UNIT-4

MICROWAVE TUBES

Introduction:

At microwave frequencies the size of electronic devices required for generation of microwave energy becomes smaller and smaller. This results in lesser power handling capability and increased noise levels.

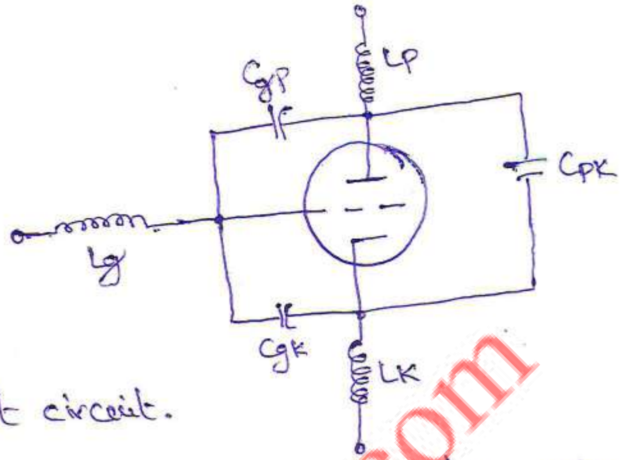
Electronic devices such as tubes and transistors will be required even at microwave frequencies. As expected, the tubes can provide high o/p power while transistors have lesser noise and better reliability.

Conventional triodes, tetrodes and pentodes are useful ~~at~~ only at low microwave frequencies. Special tubes would be required even at UHF frequencies as convential tubes have certain limitations at microwave frequencies.

HIGH Frequency Limitations of Conventional Tubes:

1. Inter Electrode Capacitance Effect
2. Lead Inductance Effect
3. Transit time Effect
4. Gain Bandwidth Product
5. Effect due to RF losses
6. Effect due to radiation losses.

① Inter Electrode Capacitance Effect (IEC)



As frequency increase, the reactance $(X_C) = \frac{1}{2\pi f C}$, decreases and the o/p voltage decreases due to shunting effect. Because at higher frequencies capacitance ~~becomes~~ act as short circuit.

Remedy: This Effect can be minimised by decreasing the area of electrodes and increasing the distance between electrodes.

② Lead Inductance (LI) Effect

As frequency increases, the reactance $X_L = 2\pi f L$ increases and hence the voltage appearing at active electrodes are less than the voltages at the base pins. This results in reduced gain for the tube amplified and the lead inductances limit the perfomance of tube.

Remedy The effect of LI can be minimised by decreasing the value of 'L'.

③ Transit Time Effect

" Transit time " is the time taken to travel from cathode to anode.

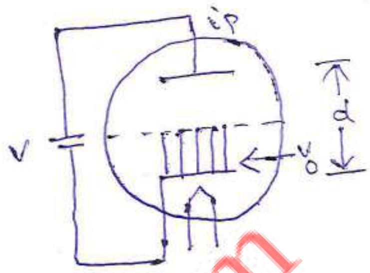
$$\text{i.e Transit time } (T) = \frac{d}{V_0} = \frac{d}{\sqrt{\frac{2eV}{m}}}$$

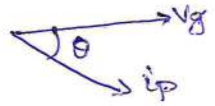
At low frequencies transit time is negligible when compared to the period of signal. i.e both i_p and v_g are in same phase. The transconductance is a large and real quantity.

i_p v_g

$$g_m = \frac{\Delta i_p}{\Delta v_g}$$

At microwave frequencies, the transit time is large compared to the period of microwave signal. i.e i_p lags v_g and g_m becomes a complex quantity. This situation indicates that o/p voltage is reduced.

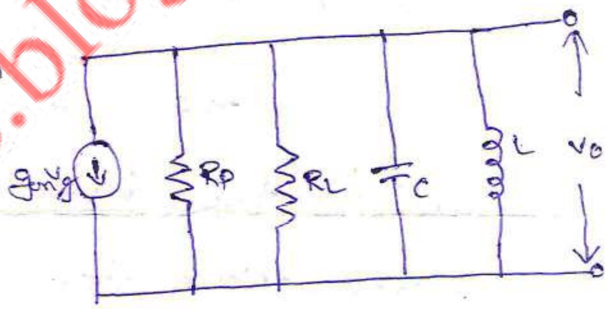




Remedy: To minimise transit time, the seperation between electrodes can be decreased.

(4) Gain Bandwidth Product

Maximum gain is achieved when the tuned circuit is at resonance.



$\therefore$ Resonant frequency $(f) = \frac{1}{2\pi\sqrt{LC}}$

The maximum gain at resonance $A_{max} = \frac{g_m}{G}$ where ; $G = 1/R$

Band width is calculate by the help of extreme frequencies

$$BW = \omega_2 - \omega_1 = \frac{G}{C}$$

Gain Bandwidth product $= A_{max} \cdot BW = \frac{g_m}{C}$

The gain bandwidth product is independent of frequency. So, higher gain can be achieved at the cost of bandwidth only. This limitations can be over come by use of

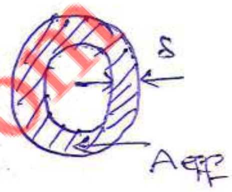
(i) re-entrant cavities

(ii) slow wave tubes for a larger gain over a larger band width.

⑤ Effect due to RF losses

ⓐ Skin Effect losses / Conductor losses

The losses occur at high frequencies at which the current has the tendency to confine it self to a smaller cross-section of the conductor towards outer surface.



Hence the losses will increase at higher frequencies. These losses can be reduced by increasing the size of conductors.

ⓑ Dielectric losses

This occurs in various type of insulating materials used in the device i.e glass envelop, silicon or plastic encapsulations etc.... The loss of material can be given by

$$P = \pi f . V_0^2 \varepsilon_r \tan\delta$$

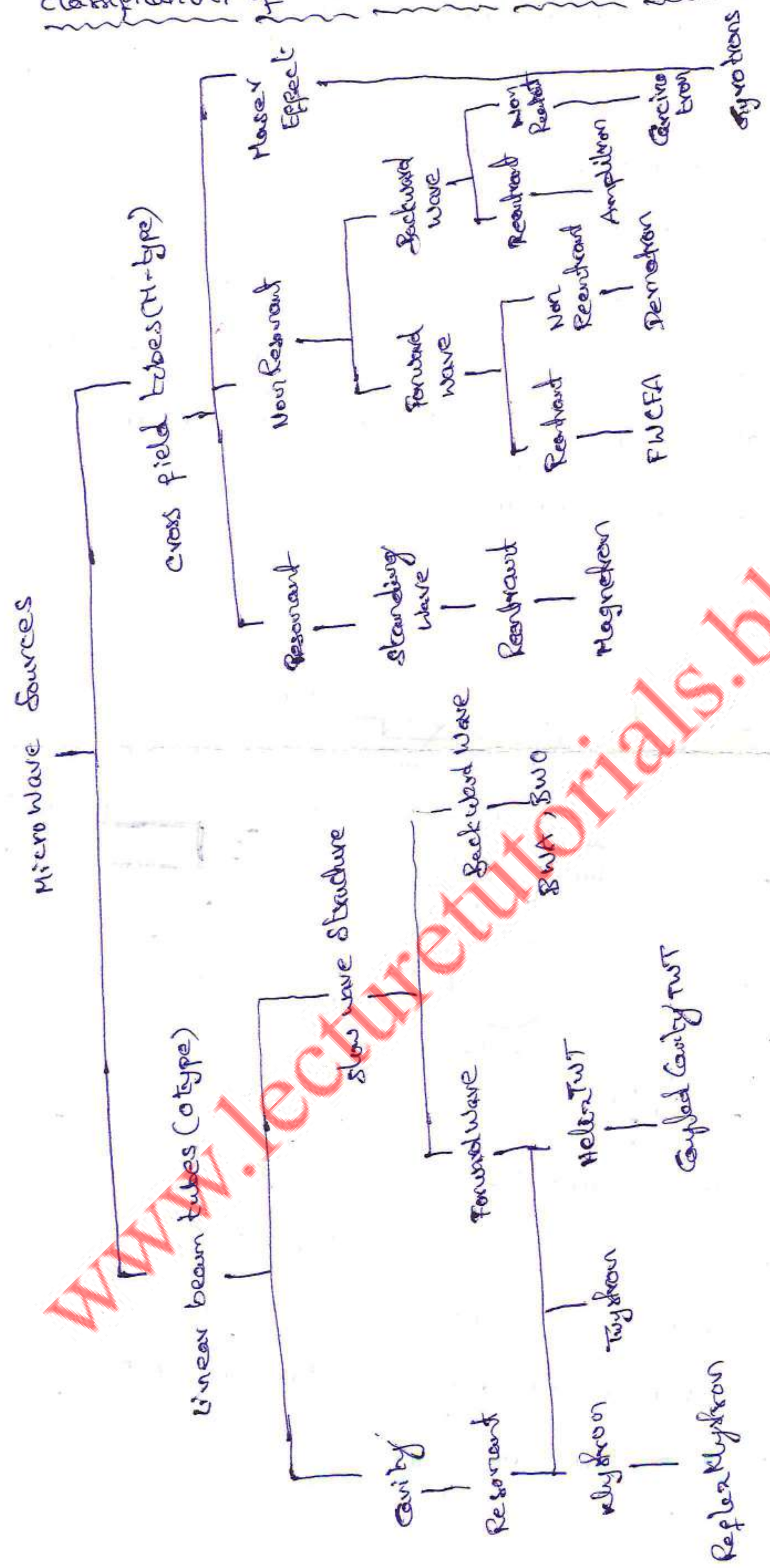
As frequency increases the power loss increases. The remedy for this is to eliminate the tube base and to reduce the surface area of glass.

ⓒ Radiation losses

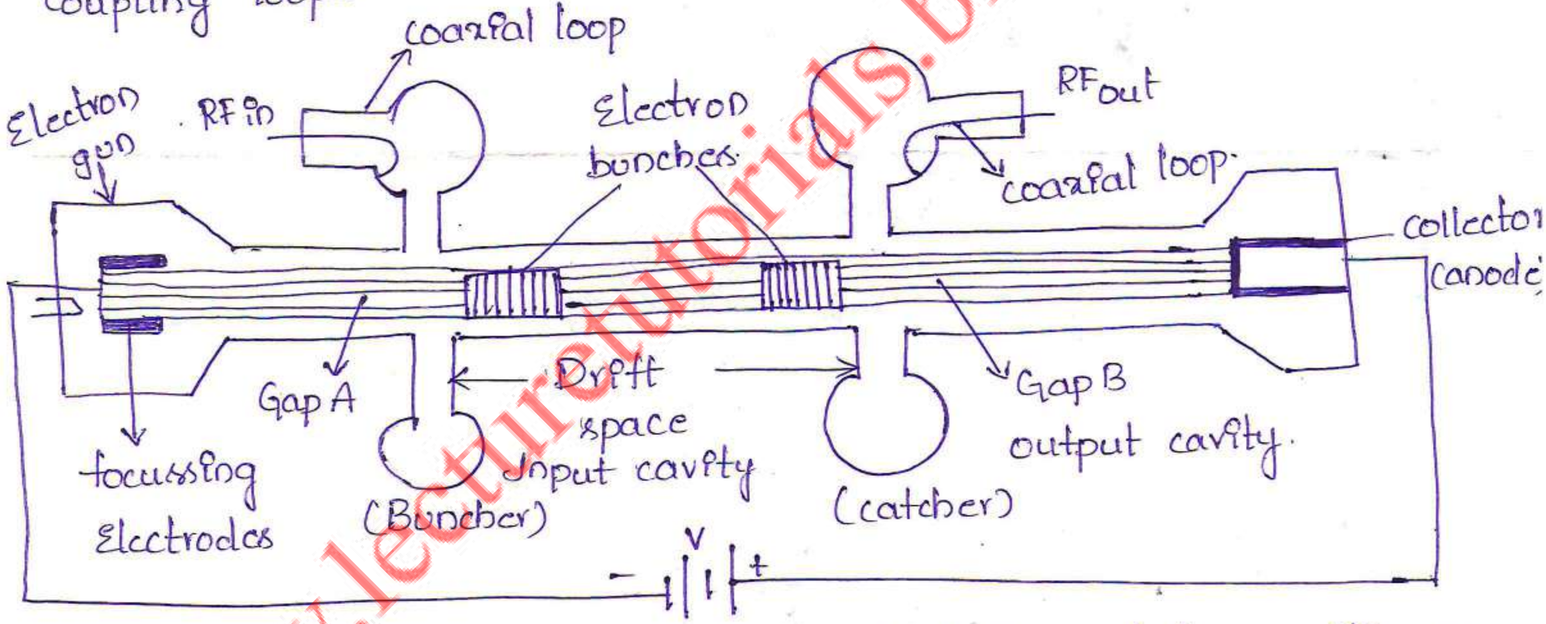
whenever the dimensions of the wire approaches the wavelength, it emits radiations. The radiation losses increase with frequency increases.

The remedy for this is to use proper shielding of of the tubes and circuitary.

classification of Microwave tubes (or) Microwave Sources



Two Cavity Klystron Amplifier:- A two cavity klystron amplifier is basically velocity modulated tube. Here a high velocity electron beam is formed and focussed and sent down along a glass tube through an input cavity (buncher cavity), a field free drift space and an output cavity (catcher) to a collector electrode or anode. The anode is kept at a +ve potential with respect to cathode. The electron beam passes through a gap-A consisting of two grids of the buncher cavity seperated by a very small distance and the other two grids of the catcher cavity with a small gap B. The i/p and o/p are taken from the tube. Via. resonant cavities with the aid of coupling loops.



Operation:- The RF signal to be amplified is used for exciting the input buncher cavity. There by developing an alternating voltage of signal frequency across the gap. Let us now consider the effect of this gap voltage on the electron beam passing through gap A.

This can be explained by an apple gate diagram.

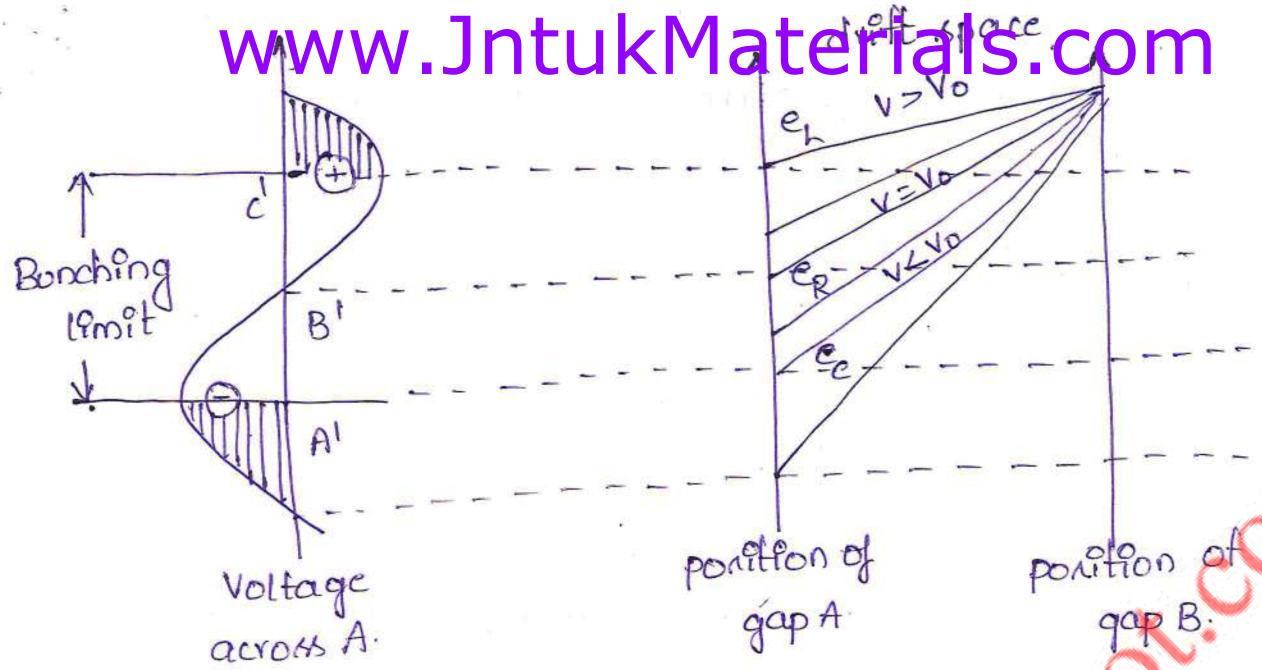


fig:- Apple gate diagram.

At point B' on the input RF cycle the alternating voltage is '0'. and going positive. At this instant the electric field across gap A is zero and an electron passes through gap A. At this instan is unaffected by the RF signal. This electron is called as the reference (e_R) which travels with an unchanged velocity $V_0 = \sqrt{\frac{2eV}{m}}$

where V - anode to cathode voltage.

At point c' of the input RF cycle, an electron which leaves gap-A later than reference electron called as late electron (e_L). Hence the electron travels towards gap-B with an increasing velocity and this electron tries to overtake the reference electron. Similarly an early electron (e_e) that passes the gap A slightly before the reference electron at maximum negative field and this electron travels with a reduced velocity V_0.

Therefore the velocity of electron varies in accordance with RF voltage resulting in Velocity modulation of the electron beam. The drift space converts the velocity modulation into current modulation. Bunching across only once per cycle centered around the C_R with proper design a little RF power applied to the buncher cavity results in larger beam currents at the catcher cavity with a considerable power gain.

Performance characteristics:

1. frequency: 250MHz to 100GHz
2. Power: 10kw – 500kW
3. Power gain:- 15dB - 70dB.
4. Bandwidths: Limited (because cavity resonators are being used) 10–60 MHz – generally used in fixed frequency applications.
5. Noise figure:- 15-20dB (Sometimes greater than 25dB)
6. Theoretical efficiency: ~~50~~ 58%

~~Reentrant Cavities~~

Applications:-

1. As power output tubes
   a) In UHF TV transmitters
   b) In troposphere scatter transmitters.
   c) Satellite Communication ground stations
   d) Radar transmitters.
2. As power oscillator (5-50 GHz) if used as a klystron oscillator.

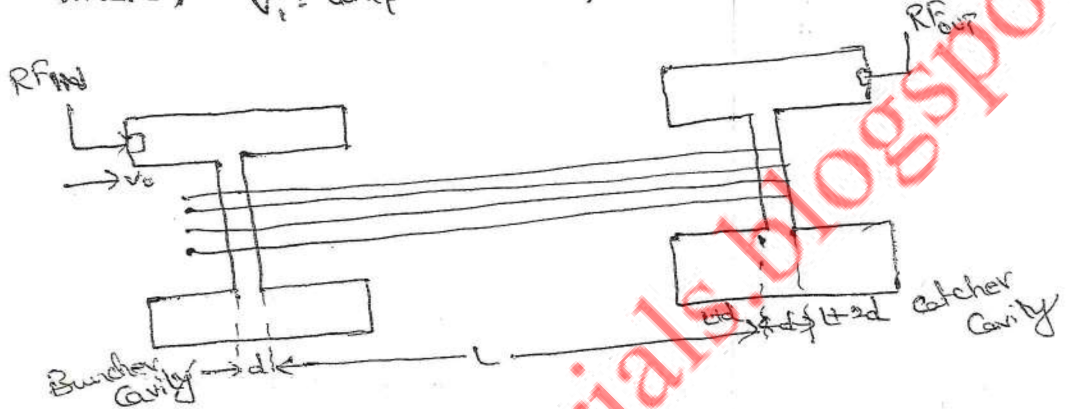
Mathematical Analysis of a Klystron Amplifier

Let the dc voltage between cathode and anode be V_0 and v_0 be the velocity of the electron, L be the drift space length and the RF input signals to be amplified by the klystron be V_s. Then

$$v_0 = \sqrt{\frac{2eV_0}{m}} \rightarrow (1)$$

$$V_s = V_1 \sin\omega t \rightarrow (2)$$

where, V_1 = amplitude of the signal $V_1 << V_0$ is assumed.



The energy of the electron at the time of leaving buncher cavity is given by,

$$\frac{1}{2}mv_1^2 = e(V_0 + V_1 \sin\omega t_1)$$

$$\Rightarrow v_1 = \sqrt{\frac{2e(V_0+V_1\sin\omega t_1)}{m}}$$

$$= \sqrt{\frac{2eV_0}{m}}\sqrt{1+\frac{V_1}{V_0}\sin\omega t_1}$$

$$v_1 = v_0\sqrt{1+\frac{V_1}{V_0}\sin\omega t_1}$$

Expanding binomically and neglecting higher powers of $\sin\omega t$ we get

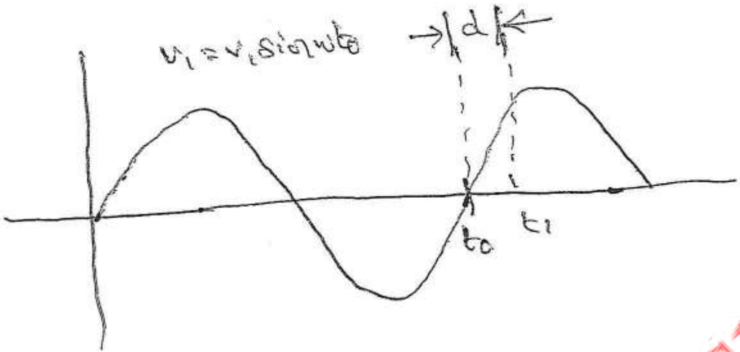
$$v_1 = v_0\left(1+\frac{V_1}{2V_0}\sin\omega t_1\right) \rightarrow (3)$$

This is the equation of velocity modulation.

$$\omega t_1 = \omega t_0 + \frac{\theta_g}{2}$$

where θ_g is the phase angle of the RF input voltage during which the electron is accelerated.

$$\theta_g = \omega t = \omega(t_1 - t_0) = \frac{\omega d}{v_0} \rightarrow (4)$$



Bunching Process of Electron

Maximum velocity occurs at $+\pi/2$ so that

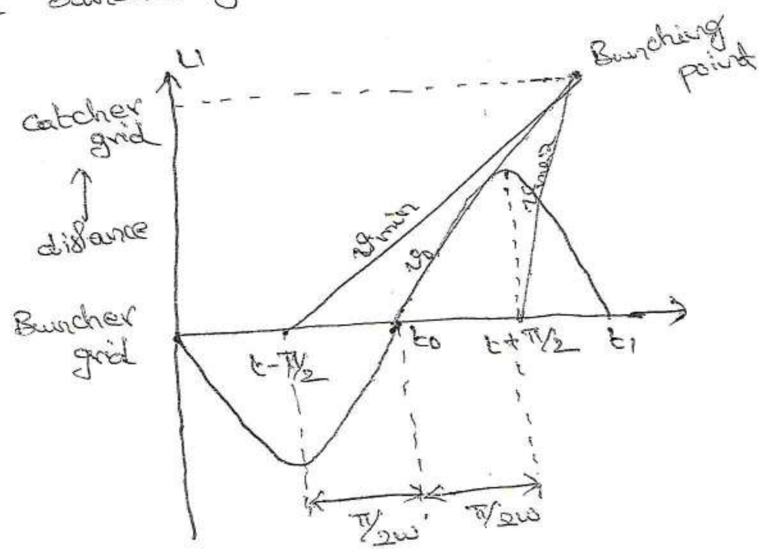
$$v_{1(max)} = v_0\left(1 + \frac{V_1}{2V_0}\right) \rightarrow (5)$$

and minimum velocity at $-\pi/2$, so that

$$v_{1(min)} = v_0\left(1 - \frac{V_1}{2V_0}\right) \rightarrow (6)$$

If the distance in the drift space at which the bunching occurs from the buncher grid at time t_1 is L_1

$$L_1 = v_0(t_1 - t_0) \rightarrow (7)$$



The distance L_1 at $t_{-\pi/2}$

$$L_1 = v_{min}(t_1 - t_{-\pi/2}) \rightarrow (8)$$

The distance L_1 at $t_{+\pi/2}$

$$L_1 = v_{min}(t_1 - t_{+\pi/2}) \rightarrow (9)$$

$$t_{-\pi/2} = t_0 - \frac{\pi}{2\omega} \quad \& \quad t_{+\pi/2} = t_0 + \frac{\pi}{2\omega} \rightarrow (10)$$

Substituting eq (6) & eq (10) in eq (7) we can get

$$L_1 \text{ at } t_{-\pi/2} = v_0\left(1-\frac{V_1}{2V_0}\right)\left(t_1-t_0+\frac{\pi}{2\omega}\right) \rightarrow (11)$$

Similarly

$$L_1 \text{ at } t_{+\pi/2} = v_0\left(1+\frac{V_1}{2V_0}\right)\left(t_1-t_0-\frac{\pi}{2\omega}\right) \rightarrow (12)$$

$$\therefore L_1 = v_0(t_1-t_0) + v_0\left[\frac{\pi}{2\omega}-\frac{V_1}{2V_0}(t_1-t_0)-\frac{V_1}{2V_0}\frac{\pi}{2\omega}\right]$$

For maximum value

$$L_1 = v_0(t_1-t_0) + v_0\left[\frac{\pi}{2\omega}-\frac{V_1}{2V_0}(t_1-t_0)-\frac{V_1}{2V_0}\frac{\pi}{2\omega}\right]$$

If the distance has to be the same for $-\pi/2, 0, +\pi/2$ bunches L_1 for all three should be equal to $v_0(t_1-t_0)$

$$\text{i.e } \frac{\pi}{2\omega}-\frac{V_1}{2V_0}(t_1-t_0)-\frac{V_1}{2V_0}\frac{\pi}{2\omega}=0$$

$$\Rightarrow -(t_1-t_0) = \frac{\pi}{2\omega}\left[\frac{V_1}{2V_0}-1\right]\cdot\frac{2V_0}{V_1}$$

$$\Rightarrow -(t_1-t_0) = \frac{\pi}{2\omega}-\frac{\pi V_0}{\omega V_1}$$

As $\frac{V_0}{V_1}$ is very high, $\frac{\pi}{2\omega}$ can be neglected

$$\Rightarrow t_1-t_0 = \frac{\pi V_0}{\omega V_1} \rightarrow (13)$$

Substitute eq (13) in eq (7)

$$L = v_0\left(\frac{\pi V_0}{\omega V_1}\right) \rightarrow (14)$$

Bunching occurs as the RF signal changes from $-\frac{\pi}{2}$ to $+\frac{\pi}{2}$ i.e for a value of $\pi = 3.682$, optimum bunching occurs and

$$L_{max} = 3.682\frac{v_0 V_0}{\omega V_1} \rightarrow (15)$$

If the beam coupling coefficient of input cavity is β, given by

$$\beta = \frac{\sin(\theta_g/2)}{\theta_g/2}$$

where $\theta_g = \frac{\omega d}{v_0}$

Then, L_{max} is given by

$$L_{max} = 3.682 \frac{v_0 V_0}{\omega \beta V_1} \rightarrow (16)$$

Efficiency is given by, $\eta = \frac{P_{out}}{P_{in}}$

<u>Output Power (P_{out}):</u>

At the catcher cavity,

RF voltage $= V_2 \sin\omega t_2$

Energy given by the electron to the bunch

$= (-e) V_2 \sin\omega t_2 = -eV_2 \sin\omega t_2$

The average energy given to the RF field in a cycle

$$P_{av} = \frac{1}{2\pi} \int_0^{2\pi} (-eV_2 \sin\omega t_2)\, d\omega t_1 \rightarrow (17)$$

In the field free space between cavities, the transit time for velocity modulated electron is given by,

$$T = t_2 - t_1 = \frac{L}{v_1} = \frac{L}{\left(v_0\left(1+\frac{V_1}{V_0}\right)\sin\omega t_1\right)^{1/2}}$$

$$T = \frac{L}{v_0}\left[\left(1+\frac{V_1}{V_0}\right)\sin\omega t_1\right]^{-1/2}$$

$$T = \frac{L}{v_0}\left[1 - \frac{V_1}{2V_0}\sin\omega t_1\right] \rightarrow (18)$$

Multiplying by ω

$$\omega T = \omega(t_2 - t_1) = \frac{\omega L}{v_0}\left[1 - \frac{V_1}{2V_0}\sin\omega t_1\right] \rightarrow (19)$$

where $\frac{L}{v_0} = T_0$ i.e the transit time without RF voltage V_1 in buncher Cavity.

$\omega T_0 = \theta_0 = 2\pi N$ is the transit angle "

N = no. of electron transit cycles in drift space.

The bunching parameter x of a klystron is defined by the equation,

$$X = \frac{V_1}{2V_0}\theta_0 \rightarrow (20)$$

which is a dimensionless quantity and propotional to input power.

So eq (17) can be written using eq (19)

$$P_{av} = \frac{-eV_2}{2\pi}\int_0^{2\pi}\sin(\omega t_1 + T)\,d\omega t_1$$

$$\Rightarrow P_{av} = \frac{-eV_2}{2\pi}\int_0^{2\pi}\sin\left[\omega t_1 + \theta_0\left(1 - \frac{V_1}{2V_0}\sin\omega t_1\right)\right]d\omega t_1$$

This is a Bessel function and its solution is given by

$$P_{av} = -eV_2 J_1(X)\sin\theta_0 \longrightarrow (21)$$

where, $J_1(X)$ = Bessel function of the first order for the argument 'x'.

For N electron transit cycles,

Energy transferred = $N P_{ave} = -Ne V_2 J_1(X)\sin\theta_0$

Where $Ne = I_0$ i.e output current

Energy transferred $= -I_0 V_2 J_1(X) \sin\theta_0 \rightarrow (22)$

But Maximum Value of $J_1(x) = 0.58$ for $X = 1.84$

For maximum energy transfer

$$P_{max} = -I_0 V_2 J_1(X) \sin\theta_0$$

$$\sin\theta_0 = -1 \quad ; \quad \theta_0 = 2n\pi - \frac{\pi}{2}$$

$\therefore$ The output power

$$P_{out} = P_{max} = I_0 V_2 (0.58) \rightarrow (23)$$

Input Power (P_{in})

The input power is ~~basically~~ the dc input given by

$$P_{in} = I_0 V_0 \rightarrow (24)$$

The efficiency $(\eta) = \dfrac{P_{out}}{P_{in}} = \dfrac{0.58 I_0 V_2}{I_0 V_0}$

$$= 0.58 \frac{V_2}{V_0}$$

Always $V_2 < V_0$, the maximum efficiency that can be attained is 0.58 (or) 58%.

Efficiency is a function of the transit angle and is maximum for a catcher gap transit angle $\theta_g = \theta_0 = -\pi/2$ and is zero for $\theta_g = 2\pi$.

However Practical efficiencies are only 15 to 30%.

Multi cavity klystron

In multicavity klystron, multiple no.of cavities are used as shown in figure. The two-cavity klystron, will suffer for gains of about 10 to 20 dB. So, a higher overall gain can be acheived by connecting multiple two cavity tubes, in cascade, feeding the output of each of the tubes to the input of the succeeding one.

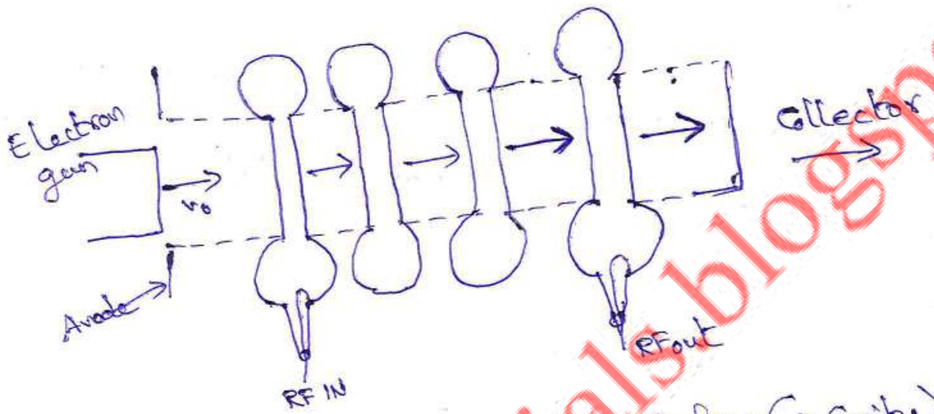


Fig: Multi cavity klystron (4 cavity)

Here each of intermediate cavities acts as a buncher with the passing electron beam including an enhanced RF voltage than the previous cavity. With four cavities, power gains of around 50 dB can be easily achieved. The cavities can all be tuned to the same frequency for narrow band operation. Bandwidth can be improved by stagger tuning of cavities upto about 80 MHz ~~of~~ with reduction in gain (to about 45 dB).

Two Cavity Klystron Oscillator

The two cavity klystron oscillator is obtained by simply providing a feed back loop between input and output cavities of a two-cavity klystron amplifier. Depending upon the cavity tuning and various d.c voltages, the feedback path between the cavity resonators is properly matched. The condition for sustained oscillations is

$$\theta + \alpha + \frac{\pi}{2} = 2n\pi \quad \longrightarrow (1)$$

where,

θ – The total phase shift in the resonators and the feed back cable

$\alpha + \frac{\pi}{2}$ – phase angle between buncher and catcher voltages

n – An integer.

If the two resonators oscillate in same phase i.e $\theta = 0$. Then maximum output power is obtained by substituting $\theta = 0$ in eq (1) i.e

$$\alpha + \frac{\pi}{2} = 2n\pi$$

$$\Rightarrow \boxed{\alpha = 2n\pi - \frac{\pi}{2}}$$

The above condition is necessary for obtaining sustained oscillations.

A critically coupled klystron oscillators has almost linear variation in frequency with accelerating voltage. High frequency stability of oscillator is obtained by controlling the temperature of the resonators and also by use of regulated power supplies.

Tuning of the oscillator is done by adjusting grid voltages, dc accelerating voltage and tuning of the two resonators.

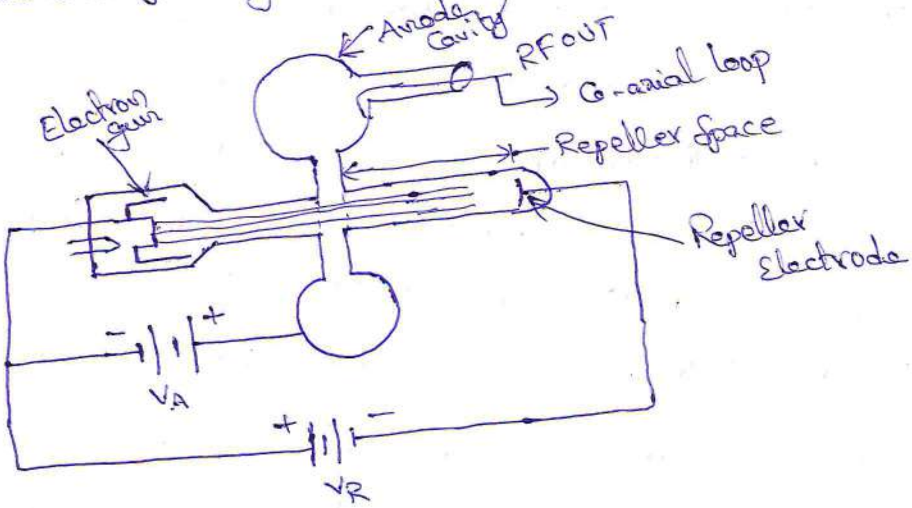
Reflex Klystron

The reflex klystron is a single cavity variable frequency microwave generator of low power and low efficiency. This is most widely used in applications where variable frequency is desired as,

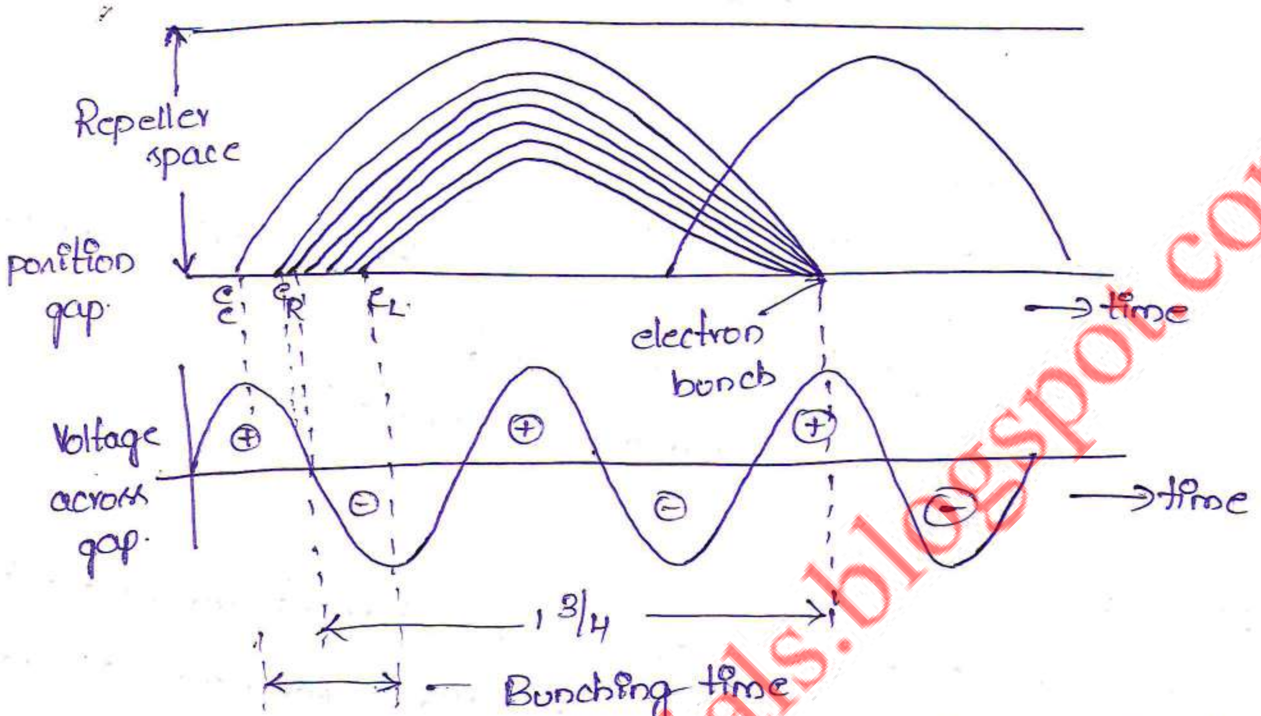
→ In radar receivers.
→ Local oscillator in microwave receivers
→ Signal source in microwave generator of variable frequency
→ Portable microwave links and
→ Pump oscillator in parametric amplifier.

Construction:

It consists of an electron gun, a filament surrounded by cathode and a focussing electrode at cathode potential as shown in fig. The electron beam is accelerated towards the anode cavity. After passing the gap in the cavity, electrons travel towards a repeller electrode which is at a high negative potential V_R. The electrons never reach the repeller because of the negative field and are returned back towards the gap. Under suitable conditions, the electrons give more energy to the gap than they took from the gap on their forward journey and oscillations are sustained.



Operation: It is assumed that the oscillations are set up in the tube initially due to noise or switching transients and these oscillations are sustained by device operation. This can be explained by apple gate diagram



The RF voltage that is produced across the gap by the cavity oscillations act on the electron beam to cause velocity modulation. 'e_R' is the reference electron that passes through the gap when the gap voltage is '0' and going negative. Electron 'e_R' is unaffected by the gap voltage. This moves towards the repeller and gets reflected by the negative voltage on the repeller. It returns and passes through the gap for a second time.

The early electron 'e_e' that passes through the gap before the reference electron 'e_R' experiences a maximum positive voltage across the gap and this electron is accelerated. It moves with greater velocity and penetrates deep into repeller space. The return time for electron 'e_e' is greater as the depth of penetration into repeller space is more. Hence 'e_e' and 'e_R' appear at the gap for the second time at the same instant.

The late electron 'e_L' that passes the gap later than reference electron 'e_R' experiences a maximum negative voltage and moves with a retarding velocity. The return time is shorter as the penetration into repeller space is less and catches up with e_R and e_L electrons forming a bunch. Bunches occur once per cycle centered around the reference electron 'e_R' and these bunches transfer maximum energy to the gap to get sustained oscillations.

For oscillations to be sustained, the time taken by the electrons to travel into the repeller space and back to the gap (called transit time) must have an optimum value. This factor is not important in a klystron amplifier but it assumes a great importance here.

The most optimum departure time is obviously centered on the reference electron which is at 180° point of sine wave voltage across the resonator gap. The cavity resonator spends energy in accelerating the electrons and gains energy in retarding them. As there are as many retarded electrons as accelerated the total energy outlay is nil. From this discussion, it is clear that the best possible time for electrons to return to the gap is at the time at which the voltage then existing ~~occurs~~ across the gap will apply maximum retardation to them. This is when the gap will apply maximum retardation to them. This is when the gap voltage is positive maximum. This causes electrons to fall through the maximum negative voltage between the gap grids, thus giving up the maximum amount of energy to the gap. Thus it appears that the best time for electrons to return to the gap is at the 90° point of sine wave gap voltage. Returning of electrons after $1\frac{3}{4}$ or $2\frac{3}{4}$ or $3\frac{3}{4}$ cycles etc. satisfies the above requirements. In general, the optimum transit time should be

$$T = n + \frac{3}{4}$$

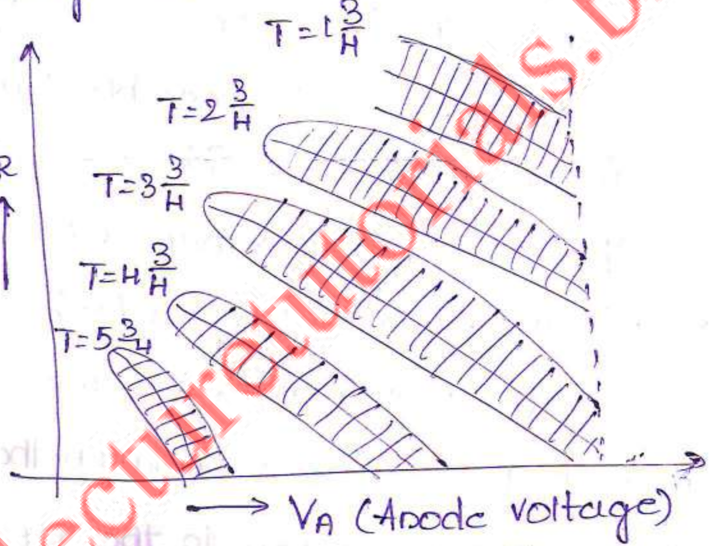
where 'n' is any integer.

This depends on repeller and anode voltages.

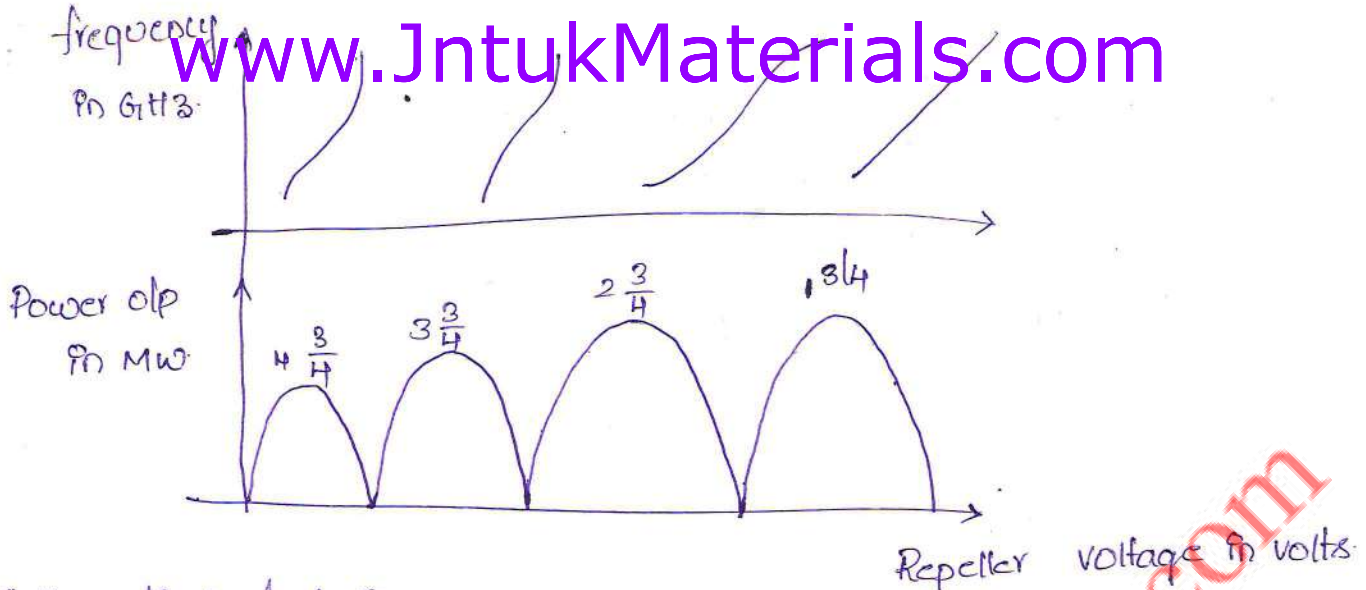
Operating characteristics

1. Voltage characteristics :- Oscillations can be obtained only for specific combinations of anode and repeller voltages that give a favourable transit time ($T = n + 3/4$)

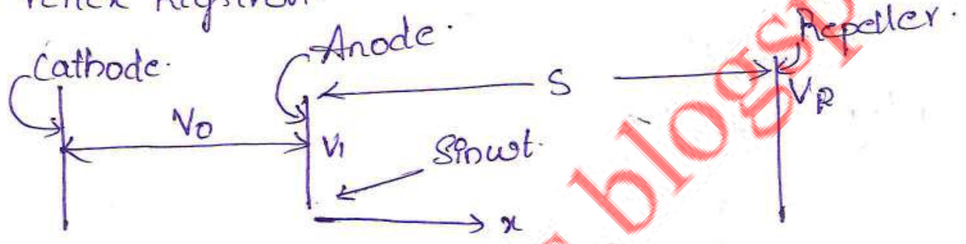
The shaded areas show possible oscillation combinations and heavy lines show optimum combinations. Each value of $n = 1, 2, 3 \ldots$ is said to correspond to a different mode for the reflex klystron oscillator. The earlier the mode the larger the output power, which is obviously an advantage. But the voltages required are also higher as shown in fig. leading to insulation problems and the possibilities of lower efficiencies. As a result the modes corresponding to $n = 2$ or $n = 3$ are most widely used.



2. Power output and frequency characteristics :- The mode curves and frequency characteristics are shown. The frequency of resonance of the cavity decides the frequency of oscillation. Variations of repeller voltage slightly changes the frequency. This amount to electronic tuning of reflex klystron and this is also depicted in fig. This makes it possible to use reflex klystron as a voltage tuned oscillator or frequency modulated oscillator.



Mathematical Analysis: we shall refer to below fig. for the mathematical analysis of reflex klystron.



V_0 – electron gun anode voltage.

$V_1 \sin\omega t$ – RF voltage at cavity gap.

V_R – Repeller voltage with respect to cathode.

s – distance b/w cavity gap and repeller electrode.

v_0 – velocity of electron in gun.

v_1 – velocity due to RF voltage in addition to the electron accelerating voltage V_0.

t_0 – time for electron entering cavity gap at $x=0$.

t_1 – time for same electron leaving cavity gap at $x=d$

t_2 – time for same electron returned by retarding field at $x=d$.

$$v_0 = \sqrt{\frac{2eV_0}{m}} \quad \left(\text{since } \tfrac{1}{2}mv_0^2 = eV_0\right) \rightarrow (1)$$

$$v_1 = V_0\sqrt{\frac{1+V_1}{V_0}}\ \sin\omega t \rightarrow (2)$$

Voltage between repeller and anode.

$$\therefore V_R - (V_0 + V_1 \sin\omega t) \approx V_R - V_0$$

Retarding electrostatic field b/w repeller and anode is given by:

$$E = -\left(\frac{V_R - V_0}{S}\right) \rightarrow ③$$

force on Electron $= -eE = +e\left(\frac{V_R - V_0}{S}\right)$

Also, force on electrons = mass × acceleration $= \frac{m d^2x}{dt^2}$

Equating the above two equations:

$$m\frac{d^2x}{dt^2} = \frac{e}{S}(V_R - V_0) \rightarrow ④$$

$$\frac{d^2x}{dt^2} = \frac{e}{mS}(V_R - V_0)$$

by integrating above eq.

$$\frac{dx}{dt} = \frac{e}{mS}(V_R - V_0)t + c \rightarrow ⑤$$

at $t = t_1$, $\frac{dx}{dt} = v_1$

$$v_1 = \frac{e}{mS}(V_R - V_0)t_1 + c$$

$$c = v_1 - \frac{e}{mS}(V_R - V_0)t_1 \rightarrow ⑥$$

substituting for 'c' in above eq, we get

$$\frac{dx}{dt} = \frac{e}{mS}(V_R - V_0)(t - t_1) + v_1 \rightarrow ⑦$$

by integrating

$$x = \frac{e}{2mS}(V_R - V_0)(t - t_1)^2 + v_1 t + c_1 \rightarrow ⑧$$

At $x = 0$, i.e. at the point of return from repeller space; $t = t_2$

$$0 = \frac{e}{2mS}(V_R - V_0)(t_2 - t_1)^2 + v_1 t_2 + c_1$$

or $$c_1 = \frac{-e}{2mS}(V_R - V_0)(t_2 - t_1)^2 - v_1 t_2$$

Using this value of c_1, we get.

$$x = \frac{e}{2ms}(V_R - V_0)\left[(t-t_1)^2 - (t_2-t_1)^2 \right] + v_1(t_1 - t_2)$$

Again when $t = t_1$, $x = 0$

i.e. $$\frac{-e}{2ms}(V_R - V_0)(t_2 - t_1)^2 - v_1(t_2 - t_1) = 0$$

$(t_2 - t_1)$ is the round trip transit time and is given by

$$(t_2 - t_1) = \frac{-2ms\,v_1}{e(V_R - V_0)} \longrightarrow (9)$$

The transit angle 'ωt' is defined as transit angle 'at time t'.

$$\omega(t_2 - t_1) = \frac{-2ms\,v_1\omega}{e(V_R - V_0)}$$

or $$\omega t_2 = \omega t_1 - \frac{2ms\,v_1\omega}{e(V_R - V_0)} \longrightarrow (10)$$

from Eq. (2)

$$v_1 = v_0\left(1 + \frac{V_1}{V_0}\sin\omega t\right)^{1/2}$$

Since $V_1 << V_0$

$$v_1 = v_0\left(1 + \frac{V_1}{2V_0}\sin\omega t\right)$$

Substituting for v_1 in Eq. (10)

$$\omega t_2 = \omega t_1 - \frac{2ms\,\omega}{e(V_R - V_0)} \cdot v_0\left(1 + \frac{V}{2V_0}\sin\omega t\right) \rightarrow (11)$$

let $$\frac{-2ms\,\omega v_0}{e(V_R - V_0)} = \omega T_0' X = \theta_0' \rightarrow (12)$$

where θ_0' is the round trip dc transit angle of centre of bunch electron. let

$$\frac{V_1}{2V_0}\theta_0' = X' \longrightarrow (13)$$

where X' is the bunching parameter.

substituting in Eq. (11)

$$\omega t_2 = \omega t_1 + \theta_0'\left(1 + \frac{V_1}{2V_0}\sin\omega t\right) \longrightarrow (14)$$

Relation between Repeller voltage and Accelerating voltage:

consider Eq. (14) and when $V_1 << V_0$

$\omega t_0 = \omega t_1 + \theta_0'$

Also, for maximum transfer of energy, the modes are $1\frac{3}{4}$ cycles apart i.e. $2\pi(n - 1/4)$ where $n - 1/4 = 3/4, 1\frac{3}{4}$ etc.

or $2\pi(n - 1/4) = 2\pi n - \pi/2$

$\therefore$ The optimum value of θ_0' is

$$\theta_0' = (2n\pi - \pi/2) \rightarrow (15)$$

from Eq. (12)

$$\theta_0' = -\frac{2msω}{e(V_R - V_0)} v_0 \quad \text{or} \quad v_0 = -\frac{e(V_R - V_0)}{2msω} \theta_0'$$

$$v_0^2 = \frac{e^2 (V_R - V_0)^2 (\theta_0')^2}{4ω^2 m^2 s^2} \rightarrow (16)$$

we know that $\frac{1}{2} m v_0^2 = eV_0$

or $V_0 = \frac{m}{2e} v_0^2$

substituting for v_0^2 from Eq. (16)

$$V_0 = \frac{m}{2e} \frac{e^2 (V_R - V_0)^2 (\theta_0')^2}{4ω^2 m^2 s^2}$$

where $\theta_0' = 2\pi n - \pi/2$

$$\therefore \frac{V_0}{(V_R - V_0)^2} = \frac{m}{2e} \frac{e^2}{4ω^2 m^2 s^2} \left(2\pi n - \frac{\pi}{2}\right)^2$$

Simplifying,

$$\frac{V_0}{(V_R - V_0)^2} = \frac{1}{8} \frac{1}{ω^2 s^2} \frac{e}{m} \left(2\pi n - \frac{\pi}{2}\right)^2 \rightarrow (17)$$

This gives relationship between V_0 and V_R

Expression for change in frequency due to repeller voltage variation electronic tuning of Reflex klystron:-

The Eq (17) can be written as

$$(V_R - V_0)^2 = \frac{8ms^2 V_0}{(2n\pi - \pi/2)^2 e} \cdot ω^2$$

Differentiating V_R with respect to ω

$$2(V_R - V_0)\frac{dV_R}{d\omega} = \frac{16ms^2 V_0}{(2\pi n - \pi/2)^2 e} \cdot \omega^2$$

$$\frac{dV_R}{d\omega} = \frac{8ms^2 V_0 \omega}{e(2\pi n - \frac{\pi}{2})^2} \frac{1}{(V_R - V_0)}$$

We can resubstitute the value of $(V_R - V_0)$ as,

$$(V_R - V_0) = \sqrt{\frac{8ms^2 V_0 \omega^2}{e(2\pi n - \pi/2)^2}}$$

and obtain the value of $\frac{dV_R}{d\omega}$ as,

$$\frac{dV_R}{d\omega} = \frac{8ms^2 V_0 \omega}{e(2\pi n - \pi/2)^2} \times \sqrt{\frac{e(2\pi n - \pi/2)^2}{8ms^2 V_0 \omega^2}}$$

$$= \sqrt{\frac{8ms^2 V_0}{e}} \cdot \frac{1}{(2n\pi - \pi/2)}$$

$$\therefore \frac{dV_R}{df} = \frac{2\pi s}{(2n\pi - \pi/2)} \sqrt{\frac{8mV_0}{e}} \longrightarrow (18)$$

This is a very useful relationship for electronic tuning of reflex klystron i.e., repeller voltage is crucial for frequency of oscillation.

<u>Efficiency of Reflex Klystron</u>:- Similar to klystron amplifier maximum power is transferred to the output when the returning bunched electrons arrive at the cavity and when the field is at antinode power output.

DC power supplied by beam voltage V_0 is

$$P_{dc} = V_0 I_0 \longrightarrow (19)$$

AC power delivered is

$$P_{ac} = I_0 V_2 J_1(X') \sin\theta_0'$$

As the current flows in the negative direction, the negative sign becomes positive and $\sin\theta_0'$ is 1 and V_2 is V_1 being single and sam cavity.

$$P_{ac} = I_0 V_1 J_1(x') \rightarrow (20)$$

where $x' = \frac{V_1}{2V_0} \cdot \theta_0'$ $\left(= 2n\pi - \frac{\pi}{2} \text{ from Eq. (15)}\right)$

$$\therefore \frac{V_1}{V_0} = \frac{2x'}{(2n\pi - \pi/2)}$$

Substituting for V_1 in Eq (20)

$$P_{ac} = \frac{2V_0 I_0 x' J_1(x')}{(2n\pi - \pi/2)} \rightarrow (21)$$

$$\text{Efficiency} = \frac{P_{ac}}{P_{dc}} = \frac{2V_0 I_0 x' J_1(x')}{V_0 I_0 (2n\pi - \pi/2)}$$

$$\eta = \frac{2x' J_1(x')}{(2n\pi - \pi/2)} \rightarrow (22)$$

The factor $x' J_1(x')$ reaches a maximum value of 1.252 at $x' = 2.408$ and $J_1(x') = 0.52$. The maximum power output is obtained when $n = 2$ or $1\frac{3}{4}$ mode

$\therefore$ Maximum theoretical efficiency is

$$\eta_{th(max)} = \frac{2(2.408)(0.52)}{2\pi(2) - \pi/2} = 22.78\% \rightarrow (23)$$

The practical values however are around 20%.

Power output in terms of repeller voltage V_R:-

$$P_{out} = \frac{2V_0 I_0 x' J_1(x')}{2n\pi - \pi/2}$$

$$2n\pi - \frac{\pi}{2} = \theta_0' = \omega T_0' = \frac{2 m s \omega}{e(V_R - V_0)} v_0$$

The negative sign is not taken as electron bunch travels in the reverse direction, $-x$.

$$P_{out} = \frac{2V_0 I_0 x' J_1(x')}{2m s \cdot \omega v_0} \times (V_R - V_0) e$$

Eliminating V_0

$$P_{out} = \frac{2V_0 I_0 \times x' J_1(x')}{\omega s} \times (V_R - V_0)\sqrt{\frac{e}{2mV_0}}$$

for maximum value of $x' J_1(x') = 1.75$, we get

$$P_{max} = \frac{1.25\, V_0 I_0 (V_R - V_0)}{\omega s} \times \sqrt{\frac{e}{2mV_0}} \rightarrow (24)$$

Equivalent circuit of Reflex klystron :- The equivalent circuit of a reflex klystron is shown in below fig. It consists of the klystron in parallel with beam loading conductance G_b, copper cavity losses G_0, load conductance G_L and parallel tuned circuit

The electronic admittance is given by

$$Y_e = \frac{I_2}{V_2}$$

where I_2 and V_2 are output current and voltage respectively.

i.e., $$Y_e = \frac{2I_0 J_1(X_1') e^{-j\theta_0'}}{V_1 e^{-j\pi/2}}$$

(Since $I_2 = 2I_0 J_1(X_1') . e^{-j\theta_0'}$ in exponential form and $V_2 = V_1 e^{-j\pi/2}$)

Putting $$V_1 = \frac{2V_0 X'}{\theta_0'}$$

$$Y_e = \frac{I_0}{V_0}\frac{\theta_0'}{X'} e^{j(\pi/2 - \theta_0')} \rightarrow (25)$$

Oscillations will take place where net conductance is less than zero (negative resistance)

$Y_e = G_e \pm JB_e$

for oscillation, $|-G_e| \geq G$

where $G = \frac{1}{R_{sh}} = G_0 + G_L + G_b$

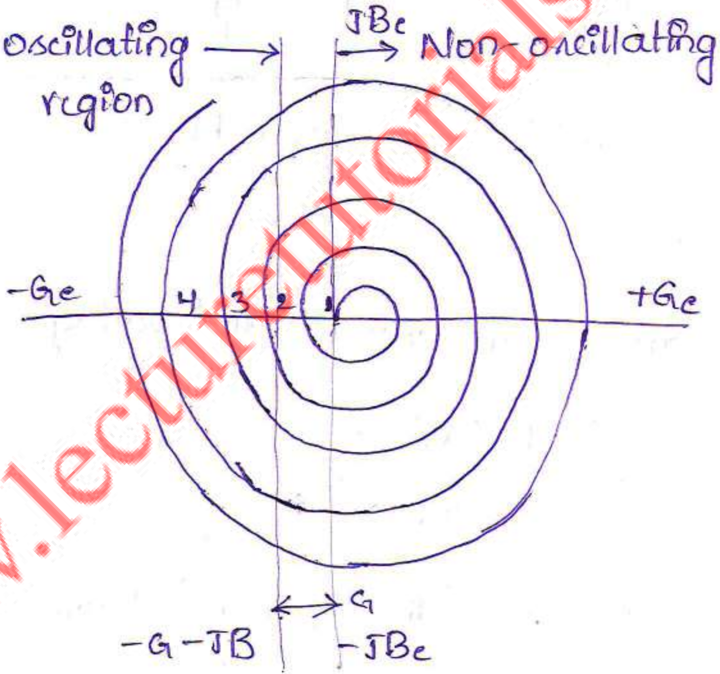
and R_{sh} = shunt resistance

A rectangular plot of $G_e + JB_e$ will be a spiral. The electronic admittance Y_e is a function of transit angle θ_0'. Its phase is $\pi/2$ when θ_0' is 0.

$n=1,\ \theta_0' = 2\pi n - \frac{\pi}{2} = \frac{3\pi}{2}$; $n=2,\ \theta_0' = \frac{7\pi}{2}$;

$n=3,\ \theta_0' = \frac{11\pi}{2}$; $n=4,\ \theta_0' = \frac{15\pi}{2}$

Practically the reflex klystron oscillations are possible, upto $n=7$



Performance characteristics of reflex klystron

1. frequency range : 4 to 200 GHz
2. Output power : 1.0mW to 2.5W
3. Theoretical η : 22.78%
4. Practical η : 10 to 20%
5. Tuning range : 5GHz at 2 watts to 30GHz at 10mW

Problems

① An x-band pulsed cylindrical magnetron has $V_0 = 30kv$, $I_0 = 80A$, $B_0 = 0.01\ Wb/Sq.m$, $a = 4\,Cm$, $b = 8\,Cm$. Calculate ⓐ cyclotron angular frequency, ⓑ cut-off voltage and ⓒ cut-off magnetic flux density

Sol:

ⓐ cycltron angular frequency is given by,

$$\omega = \frac{eB_0}{m} = \frac{1.6\times10^{-19}\times0.01}{9.1\times10^{-31}} = 1.758\times10^{9}\ rad/s$$

ⓑ Hull cutt-off voltage is given by

$$V_{HC} = \frac{eB_0^2 b^2}{8m}\left[1-\frac{a^2}{b^2}\right]^2$$

$$V_{HC} = \frac{1}{8}\times1.759\times10^{11}\times(0.01)^2\times(8\times10^{-2})^2\left[1-\frac{4^2}{8^2}\right]^2$$

$$V_{HC} = 7.9155\,kV$$

ⓒ cutt-off Magnetic flux density

$$B_C = \frac{(8V_0 m/e)^{1/2}}{b\left[1-\frac{a^2}{b^2}\right]} = \left(\frac{8\times30\times10^3}{1.759\times10^{11}}\right)^{1/2}\frac{1}{8\times10^{-2}\left(1-\frac{4^2}{8^2}\right)}$$

$$B_C = 19.468\ m\,Wb/m^2$$

② A magnetron operates with following Parameters $V_0 = 25kV$, $I_0 = 25A$, $B_0 = 0.34T$, Diameter of Cathode $= 8\,Cm$, Radius of vane edge to centre $= 8\,Cm$, Find the Cycltron frequency(f) = ? and cut-off voltage ?

Sol:

$$\omega = \frac{eB_0}{m} = 0.0598\times10^{12}\ rad/sec$$

$$f = \frac{0.0598\times10^{12}\ rad/sec}{2\pi} = 9.52\,GHz$$

cutt-off voltage

$$V_C = \frac{eB_0^2 b^2}{8m}\left[1-\frac{a^2}{b^2}\right]^2 = 9146.37\,KV$$

(3) A two-cavity klystron amplifier has the following parameters

Beam voltage (V_0) = 900V, Beam Current I_0 = 30 mA, Frequency (f) = 8GHz

Gap spacing in either cavity d = 1mm, Spacing between centres of cavities L = 4 cm, Effective shunt impedance: R_{sh} = 40kΩ. Determine

(a) The electron velocity?

(b) The d.c electron transit time?

(c) The input voltage for maximum output voltage?

(d) The voltage gain in decibles?

Sol:

(a) $v_0 = \sqrt{\frac{2eV_0}{m}} = 0.593 \times 10^6 \sqrt{900} = 1.8 \times 10^7$ m/s

(b) Transit time $= \frac{d}{v_0} = \frac{1 \times 10^{-3}}{1.8 \times 10^7} = 0.55 \times 10^{-10}$ s

(c) $$V_{1(max)} = \frac{2 \times V_0}{\beta_i \theta_0}$$

For maximum o/p voltage

$x = 1.841$; $J_1(x) = 0.582$

d.c transit angle $(\theta_0) = \frac{\omega L}{v_0} = \frac{2\pi f (4 \times 10^{-2})}{1.8 \times 10^7} = 111.64$

Beam Coupling Co-efficients $\beta_i = \frac{\sin(\theta_g/2)}{(\theta_g/2)}$

Gap transit angle $\theta_g = \frac{\omega d}{v_0} = 2.8$

$$\beta_0 = \beta_i = \frac{\sin(2.8/2)}{(2.8/2)} = 0.704$$

(2) A Replex klystron has following parameters $V_0 = 800V$, $L = 1.5mm$, $R_{sh} = 15k\Omega$, $f = 9GHz$. Calculate,

(a) The Repeller voltage for which the tube can oscillate in $1\frac{3}{4}$ mode?

(b) The direct current necessary to give a microwave gap voltage of 200V?

(c) [Electron efficiency] . Assume $\beta_0 = 1$

$n = 1\frac{3}{4}$ (or) n

Sol: (i) Repeller voltage,

we know that

$$\frac{V_0}{(V_R - V_0)^2} = \frac{1}{8}\,\frac{1}{\omega^2 s^2}\,.\,\frac{e}{m}\left[2n\pi - \pi/2\right]^2$$

$$= \frac{1}{8}\,\frac{(1.759\times10^{11})(2\pi\times2 - \pi/2)^2}{(2\pi\times9)^2(1.5\times10^{-3})^2}$$

$$= \frac{1}{8}\times\frac{2.13\times10^{13}}{7.19\times10^{15}}$$

$$\frac{V_0}{(V_R - V_0)^2} = 0.00037 \Rightarrow (V_R - V_0)^2 = \frac{V_0}{0.00037}$$

$$\Rightarrow V_R - V_0 = 1470$$

$$\Rightarrow V_R = 1470 + V_0 \Rightarrow \boxed{V_R = 2270V}$$

(ii) D.C current

Given that $V_1 = 200V$ 　　Assume $\beta_0 = 1$

$$V_1 = R_{sh} I_2 = 2 I_0 \beta_0 J_1(x') . R_{sh}$$

$$\Rightarrow I_0 = \frac{V_1}{2 J_1(x') R_{sh}} = \frac{200}{2\times0.582\times15\times10^3}$$

$$\boxed{I_0 = 12.82 mA}$$

(3) A Reflex klystron operates under the following conditions

$V_0 = 600V$, $I_0 = 11.45 mA$, $L = 1mm$, $R_{sh} = 15K$, $f = 9 GHz$. Assume $\beta_0 = 1$

& The tube is oscillating at f at the peak of $n = 1\frac{3}{4}$ mode. Calculate

(a) Microwave Gap voltage

(b) Repeller voltage for the mode $1\frac{3}{4}$.

Sol: (a) Microwave Gap voltage

$$V_1 = 2I_0 J_1(X') R_{sh}$$

$$= 2 \times 11.45 \times 10^{-3} \times 0.52 \times 15 \times 10^{3}$$

$$V_1 \approx 178.62V$$

(b) Repeller voltage

$$\frac{V_0}{(V_R - V_0)^2} = \frac{1}{8} \frac{1}{\omega^2 L^2} \frac{e}{m} \left[2n\pi - \pi/2\right]^2$$

$$= \frac{1}{8} \times \frac{(1.759 \times 10^{11})(2 \times 2 \times \pi - \pi/2)^2}{(2\pi \times 9 \times 10^9)^2 (10^{-3})^2}$$

$$= 0.0008307$$

$$(V_R - V_0)^2 = 722.282 \times 10^3$$

$$V_R - V_0 = 849.87V$$

$$\boxed{V_R = 1449.87V}$$

(4) A Reflex klystron having an accelerated field of 300V oscillates at a frequency of 10 GHz with a retarding field of 500V. If its cavity is retuned to 9GHz, what must be the new value of retarding field for oscillations in the same mode to take place?

Sol: Given,

$V_0 = 300V$

$V_{R_1} = 500V$ & $V_{R_2} = ?$

$f_1 = 10\,GHz$

$f_2 = 9\,GHz$

We know that

$$\frac{V_0}{(V_R - V_0)^2} = \frac{1}{8}\frac{e}{m}\frac{1}{\omega^2 S^2}(2n\pi - \pi/2)^2$$

$$\Rightarrow \frac{V_0}{(V_R - V_0)^2} \propto \frac{1}{f^2}$$

$$\Rightarrow \frac{\dfrac{V_0}{(V_{R_1} - V_0)^2}}{\dfrac{V_0}{(V_{R_2} - V_0)^2}} = \frac{f_2^2}{f_1^2}$$

$$\Rightarrow \frac{(V_{R_2} - V_0)^2}{(V_{R_1} - V_0)^2} = \frac{f_2^2}{f_1^2} \Rightarrow \frac{V_{R_2} - V_0}{V_{R_1} - V_0} = \frac{f_2}{f_1}$$

$$\Rightarrow \frac{V_{R_2} - 300}{500 - 300} = \frac{9}{10}$$

$$\Rightarrow 10 V_{R_2} - 3000 = 1800$$

$$\Rightarrow 10 V_{R_2} = 4800$$

$$\Rightarrow \boxed{V_{R_2} = 480V}$$

$$V_1(max) = \frac{2 X V_1}{\beta_i \theta_0}$$

$$= \frac{2(1.841)(900)}{(0.704)(111.64)} = (42.16V)\ 41.7V$$

ⓓ voltage gain $(A_v) = \dfrac{V_2}{V_1(max)} = \dfrac{1396.8}{(\cancel{42.16})\ 41.7V} = 33.49$

$= 20 \log(33.49) = 30.49 dB$

$V_2 = \beta_0 I_2 R_{sh}$

But $I_2 = 2 I_0 J_1(X)$

$V_2 = \beta_0 2 I_0 J_1(X) R_{sh}$

$V_2 = (0.74) \times 2 \times 30 \times 10^{-3} \times 0.582 \times 40 \times 10^3)$

$V_2 = 1396.8V$