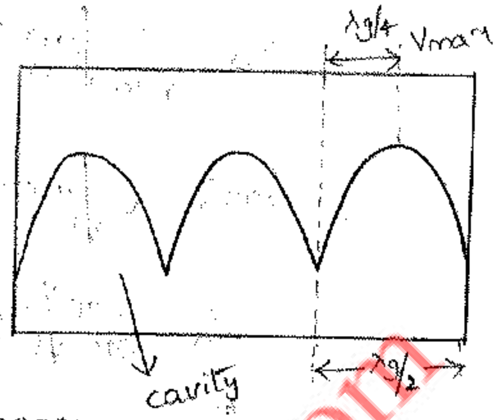
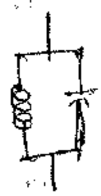
UNIT-II

Cavity Resonators:-

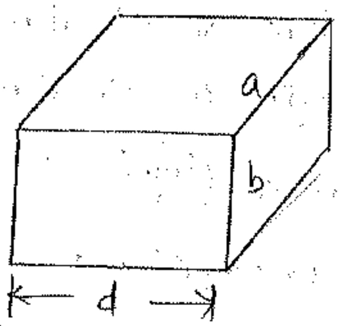


If one end of Waveguide is terminated in a shorting plate there will be reflections and hence standing waves are exist as shown in fig. When another shorting plate is kept at a distance of multiplies of $\lambda g/2$ signal bounces front & back between two shorting plates. Thus the results in a resonance, and hence hollow space is called "cavity" and resonator as cavity resonator.

cavity resonator is similar to tuned ckt at lower frequencies [having resonant freq $f_0 = \frac{1}{2\pi\sqrt{LC}}$]. cavity resonator can resonate only at particular frequencies as parallel resonant only circuit as shown in fig.



Expression for 'f_0' of rectangular cavity resonator:-



In Rectangular Waveguides,

$$P^2 + \omega^2\mu\varepsilon = \left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2$$

$$\omega^2\mu\varepsilon = \left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2 - P^2$$

For propogation $P = j\beta$

$$\omega^2\mu\varepsilon = \left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2 + \beta^2$$

$$= \left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2 + \left(\frac{2\pi}{\lambda g}\right)^2$$

The Condition for resonator is to resonate is

$$d = \frac{P\lambda g}{2} \Rightarrow \lambda g = \frac{2d}{P}$$

$$\beta = \frac{2\pi}{\lambda g} = \frac{2\pi}{\frac{2d}{P}} = \frac{P\pi}{d}$$

where $P = 1, 2, 3 \dots \infty$

when $\beta = \frac{P\pi}{d}$, $f = f_0$

$$\omega_0^2\mu\varepsilon = \left(\frac{m\pi}{a}\right)^2+\left(\frac{n\pi}{b}\right)^2+\left(\frac{p\pi}{d}\right)^2$$

$$\omega_0 = \frac{1}{\sqrt{\mu\varepsilon}}\sqrt{\left(\frac{m\pi}{a}\right)^2+\left(\frac{n\pi}{b}\right)^2+\left(\frac{p\pi}{d}\right)^2}$$

$$2\pi f_0 = c\sqrt{\left(\frac{m\pi}{a}\right)^2+\left(\frac{n\pi}{b}\right)^2+\left(\frac{p\pi}{d}\right)^2}$$

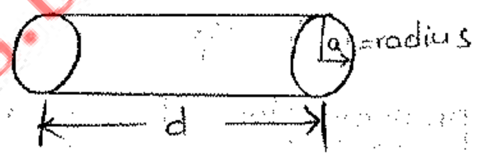
$$f_0 = \frac{c\pi}{2\pi}\sqrt{\left(\frac{m}{a}\right)^2+\left(\frac{n}{b}\right)^2+\left(\frac{p}{d}\right)^2}$$

$$\boxed{f_0 = \frac{c}{2}\sqrt{\left(\frac{m}{a}\right)^2+\left(\frac{n}{b}\right)^2+\left(\frac{p}{d}\right)^2}}$$

Note:- In rectangular cavity, the wave is designated as TE_{mnp} or TM_{mnp}

Expression for 'f_0' in circular cavity Resonator:-

In circular Waveguides



$$P^2+\omega^2\mu\varepsilon = \left(\frac{P_{nm}}{a}\right)^2$$

where P_{nm} = Constant depends on n & m

a = radius of circular cavity resonator

d = length of circular cavity resonator

$$\omega^2\mu\varepsilon = \left(\frac{P_{nm}}{a}\right)^2 - P^2$$

For propogation $P = j\beta$

$$\omega^2\mu\varepsilon = \left(\frac{P_{nm}}{a}\right)^2 + \beta^2$$

$$= \left(\frac{P_{nm}}{a}\right)^2 + \left(\frac{2\pi}{\lambda g}\right)^2$$

Condition for resonator is to resonate is

$$d = \frac{P\lambda g}{2} \Rightarrow \lambda g = \frac{2d}{P} \quad \& \quad \beta = \frac{2\pi}{\lambda g} = \frac{P\pi}{d}$$

$$\omega_0^2\mu\varepsilon = \left(\frac{P_{nm}}{a}\right)^2 + \left(\frac{P\pi}{d}\right)^2$$

$$\omega_0 = \frac{1}{\sqrt{\mu\varepsilon}}\sqrt{\left(\frac{P_{nm}}{a}\right)^2 + \left(\frac{P\pi}{d}\right)^2}$$

$$\boxed{f_0 = \frac{c}{2\pi}\sqrt{\left(\frac{P_{nm}}{a}\right)^2 + \left(\frac{P\pi}{d}\right)^2}}$$

Note:- In circular Cavity resonator, The wave is designated as TE_{nmp} or TM_{nmp}

Problem:-

1) calculate The resonant freq of rectangular resonator of dimensions a=3cm, b=2cm, l=4cm when mode of propogation is TE_{101}?

Here m=1, n=0, P=1

$$f_0 = \frac{c}{2}\sqrt{\left(\frac{1}{3}\right)^2 + 0 + (1/4)^2}$$

$$= \frac{3\times10^{10}}{2}\sqrt{\left(\frac{1}{3}\right)^2 + (1/4)^2}$$

$$f_0 = 6.25\ GH_z$$

2) calculate lowest resonant freq. of rectangular cavity resonator of dimensions a=2cm, b=1cm, l=3cm

Here m=1, n=0, p=1

$$f_0 = \frac{c}{2}\sqrt{\left(\frac{1}{2}\right)^2 + 0 + (1/3)^2}$$

$$= \frac{3\times10^{10}}{2}\sqrt{1/4 + 1/9}$$

$$f_0 = 9\ GH_z$$

Note: TE_{101} has lowest resonant frequency in rectangular Cavity resonators

3) Calculate The resonant frequency of circular cavity resonator of following dimensions: Diameter D=12.5 cm, length d=5cm, for TE_{012} mode?

$$f_0 = \frac{c}{2\pi}\sqrt{\left(\frac{P_{nm}}{a}\right)^2 + \left(\frac{P\pi}{d}\right)^2} \qquad P_{01} = 2.4$$

$$= \frac{c}{2\pi}\sqrt{\left(\frac{P_{01}}{a}\right)^2 + \left(\frac{P\pi}{d}\right)^2}$$

$$a = \frac{D}{2} = \frac{12.5}{2} = 6.25cm$$

$$f_0 = \frac{3\times10^{10}}{2\pi}\sqrt{\left(\frac{2.4}{6.25}\right)^2 + \left(\frac{2\pi}{500}\right)^2} = 1.83\ GH_z$$

4) A circular waveguide has radius of 3cm and is used as resonator for TM_{011} mode at $f_0 = 10GHz$ by placing two perfectly conducting plates at its two ends. Determine minimum distance b/w two end plates?

$$f_0 = \frac{c}{2\pi}\sqrt{\left(\frac{P_{nm}}{a}\right)^2 + \left(\frac{p\pi}{d}\right)^2}$$

$$\frac{4\pi^2 f_0^2}{c^2} = \left(\frac{P_{nm}}{a}\right)^2 + \left(\frac{p\pi}{d}\right)^2$$

$$\frac{4\pi^2 f_0^2}{c^2} - \frac{P_{01}^2}{a^2} = \frac{p^2\pi^2}{d^2}$$

$$\frac{\pi^2}{d^2} = \frac{4\pi^2 (10G)^2}{(3\times10^{10})^2} - \frac{(2.4)^2}{3^2}$$

$$= 3.746.$$

$$d = \sqrt{\pi^2/3.746} = 1.623cm$$

Quality factor (Q) of cavity resonator:-

The quality factor of resonant or anti resonant circuit measures frequency selectivity and is defined as

$$Q = \frac{\omega_0 W}{P}$$

ω_0 = resonant freq.

W = Max energy stored

P = ~~Power~~ energy dissipated per cycle.

$$Q = \frac{2\pi f_0 W}{P} = \frac{2\pi \times \text{max energy stored per cycle}}{\text{Energy dissipated per cycle}}$$

The Q of a ideal cavity resonator is infinite since P would be zero and once ~~recognized~~ energized it would resonate forever.

Normally coupling loops are used to couple energy in and out of cavity resonator. This coupling effect the load of cavity and changes the value of Q. This Q taken into account the coupling b/w cavity and coupling path is known as loaded cavity factor (Q_L).

The loaded Quality factor Q_L is given by

$$\frac{1}{Q_L} = \frac{1}{Q_o} + \frac{1}{Q_{ext}}$$

where Q_o = Quality factor of unloaded cavity

Q_{ext} = Q due to coupling

$Q_{ext} = \frac{Q_o}{K}$ where K = Coupling coefficient

There are 3 types of Coupling

* critically Coupling $(K=1)$
* Under Coupling $(K<1)$
* over Coupling $(K>1)$

Critically Coupled cavity:-

For critically Coupled cavity $K=1$ & $Q_{ext} = Q_o$

$$\frac{1}{Q_L} = \frac{1}{Q_o} + \frac{1}{Q_o} = \frac{2}{Q_o}$$

$$Q_L = \frac{Q_o}{2}$$

Under Coupled cavity:-

For under Coupled cavity, $K<1$. The cavity terminals are Connected at voltage minimum positions & Coupling coefficient is the reciprocal of SWR.

$$\frac{1}{Q_L} = \frac{1}{Q_o} + \frac{1}{Q_o \cdot \rho}$$

$$= \frac{1}{Q_o}\left[1 + \frac{1}{\rho}\right)$$

$$\frac{1}{Q_L} = \frac{1}{Q_o}\left(\frac{1+\rho}{\rho}\right)$$

$$\boxed{Q_L = \frac{Q_o \cdot \rho}{1+\rho}}$$

Over Coupled cavity:-

For over Coupled cavity $K>1$. The cavity terminals are Connected at voltage max. positions & Coupling Coefficient is equal to SWR.

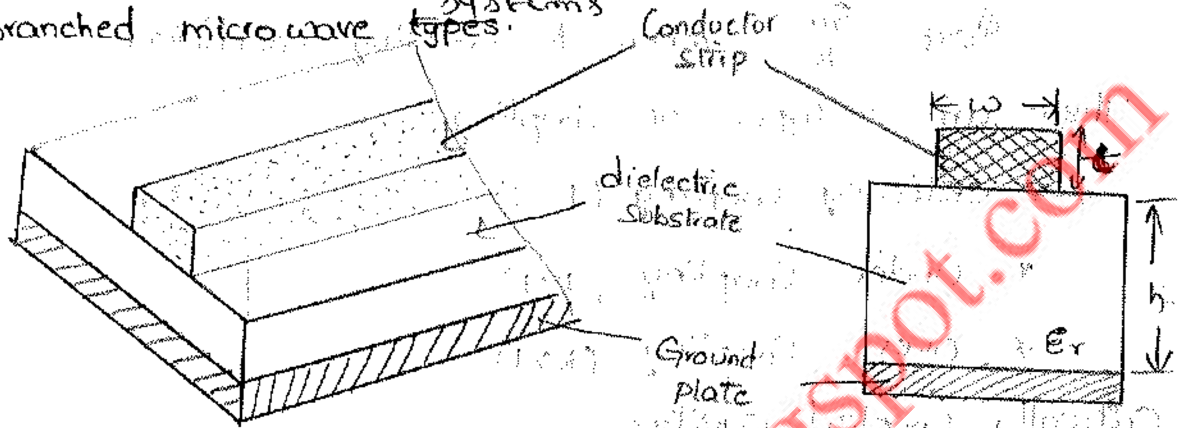
$$\frac{1}{Q_L} = \frac{1}{Q_o} + \frac{1}{Q_o/\rho} \qquad \because K = \rho$$

$= \frac{1}{Q_0}(1+\beta)$

$$\boxed{Q_L = \frac{Q_0}{1+\beta}}$$

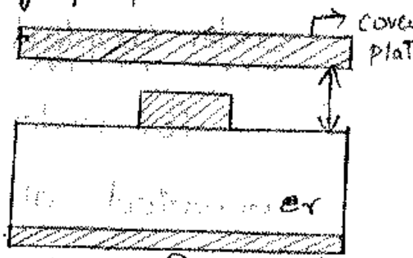
Microstrip lines:-

Microstrip lines mainly serve to interconnect microwave devices into a branched microwave ~~types~~ systems.

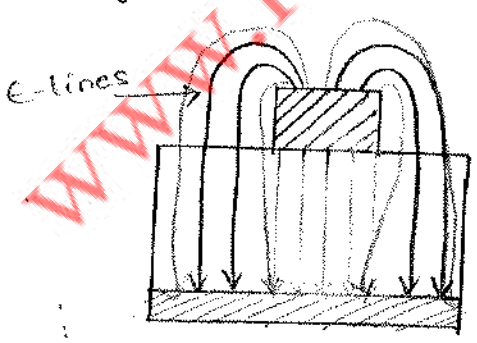


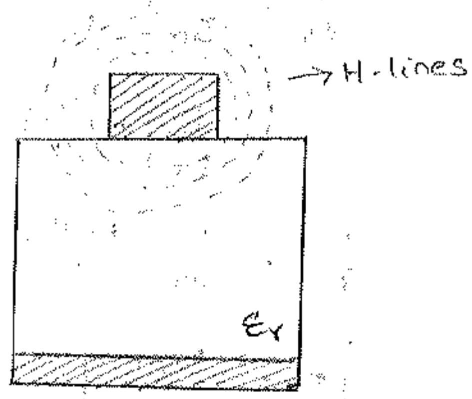
Microstrip lines are planar transmission lines. It is nothing but parallel plate TX line with dielectric substrate one face of which is metallised ground & other face has a thin conducting strip of a certain width w and thickness t as shown in fig.

Sometimes a Coverplate is used for shielding purpose but it is kept much furthur away from ground plate. So as to not affect the microstrip field lines as shown in fig.



The approximate distribution of electric & magnetic fields shown in fig. below.





Advantages of Microstrip lines:-

1. Complete conductor pattern may be deposited & processed on a dielectric substrate in single step. Thus fabrication cost is lower than strip line, Co-axial cable or Waveguide circuits.

2. Both packaged & unpackaged semiconductor chips or modules may be conveniently attatched to strip element

3. There is an easy access to the top surface making it easy to mount passive or active discrete devices, minor adjustments can be done after circuit has been fabricated or completed.

Limitations:-

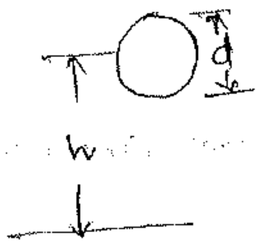
1. Due to openness of microstrip line surface, they have higher radiation losses (or) interference due to near by conductors. These can be minimized by choosing thin substrates with higher dielectric constants &

2. Due to discontinuity b/w dielectric & microstrip conductors, microstrip line propogate unpure TEM modes. It makes the analysis complicated.

Characteristic impedance (Z_0):-

The characteristic impedance of strip is function of the strip width W, thickness t and distance b/w strip line & ground plate is h. Empirical relation for Z_0 for microstrip line is given by

$$Z_0 = \frac{60}{\sqrt{\epsilon_{re}}} \ln\left(\frac{4h}{d}\right) \quad \text{for } h >> d$$



where ϵ_{re} = effective dielectric constant of or dielectric substrate

d = diameter of wire

h = distance b/w strip & ground plate

wire over ground

The effective dielectric constant ϵ_{re} has an empirical relation as

$$\boxed{\epsilon_{re} = 0.475\,\epsilon_r + 0.67}$$

$$\boxed{\epsilon_{re} = 0.475\,(\epsilon_r + 1.41)}$$

Since the cross section of strip is rectangular, diameter d also has an empirical relation.

$$\boxed{d = 0.67W\left(0.8 + \frac{t}{W}\right)} \Rightarrow \boxed{d = 0.67\,(0.8W + t)}$$

Taking relationships for ε_r and [illegible] the Z_0 can be written as

$$Z_0 = \frac{87}{\sqrt{\varepsilon_r + 1.41}} \ln\left[\frac{5.98h}{0.8W+t}\right]$$

or $\omega >> h$

$$\boxed{Z_0 = \frac{377}{\sqrt{\varepsilon_r}} \frac{h}{\omega}}$$

Quality factor (Q):-

The quality factor of Microstrip line is very high. The quality factor may be limited by radiation losses of substrate and with low dielectric constant.

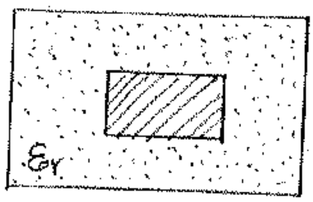
The Q of microstrip line is given by

$$\boxed{Q = \frac{1}{\tan\theta}}$$

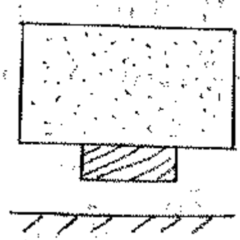
where θ is dielectric loss tangent.

Types of Microstrip lines:-

i) Embedded Microstrip line

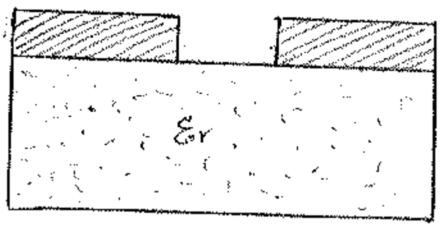


ii) Standard inverted Microstrip line

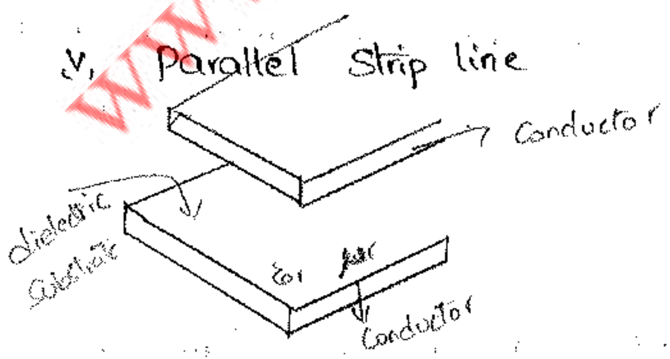


(iii) Suspended Microstrip line

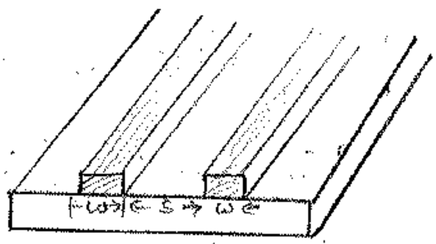
iv) Slotted microstrip line



v) Parallel Strip line



vi) Coplanar microstrip line



These are many variety of microstrip lines that have been used in practice such as embedded micro strip, standard inverted micro strip, suspended micro strip and slotted micro strip.

The cross sectional views of these are shown in fig. In addition to all these lines some other TEM lines such as parallel strip lines, coplanar strip lines also have been used for MIC.

Circular Waveguides:-

If cross section of hollow conducting tube has circular cross section, it is called circular or cylindrical waveguide. The behaviour of wave in circular waveguide is similar to their propogation in rectangular waveguides.

Unlike rectangular waveguide, In circular waveguide the wave is designated as TE_{nm} or TM_{nm} where n indicates no. of full wave variations around the circumference where as m indicates the no. of half wave variations radially out from centre to the wall.

cylindrical co-ordinate system is used for analysis of wave in circular waveguide.

All the equations of rectangular waveguide can be applied for O^{lr} waveguide except cut off frequency.

Expression for f_c (or) λ_c:-

At $f = f_c$, $P = \alpha + j\beta = 0$

$$\beta = \sqrt{\omega^2 \mu\varepsilon - \omega_c^2 \mu\varepsilon} \quad \text{for propogation}$$

$$\alpha = \sqrt{\omega_c^2 \mu\varepsilon - \omega^2 \mu\varepsilon} \quad \text{for attenuation}$$

$$\omega_c = 1/2\pi f_c \quad , \quad \mu\varepsilon = \frac{1}{c^2}$$

$$\beta = \sqrt{\omega^2 \mu\varepsilon - \left(\frac{2\pi f_c}{c}\right)^2}$$

$$= \sqrt{\omega^2 \mu\varepsilon - \left(\frac{2\pi}{\lambda_c}\right)^2}$$

$$\beta = \sqrt{\omega^2 \mu\varepsilon - h^2}$$

h depends on n, m and k. h is constant

$h = \frac{P'_{nm}}{a}$ for TE wave & $\frac{P_{nm}}{a}$ for TM wave

For TE Wave

$$h = \frac{P'_{nm}}{a} = \frac{2\pi}{\lambda_c}$$

$$\lambda_c = \frac{2\pi a}{P'_{nm}}$$

$$\boxed{f_c = \frac{c P'_{nm}}{2\pi a}}$$

For TM Wave,

$$h = \frac{P_{nm}}{a} = \frac{2\pi}{\lambda_c}$$

$$\lambda_c = \frac{2\pi a}{P_{nm}}$$

$$\boxed{f_c = \frac{c P_{nm}}{2\pi a}}$$

minimum value of P'_{nm} is 1.841 for n=1, m=1

∴ Cut off frequency $f_c = \frac{c(1.841)}{2\pi a}$

Note:- TE_{11} is dominant mode in circular waveguides because it has lowest cut off frequency and highest cut off wavelength

Problem:-

Show that resonant frequency f in rectangular cavity resonator for TE_{101} mode is $f_0 = \frac{c}{2d}\sqrt{1+\left(\frac{d}{a}\right)^2}$

$$f_0 = \frac{c}{2}\sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2 + \left(\frac{p}{d}\right)^2}$$

m=1, n=0, p=1

$$f_0 = \frac{c}{2}\sqrt{\left(\frac{1}{a}\right)^2 + \left(\frac{0}{b}\right)^2 + \left(\frac{1}{b}\right)^2}$$

$$\boxed{f_0 = \frac{c}{2d}\sqrt{1+\left(\frac{d}{a}\right)^2}}$$

Comparision between circular WGs and Rectangular WGs:-

Rectangular WG	Circular WG
* If cross section hollow metallic tube is rectangle That Wave guide is called Rectangular Waveguide.	* If cross section of hollow metallic tube is circular That Wave guide is called circular Wave guide.
* Simple to manufacture	* Complicated to manufacture
* Attenuation is less	* Attenuation is more
* mode Convertion is less	* Mode Conversion is high
* Cross Sectional area is less when Compared to circular WG to carry Same signal	* Cross Sectional area is more when Compared to Rectangular WG.
* The Wave is designated as TE_{mn} (or) TM_{mn}	* The Wave is designated as TE_{nm} (or) TM_{nm}.
* In rectangular WG, The dominant mode TE_{10}	* In circular WG, The dominant mode is TE_{11}
* It uses Cartessian Coordinate System	* It uses Cylindrical Coordinate System.

Problem:-

1) An air filled in circular WG of two cm inside radius is operated in TE_{01} mode. calculate i, f_c ii, If WG is filled with a dielectric material of $\varepsilon_r = 2.25$ to what values must its radius we changed in order to maintain the f_c as its original value?

$(P'_{01} = 2.405)$

$a = 2cm$

mode $= TE_{01}$

$$\text{i, } f_c = \frac{CP'_{01}}{2\pi a} = \frac{3\times10^{10}\times 2.405}{2\times 3.14\times 2}$$

$f = 5.74$ GHz

ii, $f_c = \frac{1}{\sqrt{\mu \varepsilon}} \frac{P_{01}'}{2\pi a}$

$$= \frac{1}{\sqrt{\mu_0 \mu_r \varepsilon_0 \varepsilon_r}} \frac{P_{01}'}{2\pi a}$$

$$= \frac{1}{\sqrt{\mu_0 \varepsilon_0} \sqrt{\varepsilon_r}} \frac{P_{01}'}{2\pi a}$$

$$f_c = \frac{c}{\sqrt{2.25}} \cdot \frac{P_{01}'}{2\pi a}$$

$$5.74 \times 10^9 = \frac{3 \times 10^{10}}{\sqrt{2.25}} \cdot \frac{2.405}{2 \times 3.14 \times a}$$

$$a = 1.3\,\text{cm}.$$

Wave equation in rectangular cavity resonator :-

TE waves

TE_{mnp} : $H_z = H_{oz} \cos\left(\frac{m\pi}{a}x\right) \cos\left(\frac{n\pi y}{b}\right) \sin\left(\frac{p\pi z}{d}\right)$.

TM waves TM_{mnp} $E_z = E_{oz} \sin\left(\frac{m\pi}{a}x\right) \sin\left(\frac{n\pi y}{b}\right) \cos\left(\frac{p\pi z}{d}\right)$