constant and varying its frequency and noting the output at each frequency. At low frequencies the output is equal to the input and hence the gain is unity. As the frequency increases, the output decreases and hence the gain decreases. The frequency at which the gain is $1/\sqrt{2}$ ($= 0.707$) of its maximum value is called the cut-off frequency. For a low-pass circuit, there is no lower cut-off frequency. It is zero itself. The upper cut-off frequency is the frequency (in the high-frequency range) at which the gain is $1/\sqrt{2}$ i.e. 70.7%, of its maximum value. The bandwidth of the low-pass circuit is equal to the upper cut-off frequency f_2 itself.

For the network shown in Figure 1.2(a), the magnitude of the steady-state gain A is given by

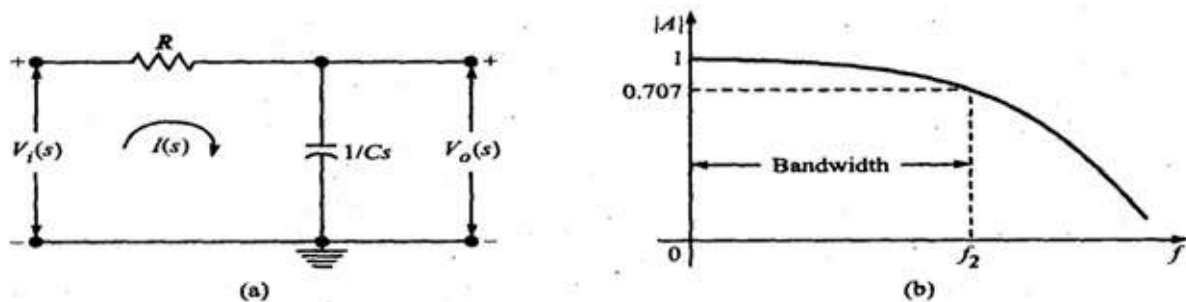


Figure (a) Laplace transformed low-pass RC circuit and (b) its frequency response.

$$A = \frac{V_o(s)}{V_i(s)} = \frac{\frac{1}{Cs}}{R + \frac{1}{Cs}} = \frac{1}{1 + RCs} = \frac{1}{1 + j\omega RC} = \frac{1}{1 + j2\pi fRC}$$

$$\therefore |A| = \frac{1}{\sqrt{1 + (2\pi fRC)^2}}$$

At the upper cut-off frequency f_2 , $|A| = \frac{1}{\sqrt{2}}$

$$\therefore \frac{1}{\sqrt{2}} = \frac{1}{\sqrt{1 + (2\pi f_2 RC)^2}}$$

Squaring both sides and equating the denominators,

$$2 = 1 + (2\pi f_2 RC)^2$$

$$\therefore \text{The upper cut-off frequency, } f_2 = \frac{1}{2\pi RC}$$

So $A = \frac{1}{1 + j\frac{f}{f_2}}$ and $|A| = \frac{1}{\sqrt{1 + \left(\frac{f}{f_2}\right)^2}}$

The angle θ by which the output leads the input is given by

$$\theta = \tan^{-1} \frac{f}{f_2}$$

Step-Voltage Input

A step signal is one which maintains the value zero for all times $t < 0$, and maintains the value V for all times $t > 0$. The transition between the two voltage levels takes place at $t = 0$ and is accomplished in an arbitrarily small time interval. Thus, in Figure (a), $v_i = 0$ immediately before $t = 0$ (to be referred to as time $t = 0^-$) and $v_i = V$, immediately after $t = 0$ (to be referred to as time $t = 0^+$). In the low-pass RC circuit shown in Figure 1.1, if the capacitor is initially uncharged, when a step input is applied, since the voltage across the capacitor cannot change instantaneously, the output will be zero at $t = 0$, and then, as the capacitor charges, the output voltage rises exponentially towards the steady-state value V with a time constant RC as shown in Figure (b).

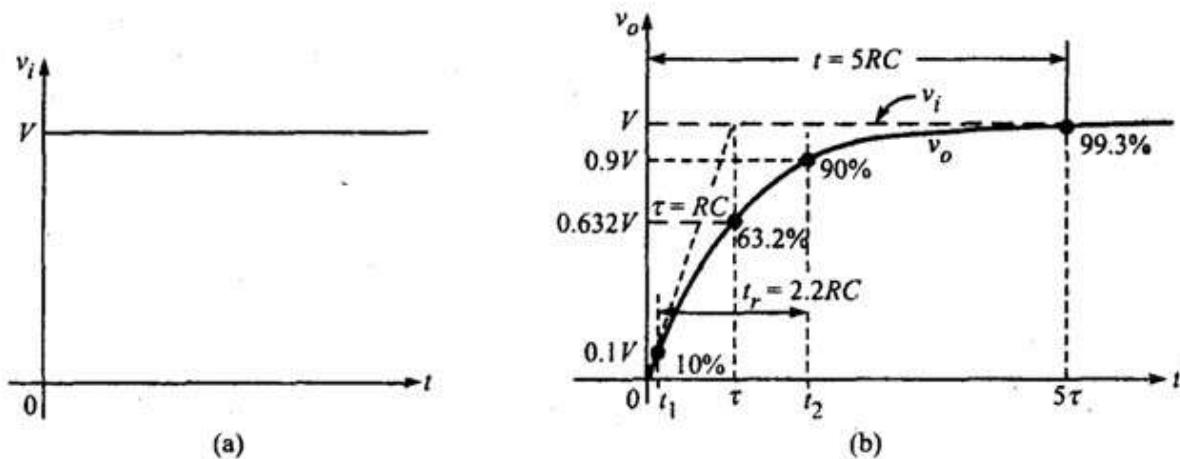


Figure (a) Step input and (b) step response of the low-pass RC circuit.

Let V' be the initial voltage across the capacitor. Writing KVL around the loop in Fig 1.1.

$$v_i(t) = Ri(t) + \frac{1}{C} \int i(t) dt$$

Differentiating this equation,

$$\frac{dv_i(t)}{dt} = R \frac{di(t)}{dt} + \frac{1}{C} i(t)$$

Since $v_i(t) = V, \quad \frac{dv_i(t)}{dt} = 0$

$$\therefore 0 = R \frac{di(t)}{dt} + \frac{1}{C} i(t)$$

Taking the Laplace transform on both sides,

$$0 = R [sI(s) - I(0^+)] + \frac{1}{C} I(s)$$

$$\therefore I(0^+) = I(s) \left(s + \frac{1}{RC} \right)$$

The initial current $I(0^+)$ is given by

$$I(0^+) = \frac{V - V'}{R}$$

$$\therefore I(s) = \frac{I(0^+)}{s + \frac{1}{RC}} = \frac{V - V'}{R \left(s + \frac{1}{RC} \right)}$$

and $V_o(s) = V_i(s) - I(s)R = \frac{V}{s} - \frac{(V - V')R}{R \left(s + \frac{1}{RC} \right)} = \frac{V}{s} - \frac{V - V'}{s + \frac{1}{RC}}$

Taking the inverse Laplace transform on both sides,

$$v_o(t) = V - (V - V')e^{-t/RC}$$

where V' is the initial voltage across the capacitor (V_{initial}) and V is the final voltage (V_{final}) to which the capacitor can charge.

So, the expression for the voltage across the capacitor of an RC circuit excited by a step input is given by

$$v_o(t) = V_{\text{final}} - (V_{\text{final}} - V_{\text{initial}})e^{-t/RC}$$

If the capacitor is initially uncharged, then $v_o(t) = V(1 - e^{-t/RC})$

Expression for rise time

When a step signal is applied, the rise time t_r is defined as the time taken by the output voltage waveform to rise from 10% to 90% of its final value: It gives an indication of how fast the circuit can respond to a discontinuity in voltage. Assuming that the capacitor is initially uncharged, the output voltage shown in Figure (b) at any instant of time is given by

$$v_o(t) = V(1 - e^{-t/RC})$$

At $t = t_1$, $v_o(t) = 10\%$ of $V = 0.1V$

$$\therefore 0.1V = V(1 - e^{-t_1/RC})$$

$$\therefore e^{-t_1/RC} = 0.9 \quad \text{or} \quad e^{t_1/RC} = \frac{1}{0.9} = 1.11$$

$$\therefore t_1 = RC \ln(1.11) = 0.1RC$$

At $t = t_2$, $v_o(t) = 90\%$ of $V = 0.9V$

$$\therefore 0.9V = V(1 - e^{-t_2/RC})$$

$$\therefore e^{-t_2/RC} = 0.1 \quad \text{or} \quad e^{t_2/RC} = \frac{1}{0.1} = 10$$

$$\therefore t_2 = RC \ln 10 = 2.3RC$$

$$\therefore \text{Rise time, } t_r = t_2 - t_1 = 2.2RC$$

This indicates that the rise time t_r is proportional to the time constant RC of the circuit. The larger the time constant, the slower the capacitor charges, and the smaller the time constant, the faster the capacitor charges.

Relation between rise time and upper 3-dB frequency

We know that the upper 3-dB frequency (same as bandwidth) of a low-pass circuit is

$$f_2 = \frac{1}{2\pi RC} \quad \text{or} \quad RC = \frac{1}{2\pi f_2}$$

$$\therefore \text{Rise time, } t_r = 2.2RC = \frac{2.2}{2\pi f_2} = \frac{0.35}{f_2} = \frac{0.35}{\text{BW}}$$

Thus, the rise time is inversely proportional to the upper 3-dB frequency. The *time constant* ($T = RC$) of a circuit is defined as the time taken by the output to rise to 63.2% of the amplitude of the input step. It is same as the time taken by the output to rise to 100% of the amplitude of the input step, if the initial slope of rise is maintained. See Figure (b). The Greek letter T is also employed as the symbol for the time constant.

Pulse Input

The pulse shown in Figure (a) is equivalent to a positive step followed by a delayed negative step as shown in Figure (b). So, the response of the low-pass RC circuit to a pulse for times less than the pulse width tp is the same as that for a step input and is given by $v_o(t) = V(1 - e^{-t/RC})$. The responses of the low-pass RC circuit for time constant $RC \gg tp$, RC smaller than tp and RC very small compared to tp are shown in Figures (c), (d), and (e) respectively.

If the time constant RC of the circuit is very large, at the end of the pulse, the output voltage will

be $V_p(t) = V(1 - e^{-tp/RC})$ and the output will decrease to zero from this value with a time constant

RC as shown in Figure (c). Observe that the pulse waveform is distorted when it is passed through a linear network. The output will always extend beyond the pulse width tp , because whatever charge has accumulated across the capacitor C during the pulse cannot leak off instantaneously.

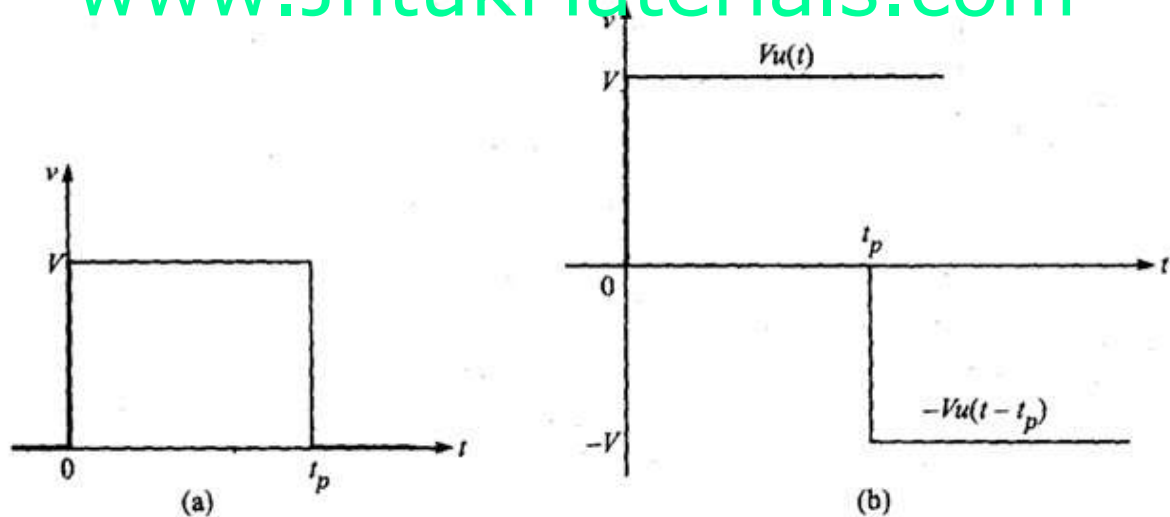


Figure (a) A pulse and (b) a pulse in terms of steps.

If the time constant RC of the circuit is very small, the capacitor charges and discharges very quickly and the rise time t_r will be small and so the distortion in the wave shape is small. For minimum distortion (i.e. for preservation of wave shape), the rise time must be small compared to the pulse width t_p . If the upper 3-dB frequency f_2 is chosen equal to the reciprocal of the pulse width t_p , i.e. if $f_2 = 1/t_p$ then $t_r = 0.35t_p$ and the output is as shown in Figure 1.5(b), which for many applications is a reasonable reproduction of the input. As a rule of thumb, we can say:

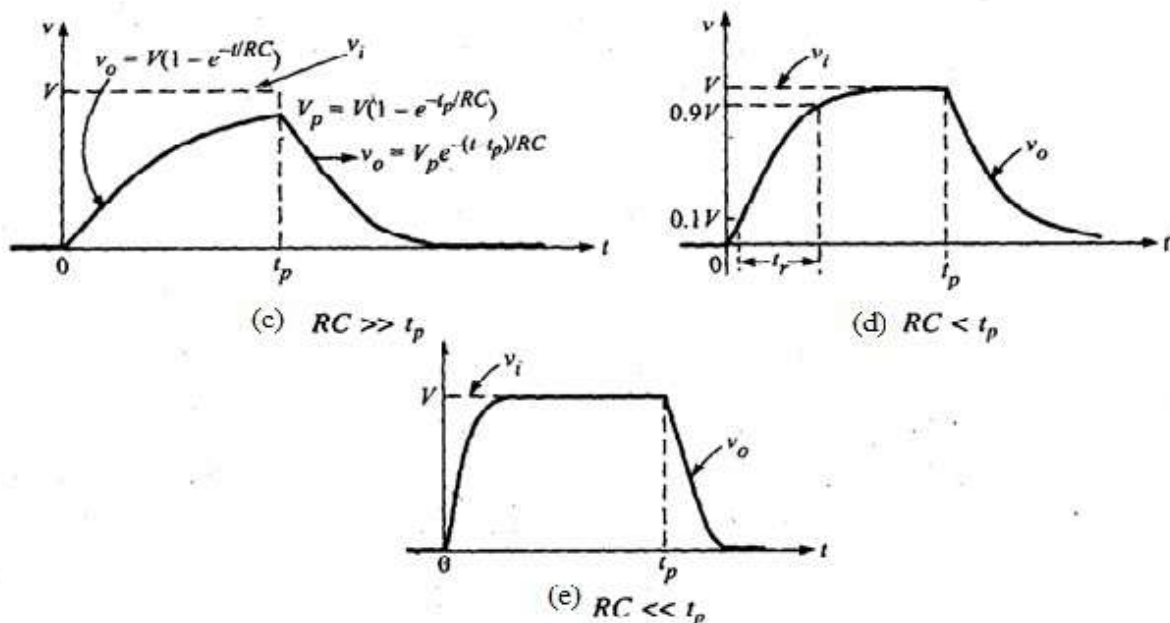


Figure Pulse response for $RC \gg t_p$, $RC < t_p$, and $RC \ll t_p$.

A pulse shape will be preserved if the 3-dB frequency is approximately equal to the reciprocal of the pulse width.

Thus to pass a $0.25 \mu\text{s}$ pulse reasonably well requires a circuit with an upper cut-off frequency of the order of 4 MHz.

Square-Wave Input

A square wave is a periodic waveform which maintains itself at one constant level V' with respect to ground for a time T_1 and then changes abruptly to another level V'' , and remains constant at that level for a time T_2 , and repeats itself at regular intervals of $T = T_1 + T_2$. A square wave may be treated as a series of positive and negative steps.

The shape of the output waveform for a square wave input depends on the time constant of the circuit. If the time constant is very small, the rise time will also be small and a reasonable reproduction of the input may be obtained.

For the square wave shown in Figure (a), the output waveform will be as shown in Figure (b) if the time constant RC of the circuit is small compared to the period of the input waveform. In this case, the wave shape is preserved. If the time constant is comparable with the period of the input square wave, the output will be as shown in Figure (c). The output rises and falls exponentially. If the time constant is very large compared to the period of the input waveform, the output consists of exponential sections, which are essentially linear as indicated in

Figure (d). Since the average voltage across R is zero, the dc voltage at the output is the same as that of the input. This average value is indicated as V_{dc} in all the waveforms.

In Figure 1.6(c), the equation for the rising portion is

$$v_{01} = V' - (V' - V_2)e^{-t/RC}$$

where V_2 is the voltage across the capacitor at $t = 0$, and V' is the level to which the capacitor can charge.

The equation for the falling portion is

$$v_{02} = V'' - (V'' - V_1)e^{-(t - T_1)/RC}$$

where V_1 is the voltage across the capacitor at $t = T_1$ and V'' is the level to which the capacitor can discharge.

Setting $v_{01} = V_1$ at $t = T_1$,

$$V_1 = V' - (V' - V_2)e^{-T_1/RC} = V'(1 - e^{-T_1/RC}) + V_2e^{-T_1/RC}$$

Setting $v_{02} = V_2$ at $t = T_1 + T_2$,

$$V_2 = V'' - (V'' - V_1)e^{-(T_1+T_2-T_1)/RC} = V''(1 - e^{-T_2/RC}) + V_1e^{-T_2/RC}$$

Substituting this value of V_2 in the expression for V_1 ,

$$V_1 = V'(1 - e^{-T_1/RC}) + [V''(1 - e^{-T_2/RC}) + V_1e^{-T_2/RC}]e^{-T_1/RC}$$

i.e.

$$V_1 = \frac{V'(1 - e^{-T_1/RC}) + V''(1 - e^{-T_2/RC})e^{-T_1/RC}}{1 - e^{-(T_1+T_2)/RC}}$$

Similarly substituting the value of V_1 in the expression for V_2 ,

$$V_2 = \frac{V''(1 - e^{-T_2/RC}) + V'(1 - e^{-T_1/RC})e^{-T_2/RC}}{1 - e^{-(T_1+T_2)/RC}}$$

For a symmetrical square wave with zero average value,

$$T_1 = T_2 = \frac{T}{2} \quad \text{and} \quad V' = -V'' = \frac{V}{2}. \quad \text{So, } V_2 \text{ will be equal to } -V_1$$

$$V_1 = \frac{\frac{V}{2}(1 - e^{-T/2RC}) - \frac{V}{2}(1 - e^{-T/2RC})e^{-T/2RC}}{1 - e^{-T/RC}}$$

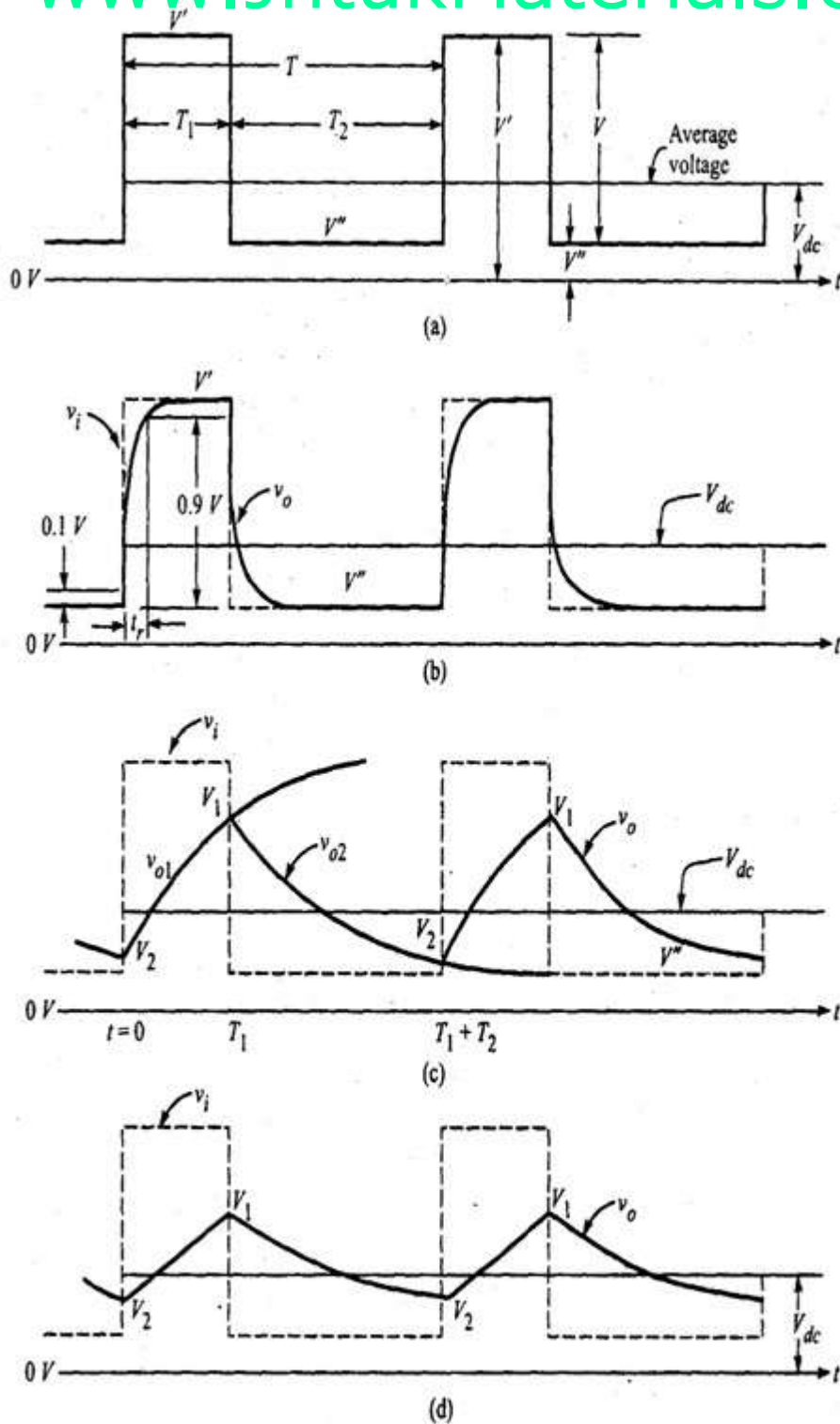


Figure Response of a low-pass RC circuit to a square wave input: (a) square-wave input wave form, (b) output waveform for $RC \ll T$, (c) output waveform for $RC = T$, and (d) output waveform for $RC \gg T$.

$$\begin{aligned}
 &= \frac{V}{2} \frac{1 - e^{-T/2RC} - e^{-T/2RC} + e^{-T/RC}}{1 - e^{-T/RC}} \\
 &= \frac{V}{2} \frac{(1 - e^{-T/2RC})^2}{(1 + e^{-T/2RC})(1 - e^{-T/2RC})} \\
 &= \frac{V}{2} \left(\frac{1 - e^{-T/2RC}}{1 + e^{-T/2RC}} \right) \\
 &= \frac{V}{2} \left(\frac{e^{T/2RC} - 1}{e^{T/2RC} + 1} \right) \\
 &= \frac{V}{2} \left(\frac{e^{2x} - 1}{e^{2x} + 1} \right) = \frac{V}{2} \tanh x
 \end{aligned}$$

where $x = \frac{T}{4RC}$ and T is the period of the square wave.

Now,
$$V_2 = -V_1 = -\frac{V}{2} \left(\frac{1 - e^{-T/2RC}}{1 + e^{-T/2RC}} \right) = \frac{V}{2} \left(\frac{1 - e^{-T/2RC}}{1 + e^{-T/2RC}} \right)$$

Ramp Input

When a low-pass RC circuit shown in Figure is excited by a ramp input, i.e.

$$v_i(t) = \alpha t, \text{ where } \alpha \text{ is the slope of the ramp}$$

we have,
$$V_i(s) = \frac{\alpha}{s^2}$$

From the frequency domain circuit of Figure (a), the output is given by

$$\begin{aligned}
 V_o(s) &= V_i(s) \frac{\frac{1}{Cs}}{R + \frac{1}{Cs}} = \frac{\alpha}{s^2} \cdot \frac{1}{1 + RCs} = \frac{\alpha}{RC} \frac{1}{s^2 \left(s + \frac{1}{RC} \right)} \\
 &= \frac{\alpha}{RC} \left[\frac{-(RC)^2}{s} + \frac{RC}{s^2} + \frac{(RC)^2}{s + \frac{1}{RC}} \right]
 \end{aligned}$$

$$\text{i.e. } V_o(s) = \frac{-\alpha RC}{s} + \frac{\alpha}{s^2} + \frac{\alpha RC}{s + \frac{1}{RC}}$$

Taking the inverse Laplace transform on both sides,

$$\begin{aligned}
 v_o(t) &= -\alpha RC + \alpha t + \alpha RC e^{-t/RC} \\
 &= \alpha(t - RC) + \alpha RC e^{-t/RC}
 \end{aligned}$$

If the time constant RC is very small, $e^{-t/RC} \approx 0$

$$\therefore v_o(t) = \alpha(t - RC)$$

When the time constant is very small relative to the total ramp time T , the ramp will be transmitted with minimum distortion. The output follows the input but is delayed by one time constant RC from the input (except near the origin where there is distortion) as shown in Figure (a). If the time constant is large compared with the sweep duration, i.e. if $RC/T \gg 1$, the output will be highly distorted as shown in Figure (b).

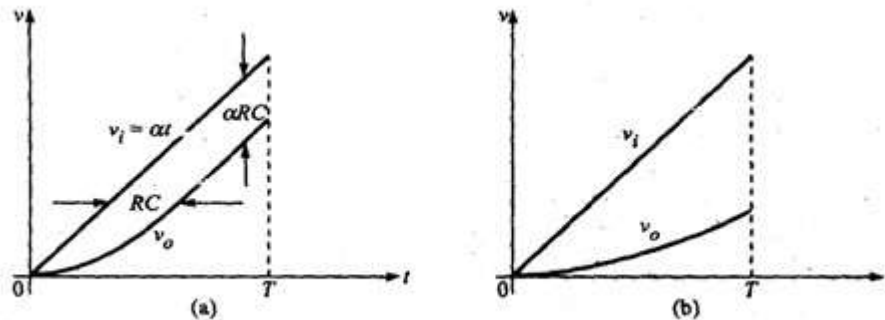


Figure Response of a low-pass RC circuit for a ramp input for (a) $RC/T \ll 1$ and (b) $RC/T \gg 1$.

Expanding $e^{-t/RC}$ in to an infinite series in t/RC in the above equation for $v_o(t)$,

$$\begin{aligned} v_o(t) &= \alpha(t - RC) + \alpha RC \left(1 - \frac{t}{RC} + \left(\frac{t}{RC} \right)^2 \frac{1}{2!} - \left(\frac{t}{RC} \right)^3 \frac{1}{3!} + \dots \right) \\ &= \alpha t - \alpha RC + \alpha RC - \alpha t + \frac{\alpha t^2}{2RC} - \dots \\ &\approx \frac{\alpha t^2}{2RC} = \frac{\alpha}{RC} \left(\frac{t^2}{2} \right) \end{aligned}$$

This shows that a quadratic response is obtained for a linear input and hence the circuit acts as an integrator for $RC/T \gg 1$.

The transmission error e_t for a ramp input is defined as the difference between the input and the output divided by the input at the end of the ramp, i.e. at $t = T$.

For $RC/T \ll 1$,

$$\begin{aligned} e_t &= \frac{\alpha t - (\alpha t - \alpha RC)}{\alpha t} \Big|_{t=T} \\ &= \frac{\alpha RC}{\alpha T} = \frac{RC}{T} = \frac{1}{2\pi f_2 T} \end{aligned}$$

where f_2 is the upper 3-dB frequency. For example, if we desire to pass a 2 ms pulse with less than 0.1% error, the above equation yields $f_2 > 80$ kHz and $RC < 2$ μ s.

Exponential Input

For the low-pass RC circuit shown in Figure 1.1, let the input applied as shown in Figure 1.8 be $v_i(t) = V(1 - e^{-t/T})$, where T is the time constant of the input waveform.

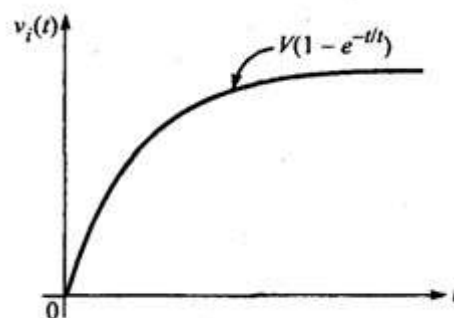


Figure Exponential input.

Writing the KVL around the loop,

$$v_i(t) = Ri(t) + v_o(t) = RC \frac{dv_o(t)}{dt} + v_o(t)$$

$$\therefore V(1 - e^{-t/\tau}) = RC \frac{dv_o(t)}{dt} + v_o(t)$$

Taking the Laplace transform on both sides and neglecting the initial conditions,

$$\frac{V}{s} - \frac{V}{s + \frac{1}{\tau}} = RCsV_o(s) + V_o(s)$$

$$\text{i.e. } V \left[\frac{\frac{1}{\tau}}{s \left(s + \frac{1}{\tau} \right)} \right] = V_o(s)(RCs + 1) = RCV_o(s) \left(s + \frac{1}{RC} \right)$$

$$\therefore V_o(s) = \frac{V}{RC\tau} \left[\frac{1}{s \left(s + \frac{1}{\tau} \right) \left(s + \frac{1}{RC} \right)} \right]$$

$$= V \left[\frac{1}{s} - \frac{1}{\left(1 - \frac{RC}{\tau} \right) \left(s + \frac{1}{\tau} \right)} + \frac{1}{\left(\frac{\tau}{RC} - 1 \right) \left(s + \frac{1}{RC} \right)} \right]$$

Taking the inverse Laplace transform on both sides and letting $RC/\tau = n$,

$$v_o(t) = V \left[1 - \frac{e^{-t/\tau}}{1 - n} + \frac{e^{-t/RC}}{\frac{1}{n} - 1} \right]$$

$$\text{If } t/\tau = x, \text{ then } v_o(t) = V \left[1 - \frac{e^{-x}}{1 - n} + \frac{n}{1 - n} e^{-x/n} \right], \text{ if } n \neq 1$$

$$\text{and } v_o(t) = 1 - (1 + x)e^{-x}, \quad \text{if } n = 1$$

These are the expressions for the voltage across the capacitor of a low-pass RC circuit excited by an exponential input of rise time $tr1 - 2.2r$.

If an exponential of rise time $tr1$ is passed through a low-pass circuit with rise time $tr2$, the rise time of the output waveform tr will be given by an empirical relation, $t_r = 1.05\sqrt{t_{r1}^2 + t_{r2}^2}$. This is same as the rise time obtained when a step is applied to a cascade of two circuits of rise times t_{r1} and t_{r2} assuming that the second circuit does not load the first.

THE LOW-PASS RC CIRCUIT AS AN INTEGRATOR

If the time constant of an RC low-pass circuit is very large, the capacitor charges very slowly and so almost all the input voltage appears across the resistor for small values of time.

$$v_o(t) = \frac{1}{C} \int i(t) dt = \frac{1}{C} \int \frac{v_i(t)}{R} dt = \frac{1}{RC} \int v_i(t) dt$$

Hence the output is the integral of the input, i.e. if $v_i(t) = \alpha t$, then

$$v_o(t) = \frac{\alpha t^2}{2RC}$$

As time increases, the voltage drop across C does not remain negligible compared with that across R and the output will not remain the integral of the input. The output will change from a quadratic to a linear function of time.

If the time constant of an RC low-pass circuit is very large in comparison with the time required for the input signal to make an appreciable change, the circuit acts as an integrator.

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A criterion for good integration in terms of steady-state analysis is as follows. The low-pass circuit acts as an integrator provided the time constant of the circuit $RC > 15T$, where T is the period of the input sine wave. When $RC > 15T$, the input sinusoid will be shifted at least by 89.4° (instead of the ideal 90° shift required for integration) when it is transmitted through the network.

An RC integrator converts a square wave into a triangular wave. Integrators are almost invariably preferred over differentiators in analog computer applications for the following reasons:

1. It is easier to stabilize an integrator than a differentiator because the gain of an integrator decreases with frequency whereas the gain of a differentiator increases with frequency.
2. An integrator is less sensitive to noise voltages than a differentiator because of its limited bandwidth.
3. The amplifier of a differentiator may overload if the input waveform changes very rapidly.
4. It is more convenient to introduce initial conditions in an integrator.

Attenuators

An attenuator is a circuit that reduces the amplitude of the signal by a finite amount. A simple resistance attenuator is represented in the below Fig. The output of the attenuator shown in Fig. is given by the relation:

$$v_o = v_i \times \frac{R_2}{R_1 + R_2} = \alpha v_i$$

From this equation, it is evident that the output is smaller than the input, which is the main purpose of an attenuator—to reduce the amplitude of the signal. Attenuators are used when the signal amplitude is very large. Let us measure a voltage, say, 5000 V, using a CRO; such a large voltage may not be handled by the amplifier in a CRO. Therefore, to be able to measure such a voltage we first attenuate the voltage by a known amount, say by a factor of 10 ($\alpha = 0.1$), so that the voltage that is actually connected to the CRO is only 500 V. The output of the attenuator is thus reduced depending on the choice of R_1 and R_2 .

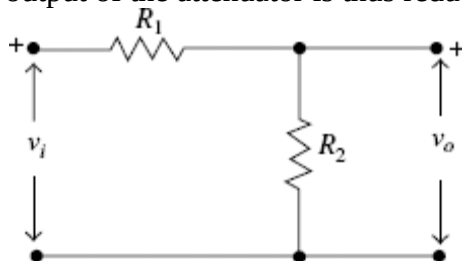
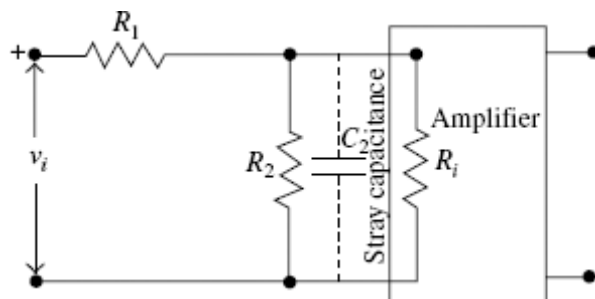


FIGURE: A resistance attenuator



FIGURE(a): The attenuator output connected to amplifier input

Uncompensated Attenuators

If the output of an attenuator is connected as input to an amplifier with a stray capacitance C_2 and input resistance R_i , as shown in Fig.(a).

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Consider the parallel combination of R_2 and R_i . If the amplifier input is not to load the attenuator output, then R_i should always be significantly greater than R_2 . The attenuator circuit is now shown in Fig.(b).

Reducing the two-loop network into a single-loop network by Thevenizing:

$$V_{Th} = v_i \times \frac{R_2}{R_1 + R_2} = \alpha v_i \quad \text{where} \quad \alpha = \frac{R_2}{R_1 + R_2}$$

And $R_{th} = R_1 || R_2$

Hence, the circuit in Fig.(b) reduces to that shown in Fig. (c).

When the input αv_i is applied to this low-pass RC circuit, the output will not reach the steady-state value instantaneously. If, for the above circuit, $R_1 = R_2 = 1 \text{ M}\Omega$ and $C_2 = 20 \text{ nF}$, the rise time is:

$$t_r = 2.2 R_{th} C_2 = 2.2 \times 0.5 \times 10^6 \times 20 \times 10^{-9}$$

$$t_r = 22 \text{ ms}$$

This means that after a time interval of approximately 22ms after the application of the input αv_i to the circuit, the output reaches the steady-state value. This is an abnormally long delay. An attenuator of this type is called an uncompensated attenuator, i.e., its output is dependent on frequency.

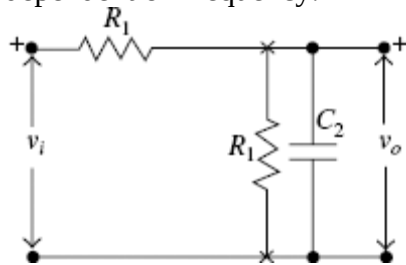


FIGURE (b): The attenuator, considering the stray capacitance at the amplifier input

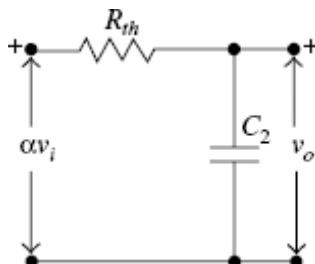


FIGURE (c): An uncompensated attenuator

Compensated Attenuators

To make the response of the attenuator independent of frequency, the capacitor C_1 is connected across R_1 . This attenuator now is called a compensated attenuator shown in below Fig. (d). This circuit in Fig. (d) is redrawn as shown in below Fig. (e).

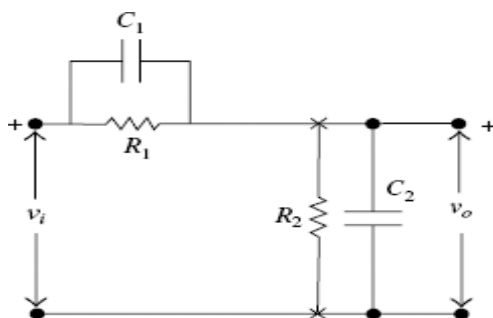


FIGURE (d) A compensated attenuator

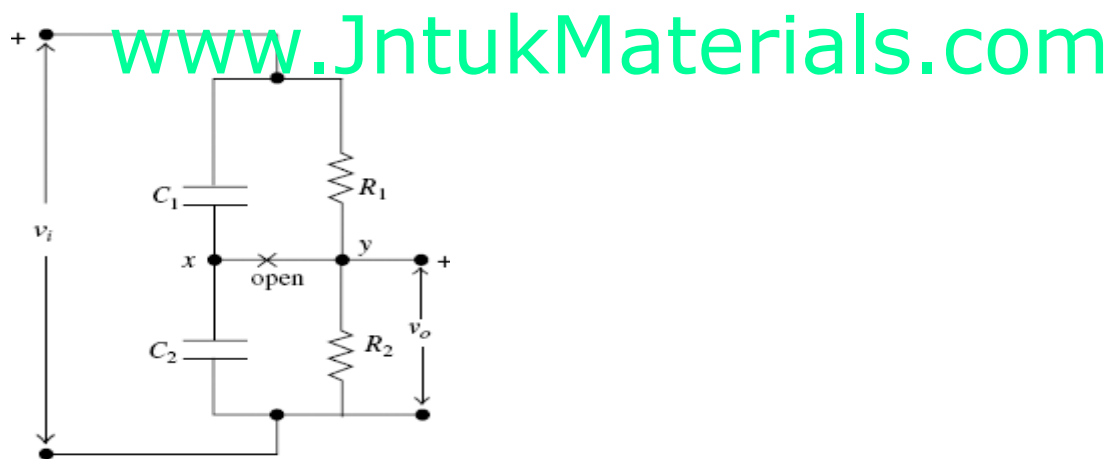


FIGURE (e) Redrawn circuit of Fig (d)

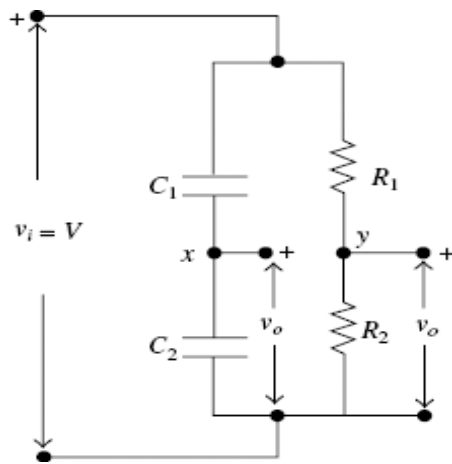


FIGURE (f) The compensated attenuator open-circuiting the xy branch

In Figs. (d) and (e), R_1 , R_2 , C_1 , C_2 form the four arms of the bridge. The bridge is said to be balanced when $R_1C_1 = R_2C_2$, in which case no current flows in the branch xy . Hence, for the purpose of computing the output, the branch xy is omitted. The resultant circuit is shown in Fig. (f).

When a step voltage with $v_i = V$ is applied as an input, the output is calculated as follows: At $t = 0^+$, the capacitors do not allow any sudden changes in the voltage; as the input changes, the output should also change abruptly, depending on the values of C_1 and C_2 .

$$v_o(0^+) = V \frac{C_1}{C_1 + C_2}$$

Thus, the initial output voltage is determined by C_1 and C_2 . As $t \rightarrow \infty$, the capacitors are fully charged and they behave as open circuits for dc. Hence, the resultant output is:

$$v_o(\infty) = V \frac{R_2}{R_1 + R_2}$$

Perfect compensation is obtained if, $v_o(0^+) = v_o(\infty)$
From this using above Eqs., we get:

$$\frac{C_1}{C_1 + C_2} V = V \frac{R_2}{R_1 + R_2}$$

$$C_1 R_1 = C_2 R_2 \quad \text{or}$$

$$C_1 = (R_2/R_1) C_2 = C_p$$

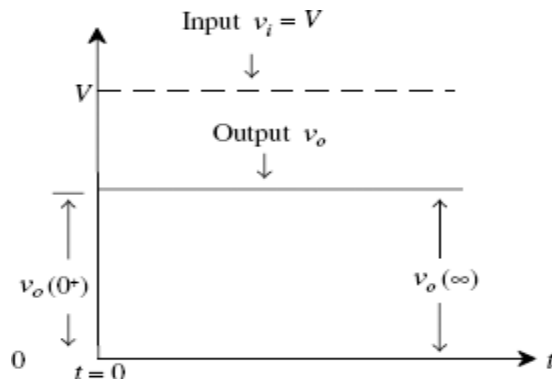


FIGURE (g) A perfectly compensated attenuator ($C_1 = C_2$)

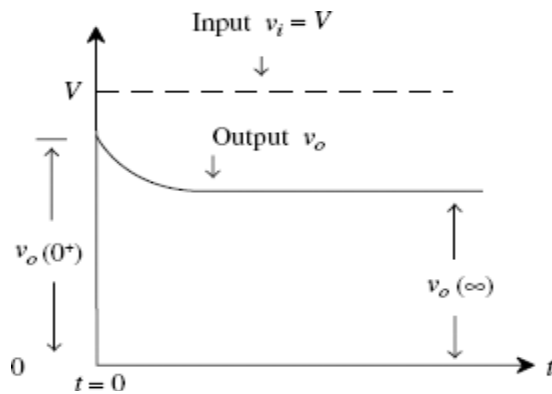


FIGURE (h) An over-compensated attenuator ($C_1 > C_2$)

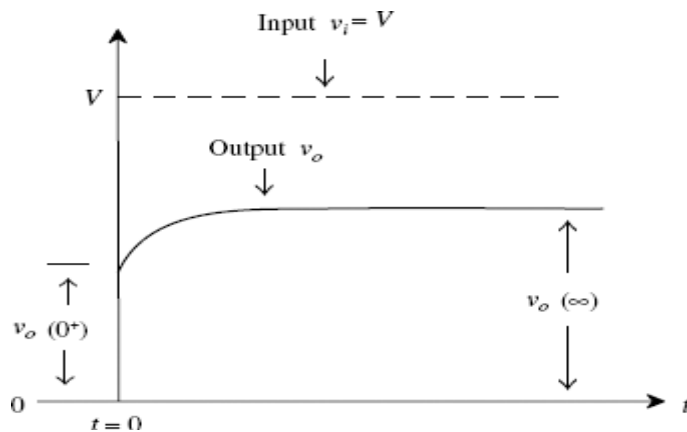


FIGURE (i) An under-compensated attenuator ($C_1 < C_2$)

Let us consider the following circuit conditions:

1. When $C_1 = C_p$, the attenuator is a perfectly compensated attenuator.
2. When $C_1 > C_p$, it is an over-compensated attenuator.
3. When $C_1 < C_p$, it is an under-compensated attenuator.

The response of the attenuator to a step input under these three conditions is shown in Figs. (g),(h) and (i), respectively.

In the attenuator circuit, as at $t = 0+$, the capacitors C_1 and C_2 behave as short circuits, the current must be infinity. But impulse response is impossible as the generator, in practice, has a finite source resistance, not ideally zero. Now consider the compensated attenuator with source resistance R_s [see Fig. (j)].

If the xy loop is open for a balanced bridge, Thevenizing the circuit, the Thevenin voltage source and its internal resistance and R' are calculated using Fig. (k).

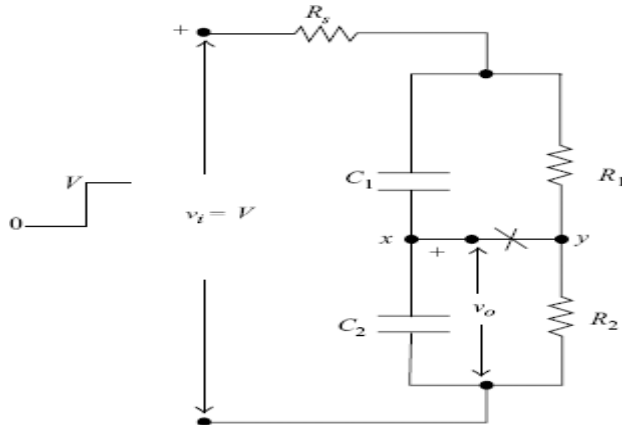


FIGURE (j) The attenuator taking the source resistance into account

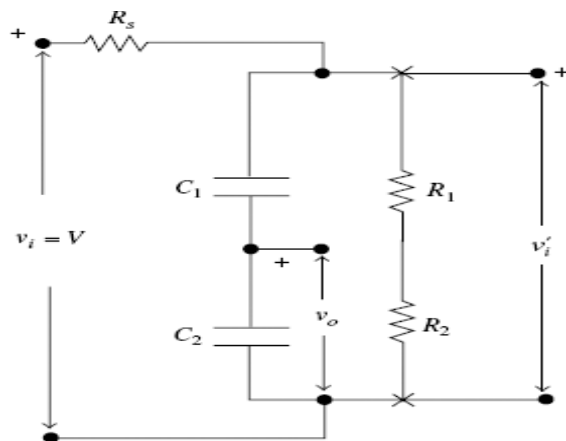


FIGURE (k) The circuit used to calculate the Thevenin voltage source and its internal resistance

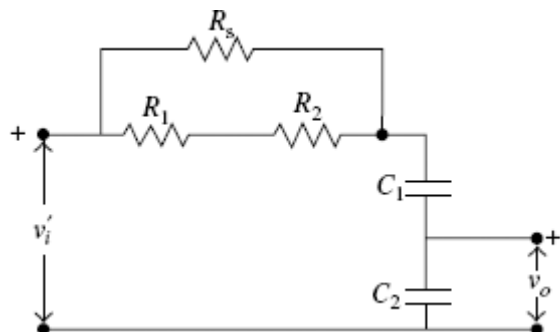


FIGURE (i) Redrawn circuit of Fig.(k)

The value of Thevenin voltage source is:

$$v'_i = \frac{v_i(R_1 + R_2)}{R_s + R_1 + R_2}$$

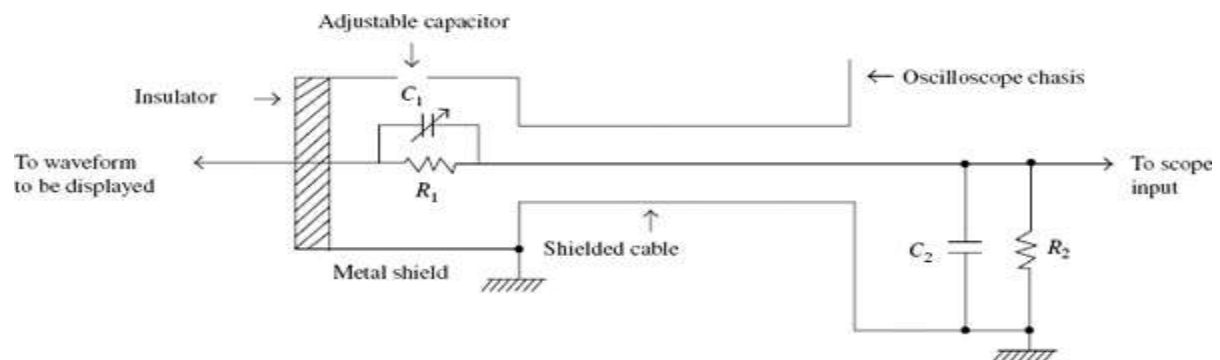
and its internal resistance is: www.IntukMaterials.com

$$R' = \frac{R_s(R_1 + R_2)}{R_s + R_1 + R_2}$$

The above circuit now reduces to that shown in above Fig. (i). Usually $R_s \ll (R_1 + R_2)$, hence, $R_s \parallel (R_1 + R_2) \approx R_s$.

This is a low-pass circuit with time constant $\tau_s = R_s C_s$, where C_s is the series combination of C_1 and C_2 ; $C_s = C_1 C_2 / (C_1 + C_2)$. The output of the attenuator is an exponential with time constant τ_s ; and if τ_s is small, the output almost follows the input.

A perfectly compensated attenuator is sometimes used to reduce the signal amplitude when the signal is connected to a CRO to display a waveform. A typical CRO probe may be represented as in below Fig.



RL and RLC circuits:

RL High pass filter: It is an electronic circuit which allows all the inputs with frequencies greater than a particular frequency called the cut-off frequency to pass through it._

Transfer Function

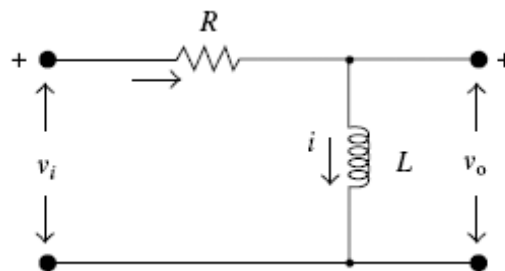


Fig. RL High Pass Circuit

$$v_o(\omega) = v_i(\omega) \cdot \frac{j\omega L}{R + j\omega L}$$

$$H(\omega) = \frac{v_o(\omega)}{v_i(\omega)} = \frac{j\omega L}{R + j\omega L}$$

$$|H(\omega)| = \left| \frac{v_o(\omega)}{v_i(\omega)} \right| = \left| \frac{j\omega L}{R + j\omega L} \right|$$

$$|H(\omega)| = \frac{\omega L}{\sqrt{R^2 + \omega^2 L^2}}$$

$$|H(\omega)| = \frac{\omega L/R}{\sqrt{1 + \omega^2 L^2/R^2}}$$

$$|H(\omega)| = \frac{2\pi f L/R}{\sqrt{1 + 4\pi^2 f^2 L^2/R^2}}$$

Frequency Response:

$$\text{If } f_l = \frac{1}{2\pi(L/R)} \quad |H(\omega)| = \frac{f/f_l}{\sqrt{1 + (f/f_l)^2}}$$

$$\text{If } f \ll f_l \quad (f/f_l)^2 \ll 1 \quad |H(\omega)| = f/f_l$$

As frequency increases $|H(\omega)|$ also increases and the rate of increase is 20dB/decade.

$$\text{If } f = f_l \quad (f/f_l)^2 = 1 \quad |H(\omega)| = \frac{1}{\sqrt{2}} \quad \text{If } f \gg f_l \quad (f/f_l)^2 \gg 1 \quad |H(\omega)| = 1$$

$$\frac{1}{0.707}$$

Fig. Frequency Response

The frequency response indicates that the frequencies below f_l will be blocked by the circuit and the frequencies above f_l will be transmitted. This shows that the circuit is indeed a Highpass filter.

Response of RL Highpass filter to a step input:

$$v_i(t) = Au(t)$$

$$v_i(s) = \frac{A}{s}$$

$$v_o(s) = v_i(s) \frac{Ls}{R + Ls}$$

$$\frac{A}{s} \frac{Ls}{R + Ls}$$

$$A \frac{1}{R/L + s}$$

$$v_0(s) = A \frac{1}{s + R/L}$$

$$v_0(t) = A \cdot e^{\frac{-tR}{L}} u(t)$$

=

A

t

A

t

Fig. Response of RL High pass filter to a step input

RL Low pass filter: It is an electronic circuit which allows all the inputs with frequencies lesser than a particular frequency called the cut-off frequency to pass through it._

Transfer Function

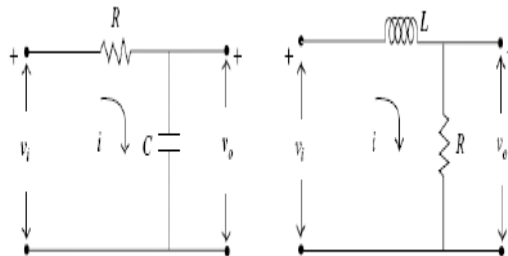


Fig. RL Low pass filter

$$v_0(\omega) = v_i(\omega) \cdot \frac{R}{R + j\omega L}$$

$$H(\omega) = \frac{v_0(\omega)}{v_i(\omega)} = \frac{R}{R + j\omega L}$$

$$|H(\omega)| = \left| \frac{v_0(\omega)}{v_i(\omega)} \right| = \left| \frac{R}{R + j\omega L} \right|$$

$$|H(\omega)| = \frac{R}{\sqrt{R^2 + \omega^2 L^2}}$$

$$|H(\omega)| = \frac{1}{\sqrt{1 + \omega^2 LC^2}}$$

$$|H(\omega)| = \frac{1}{\sqrt{1 + 4\pi^2 f^2 LC^2}}$$

Frequency Response:

$$\text{If } f_h = \frac{1}{2\pi(L/R)} \quad |H(\omega)| = \frac{1}{\sqrt{1 + LC^2 \omega^2}}$$

$$\text{If } f \ll f_h \quad (f/f_h)^2 \ll 1 \quad |H(\omega)| = 1$$

$$\text{If } f = f_h \quad (f/f_h)^2 = 1 \quad |H(\omega)| = \frac{1}{\sqrt{2}} \quad \text{If } f \gg f_h \quad (f/f_h)^2 \gg 1 \quad |H(\omega)| = f_h/f$$

As frequency increases $|H(\omega)|$ also decreases and the rate of decrease is 20dB/decade.

$$\frac{1}{0.707}$$

f

Fig. Frequency Response

The frequency response indicates that the frequencies below f_h will be transmitted by the circuit and the frequencies above f_h will be blocked. This shows that the circuit is indeed a Lowpass filter.

Response of RL Lowpass filter to a step input:

$$v_i(t) = Au(t)$$

$$v_i(s) = \frac{A}{s}$$

$$v_o(s) = v_i(s) \frac{R}{R + Ls}$$

$$\frac{A}{s} \frac{R}{R + Ls}$$

$$\frac{A}{s} \frac{R/L}{R/L + s}$$

$$i \frac{A}{s} + \frac{-A}{R/L+s}$$

$$v_0(t) = Au(t) - A \cdot e^{-\frac{tR}{L}} u(t)$$

$$v_0(t) = A(1 - e^{-\frac{tR}{L}}) u(t)$$

=

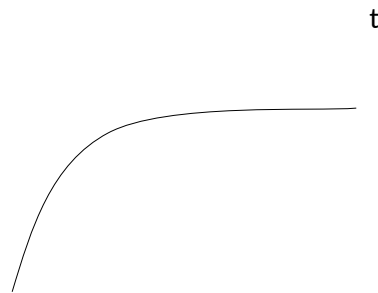


Fig. Response of RL Low pass filter to a step input

Series RLC Circuit:

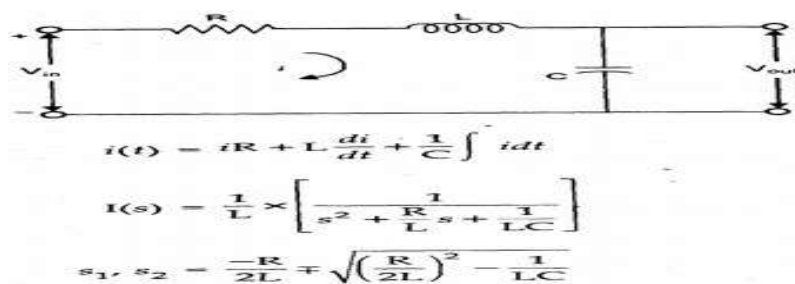


Fig. Series RLC Circuit

Response of series RLC circuit to a step input:

$$v_i(t) = Au(t)$$

$$v_i(s) = \frac{A}{s}$$

$$v_0(s) = v_i(s) \frac{\frac{1}{sC}}{R + Ls + \frac{1}{sC}}$$

$$v_0(s) = \frac{A}{s} \frac{\frac{1}{sC}}{R + Ls + \frac{1}{sC}}$$

$$\mathcal{L}^{-1} \left\{ \frac{\frac{1}{LC}}{s \left(s^2 + s \frac{R}{L} + \frac{1}{LC} \right)} \right\}$$

$$\text{If } \omega_n^2 = \frac{1}{LC}, \quad \varepsilon = \frac{R}{2} \sqrt{\frac{C}{L}}$$

$$v_0(s) = \frac{\omega_n^2}{s(s^2 + 2\varepsilon\omega_n s + \omega_n^2)}$$

Case-1:

$$\text{If } \varepsilon = 1 \quad \frac{R}{2} \sqrt{\frac{C}{L}} = 1$$

$$v_0(s) = \frac{\omega_n^2}{s(s^2 + 2\omega_n s + \omega_n^2)}$$

$$\mathcal{L}^{-1} \left\{ \frac{\omega_n^2}{s(s + \omega_n)^2} \right\}$$

$$\mathcal{L}^{-1} \left\{ \frac{A}{s} + \frac{B}{s + \omega_n} + \frac{C}{(s + \omega_n)^2} \right\}$$

$$A = 1, B = -1 \wedge C = -\omega_n$$

$$\mathcal{L}^{-1} \left\{ \frac{1}{s} + \frac{-1}{s + \omega_n} + \frac{-\omega_n}{(s + \omega_n)^2} \right\}$$

$$v_0(t) = (1 - e^{-\omega_n t} - \omega_n t e^{-\omega_n t}) u(t)$$

$$v_0(0) = 0$$

$$v_0(\infty) = 1$$

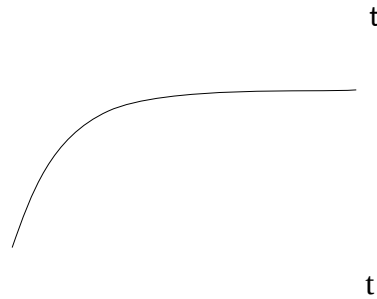


Fig. Response of series RLC circuit to a step input for case 1

Case-2:

$$\text{If } \mathcal{E} > 1 \quad \frac{R}{2} \sqrt{\frac{C}{L}} > 1$$

$$v_0(s) = \frac{\omega_n^2}{s(s^2 + 2\mathcal{E}\omega_n s + \omega_n^2)}$$

$$s^2 + 2\mathcal{E}\omega_n s + \omega_n^2 = (s - \{-\mathcal{E}\omega_n + \omega_n\sqrt{\mathcal{E}^2 - 1}\})(s - \{-\mathcal{E}\omega_n - \omega_n\sqrt{\mathcal{E}^2 - 1}\})$$

$$v_0(s) = \frac{A}{s} + \frac{B}{s - \{-\mathcal{E}\omega_n + \omega_n\sqrt{\mathcal{E}^2 - 1}\}} + \frac{C}{s - \{-\mathcal{E}\omega_n - \omega_n\sqrt{\mathcal{E}^2 - 1}\}}$$

On solving

$$A=1, B = \frac{1}{2(\mathcal{E}^2 - 1 - \mathcal{E}\sqrt{\mathcal{E}^2 - 1})} \wedge C = \frac{1}{2(\mathcal{E}^2 - 1 + \mathcal{E}\sqrt{\mathcal{E}^2 - 1})}$$

$$v_0(s) = \frac{1}{s} + \frac{1}{2(\mathcal{E}^2 - 1 - \mathcal{E}\sqrt{\mathcal{E}^2 - 1})} \frac{1}{s - \{-\mathcal{E}\omega_n + \omega_n\sqrt{\mathcal{E}^2 - 1}\}} + \frac{1}{2(\mathcal{E}^2 - 1 + \mathcal{E}\sqrt{\mathcal{E}^2 - 1})} \frac{1}{s - \{-\mathcal{E}\omega_n - \omega_n\sqrt{\mathcal{E}^2 - 1}\}}$$

$$v_0(t) = u(t) + \frac{1}{2} \left[\frac{e^{(-\mathcal{E}\omega_n + \omega_n\sqrt{\mathcal{E}^2 - 1})t}}{(\mathcal{E}^2 - 1 - \mathcal{E}\sqrt{\mathcal{E}^2 - 1})} + \frac{e^{(-\mathcal{E}\omega_n - \omega_n\sqrt{\mathcal{E}^2 - 1})t}}{(\mathcal{E}^2 - 1 + \mathcal{E}\sqrt{\mathcal{E}^2 - 1})} \right] u(t)$$

$$v_0(0) = 0$$

$$v_0(\infty) = 1$$

$$\text{If } \mathcal{E} < 1 \quad \frac{R}{2} \sqrt{\frac{C}{L}} < 1$$

$$v_0(s) = \frac{\omega_n^2}{s(s^2 + 2\mathcal{E}\omega_n s + \omega_n^2)}$$

$$s^2 + 2\mathcal{E}\omega_n s + \omega_n^2 = (s - \{-\mathcal{E}\omega_n + \omega_n\sqrt{\mathcal{E}^2 - 1}\})(s - \{-\mathcal{E}\omega_n - \omega_n\sqrt{\mathcal{E}^2 - 1}\})$$

$$v_0(s) = \frac{\omega_n^2}{s(s - \{-\mathcal{E}\omega_n + \omega_n\sqrt{\mathcal{E}^2 - 1}\})(s - \{-\mathcal{E}\omega_n - \omega_n\sqrt{\mathcal{E}^2 - 1}\})}$$

$$-\mathcal{E}\omega_n + \omega_n\sqrt{\mathcal{E}^2 - 1} \text{ in polarform can be written as } \omega_n e^{-j\alpha} \quad \alpha = \tan^{-1}\left(\frac{\sqrt{\mathcal{E}^2 - 1}}{\mathcal{E}}\right)$$

$$-\mathcal{E}\omega_n - \omega_n\sqrt{\mathcal{E}^2 - 1} \text{ in polarform can be written as } \omega_n e^{+j\alpha}$$

$$v_0(s) = \frac{\omega_n^2}{s(s - \omega_n e^{-j\alpha})(s - \omega_n e^{+j\alpha})}$$

$$v_0(s) = \frac{A}{s} + \frac{B}{s - \omega_n e^{-j\alpha}} + \frac{C}{s - \omega_n e^{+j\alpha}}$$

On solving

$$A = 1, B = \frac{e^{+j\alpha}}{-2j \sin \alpha}, C = \frac{e^{-j\alpha}}{2j \sin \alpha}$$

$$v_0(s) = \frac{1}{s} + \left(\frac{e^{+j\alpha}}{-2j \sin \alpha}\right) \left(\frac{1}{s - \omega_n e^{-j\alpha}}\right) + \left(\frac{e^{-j\alpha}}{2j \sin \alpha}\right) \left(\frac{1}{s - \omega_n e^{+j\alpha}}\right)$$

$$v_0(t) = u(t) + \left(\frac{e^{+j\alpha}}{-2j \sin \alpha}\right) e^{\omega_n e^{-j\alpha} t} u(t) + \left(\frac{e^{-j\alpha}}{2j \sin \alpha}\right) e^{\omega_n e^{+j\alpha} t} u(t)$$

$$v_0(t) = u(t) + \left(\frac{e^{+j\alpha}}{-2j \sin \alpha}\right) e^{-\mathcal{E}\omega_n t + \omega_n \sqrt{\mathcal{E}^2 - 1} t} u(t) + \left(\frac{e^{-j\alpha}}{2j \sin \alpha}\right) e^{-\mathcal{E}\omega_n t - \omega_n \sqrt{\mathcal{E}^2 - 1} t} u(t)$$

$$v_0(t) = 1 - \frac{e^{-\mathcal{E}\omega_n t}}{\sin \alpha} \left[\frac{e^{j(\omega_n t \sqrt{1 - \mathcal{E}^2} + \alpha)} + e^{-j(\omega_n t \sqrt{1 - \mathcal{E}^2} + \alpha)}}{2j} \right]$$

$$v_0(t) = 1 - \frac{e^{-\mathcal{E}\omega_n t}}{\sin \alpha} [\sin(\omega_n t \sqrt{1 - \mathcal{E}^2} + \alpha)]$$

$$v_0(t) = 1 - \frac{e^{-\alpha t}}{\sqrt{1-\xi^2}} \left[\sin(\omega_n t \sqrt{1-\xi^2} + \alpha) \right]$$

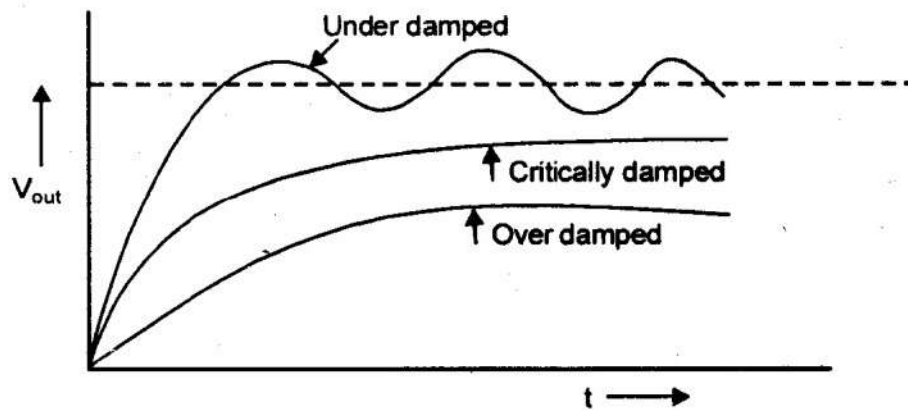
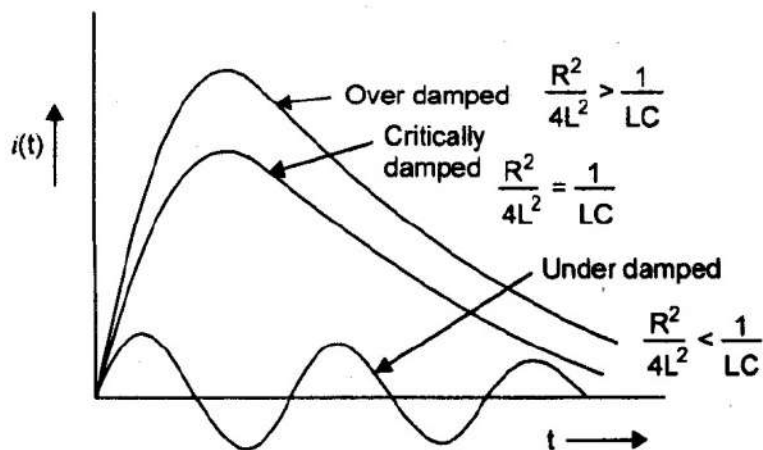


Fig. Response of series RLC circuit to a step input for three different cases

$$i_0(t) = C \frac{dv_0(t)}{dt}$$



Parallel RLC Circuit:

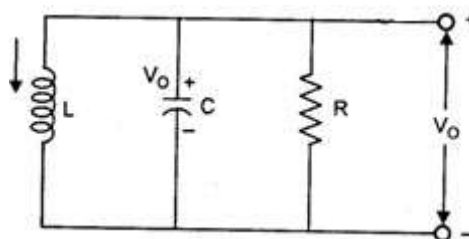
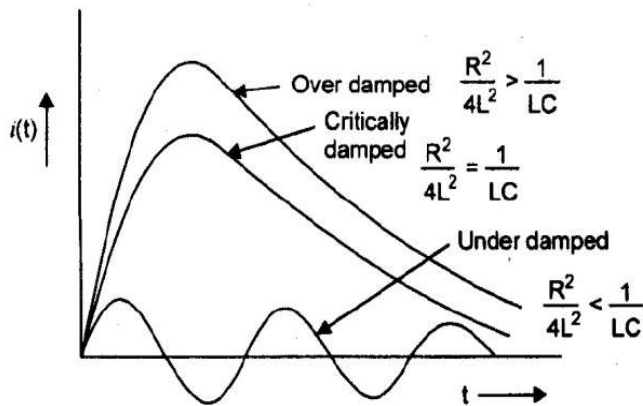


Fig. Parallel RLC circuit

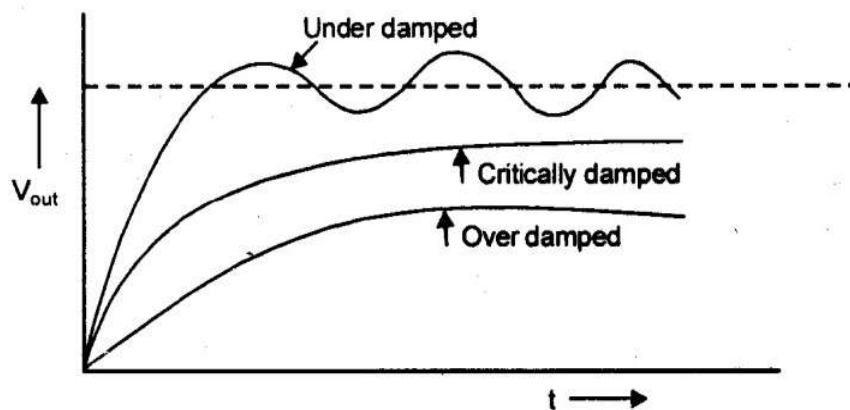
Response of Parallel RLC circuit to a step input:

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As the parallel RLC circuit is a dual of the series RLC circuit, So the output voltage of it will have the same shape as the current waveform of the series RLC circuit. The same is presented in below Fig.



Similarly the output current waveform of Parallel RLC circuit will have the same shape as the voltage waveform of the series RLC circuit. The same is presented in below Fig.



Ringling Circuit :

Ringling circuits are those which generate a sequence of pulses spaced regularly in time these circuits have un damped oscillations depending on the number of ringling duty cycles required. The parallel and Series RLC circuits can be considered as the ringling circuits.

