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UNIT-5

MULTIRATE DSP

Introduction: Single rate system is used in single sampling rate the system that requires more than one sampling rate to process the digital signals is known as multirate DSP.

The different sampling rates are used in digital systems the:

The sampling rate conversion of a digital signal can be completed in 2 methods.

1) Analog to Digital convertor
2) Digital to Analog convertor

The Basic operations perform in multirate dsp are.

1) Decimation
2) Interpolation

1) Decimation: The sampling rate compression of the signal is known as decimation and also this process is called "downsampling".

$$x(n) \rightarrow \boxed{\downarrow M} \rightarrow y(n) = x(nM)$$

2) Interpolation: The sampling rate expansion of the signal is known as interpolation, and also this process is called "upsampling".

$$x(n) \rightarrow \boxed{\uparrow M} \rightarrow y(n) = x(n/M)$$

where M is interpolation factor

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P] For $x(n) = \{1,2,4,-2,3,2,1\}$; $M=3$.

Find o/p $y(n)$ for desimation and interpolation process.

Sol) given $x(n) = \{\underset{0}{1}, \underset{1}{2}, \underset{2}{4}, \underset{3}{-2}, \underset{4}{3}, \underset{5}{2}, \underset{6}{1}\}$; $M = 3$.

By using desimation process.

$x(n) \rightarrow \boxed{\downarrow n} \rightarrow y(n) = x(nM)$

$y(n) = x(nM)$

$y(0) = x(0(3)) = x(0) = 1$

$y(1) = x(1(3)) = x(3) = -2$

$y(2) = x(2(3)) = x(6) = 1$

$y(n) = \{1, -2, 1, \ldots\}$.

By using interpolation process.

$x(n) \rightarrow \boxed{M} \rightarrow y(n) = x(n/M)$

$y(n) = x(n/M)$

$y(0) = x(0/3) = x(0) = 1$

$y(1) = x(1/3) = x(1/3) = 0$

$y(2) = x(2/3) = 0$

$y(3) = x(3/3) = x(1) = 2$

$y(4) = x(6/3) = x(2) = 4$

$y(9) = x(9/3) = x(3) = -2$

$y(12) = x(12/3) = x(4) = 4$

$y(15) = x(15/3) = x(5) = 2$

$y(18) = x(18/3) = x(6) = 1$.

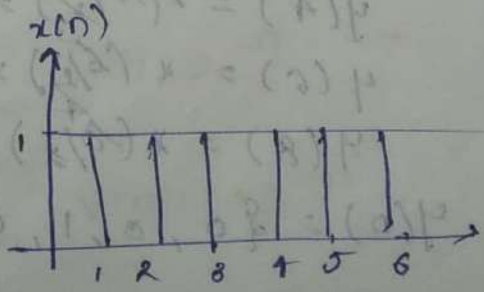
$y(n) = \{1, 0, 0, 2, 0, 0, 4, 0, 0, -2, 0, 0, 4, 0, 0, 2, 0, 0, 1\}$

2] consider a signal $x(n) = u(n)$. obtain the signal with a desimation factor $M=3$, obtain signal of interpolation factor $M=3$.

Sol) Given data $x(n) = u(n)$, $M=3$

$u(n) = 1$ for $n \geq 0$

$u(n) = 0$ for $n < 0$



for desimation process

$x(n) = \{0, 1, 2, 3 \ldots\}$

$y(0) = x(0) = 1$

$y(1) = x(1(3)) = 1$

$y(2) = x(2(3)) = 1$

$\therefore y(n) = \{1, 1, 1, 1, \ldots\}$

$\boxed{y(n) = u(n)}$

for interpolation process $y(n) = x(n/M)$.

$y(n) = x(n/M)$

$y(0) = x(0/3) = x(0) = 1$

$y(1) = x(1/3) = 1$

$y(2) = x(2/3) = 1$

$y(3) = x(3/3) = 1$

$y(4) = x(4/3)$

$y(6) = x(6/3) = 1$

$\boxed{\therefore y(n) = u(n)}$

p-3) consider a signal $x(n) = n$, for $n \geq 0$.

i) obtain a signal with desimation factor. $M = 2$.

ii) obtain a signal with interpolation factor $M = 2$.

Sol) Given data. $x(n) = n$, $n \geq 0$, $M = 2$.

$y(n) = x(nM)$

$y(0) = x(0(2)) = x(0) = 0$.

$y(1) = x(1(2)) = x(1) = 2$.

$y(2) = x(2(2)) = x(4) = 4$

$y(3) = x(3(2)) = x(6) = 6$.

$y(n) = \{0, 2, 4, 6 \ldots\}$

for interpolation. $y(n) = x(n/M)$

$y(0) = x(0/3) = x(0) = 0$

$y(1) = x(1/2) = 0$

$y(2) = x(2/2) = 1$ $\quad y(3) = x(3/2) = ?$

$y(4) = x(4/2) = 2$

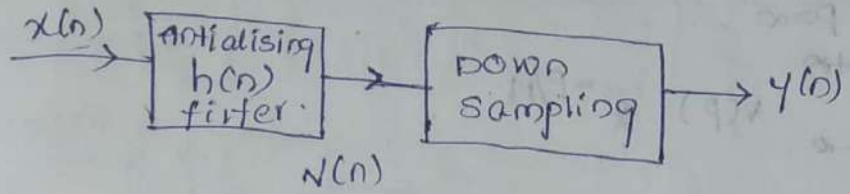
$y(6) = x(6/2) = 3$

$y(8) = x(8/2) = 4$

$y(n) = \{0, 0, 1, 0, 2, 0, 3, 0, 4 \ldots\}$

22/12/19 Decimation by a factor "M"

The decimation process is shown in figure.



⇒ The i/p sequence "$x(n)$" is passed through a low pass filter is characterized by the impulse Response $h(n)$.

⇒ The i/p sequence "$x(n)$" decimated by a factor "M" ie;

" $T'/T = M$ " $\Rightarrow T' = TM$.

⇒ The new sampling rate at the o/p sequence "$y(n)$":

$$F' = \frac{1}{T'} = \frac{1}{TM} \Rightarrow \boxed{F' = F/M} \quad [F = 1/T]$$

⇒ The o/p of the filter is a sequence "$v(n)$" is given as $v(n) = x(n) * h(n)$

$$= \sum_{k=-\infty}^{+\infty} x(k)\, h(n-k)$$

which is down sampled by the factor "M" to produce "$y(n)$" ie; $y(n) = v(nM)$

⇒ The sequence "$v(n)$" can be obtain with a periodic train of impulses "$h(n)$" with period "M" then the discrete fourier series can be represented as.

$$h(n) = \frac{1}{M} \sum_{k=0}^{M-1} e^{j2\pi kn/M}$$

Now, let us take the z-transform of the o/p sequence $y(n)$ is given by

$$Y(z) = \sum_{n=-\infty}^{+\infty} y(n) \cdot z^{-n}$$

$$= \sum_{n=-\infty}^{+\infty} v(nM) \cdot z^{-n}$$

let us take $nM = p$

$n = p/M$

$n = -\infty,\ p = -\infty$

$n = \infty,\ p = \infty.$

$$Y(z) = \sum_{p=-\infty}^{+\infty} V(p)\, z^{-p/M}.$$

let $p \to n$

$$= \sum_{n=-\infty}^{\infty} V(n) \cdot z^{-n/M}.$$

$$= \sum_{n=-\infty}^{+\infty} x(n)\, h(n)\, z^{-n/M}.$$

$$= \sum_{n=-\infty}^{+\infty} x(n) \tfrac{1}{M} \sum_{k=0}^{M-1} e^{j2\pi kn/M} \cdot z^{-n/M}$$

$$= \frac{1}{M} \sum_{k=0}^{M-1} \sum_{n=-\infty}^{\infty} x(n)\, e^{j2\pi kn/M} \cdot z^{-n/M}$$

$$= \frac{1}{M} \sum_{k=0}^{M-1} \sum_{n=-\infty}^{n=+\infty} x(n) \left[e^{-j2\pi k/M} \cdot z^{1/M}\right]^{-n}$$

let $z = e^{j\omega}$.

$$Y(z) = \frac{1}{M} \sum_{k=0}^{M-1} X\left(e^{-j2\pi k/M} \cdot z^{1/M}\right)$$

$$Y(e^{j\omega}) = \frac{1}{M} \sum_{k=0}^{M-1} X\left(e^{-j2\pi k/M}\, e^{j\omega/M}\right)$$

$$Y(e^{j\omega}) = \frac{1}{M} \sum_{k=0}^{M-1} X\left(e^{j(\omega - 2\pi k)/M}\right)$$

from this relation we can find that the fourier transform of the i/p signal of a down sampler is "$X(e^{j\omega})$". then the o/p sequence of the fourier transform "$Y(e^{j\omega})$". is a sum of "M" uniformly shifted version of $X(e^{j\omega})$ scaled by a factor "$1/M$".

23/2/19 **Interpolation by a factor "L"**

The sampling rate expansion of the signal is known as interpolation process it is also called as upsampling

$$x(n) \rightarrow \boxed{\uparrow L} \rightarrow y(n) = x(n/L)$$

→ let $x(n)$ be the i/p signal is interpolating by a factor "L" then $T'/T = 1/L \Rightarrow T' = T/L$

→ The new sampling frequency rate $F' = \frac{1}{T'} \Rightarrow \frac{1}{T/L}$

$= L/T \Rightarrow LF = F'$

→ The o/p signal $y(n)$ is obtain by applying z-transform

$$Y(z) = \sum_{n=-\infty}^{\infty} y(n) \cdot z^{-n}$$

$$= \sum_{n=-\infty}^{+\infty} x(n/L) \cdot z^{-n} \quad [\because \text{let } n/L = p,\ n = Lp]$$

$$= \sum_{p=-\infty}^{+\infty} x(p)\, z^{-Lp}$$

Replace $p \rightarrow n$

$$= \sum_{n=-\infty}^{+\infty} x(n)\, z^{-Ln}$$

$$= \sum_{n=-\infty}^{+\infty} x(n)\, (z^{L})^{-n}$$

$$Y(z) = X(z^{L})$$

Substitute $z = e^{j\omega}$

$$Y(e^{j\omega}) = X(e^{j\omega L})$$

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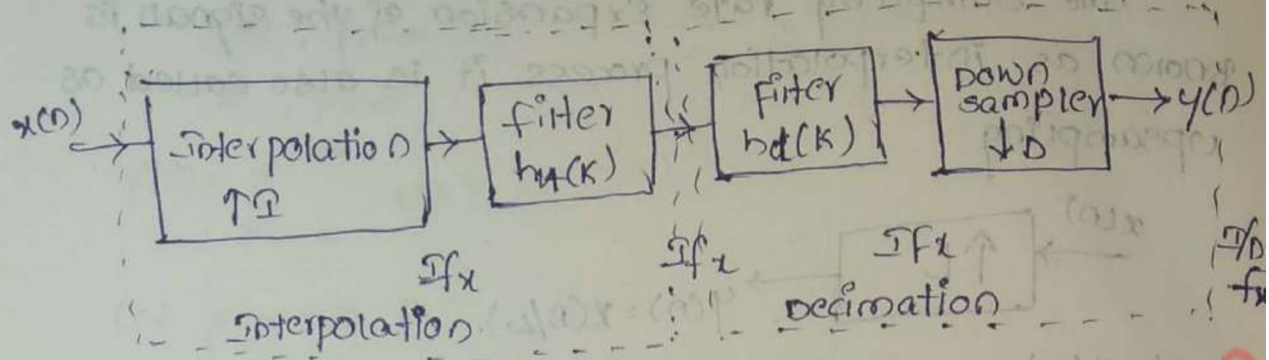
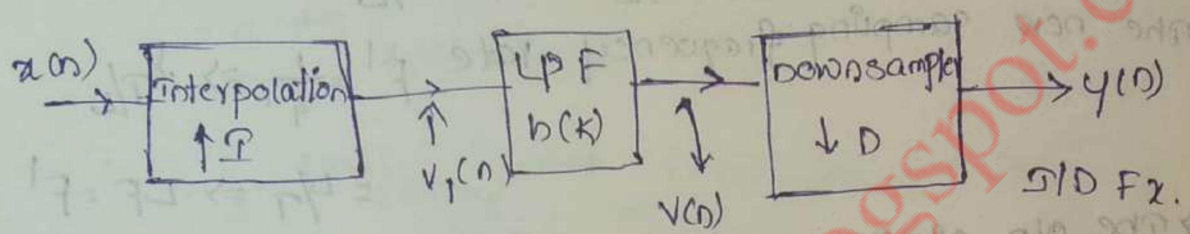
Sampling Rate Conversion by a factor (I/D):





⇒ The sampling rate conversion can be achieved by the

⇒ first performing by the interpolation factor "I" by dessimation.

by the factor "D" in otherwords the sampling rate conversion by the rational factor "I/D" is completed by cascading by interpolator with a dessimator as shown in fig.

we can notice the importance of performing the interpolation first in the dessimation second to preserve the desired spectral characteristics of x(n) from the fig the two filters with impulse response $h_u(k)$ and $h_D(k)$ the operated at the same rate ie; I Fx is two filters are combined into a single lowpass filter with impulse response h(k) as shown in fig.

The frequency response of the upsampler filter can be defined as

$$H_u(\omega_y) = \begin{cases} I, & |\omega_y| \le \pi/I \\ 0, & \text{otherwise.} \end{cases}$$

The freq response of the downsampling filter (anti aliasing filter) can be defined as.

$$H_d(w_y) = \begin{cases} 1, & |w_y| \le \pi/D \\ 0, & \text{otherwise.} \end{cases}$$

The freq response of $H(w_y)$ the combined filter must incorporate the filtering operations for both interpolation and dessimation. and Hence we should ideally passes the freq response is

$$H(w_y) = \begin{cases} I, & 0 \le |w_y| \le \min(\pi/I, \pi/D) \\ 0, & \text{otherwise.} \end{cases}$$

→ π/I is minimum the low pass filter can acts as a anti imaging filter

→ π/D is minimum the low pass filter can acts as a anti aliasing filter

In the time domain : In the time Domain.

the o/p of the op sampler is given by

$$v_1(n) = x(n/I)$$

~~and~~ the o/p of low pass filter

$$v_2(n) = v_1(n) * h(k)$$

$$= \sum_{k=-\infty}^{+\infty} v_1(k)\, h(n-k)$$

$$v_1(n) = x(n/I)$$

Let $k = n/I$, $\therefore \boxed{n = kI}$

$$= \sum_{k=-\infty}^{+\infty} x(k)\, h(kI-k)$$

After the o/p of the Down sampler i.e; "$y(n)$"=

$$y(n) = \sum_{k=-\infty}^{+\infty} x(k)\, h(kI-Dk)$$

Frequency Domain:-

In the freq Domain the freq response of the o/p signal $x(n)$ is given by $Y(e^{j\omega}) = \frac{1}{D} \sum_{k=0}^{D-1} V(e^{\frac{j\omega - 2\pi k}{D}})$

where $V(e^{j\omega}) = H(e^{j\omega}) \cdot V_1(e^{j\omega})$

Here $V_1(e^{j\omega})$ = the frequency response of the interpolation ie; $X(e^{j\omega I})$

$$\therefore Y(e^{j\omega}) = \frac{1}{D} \sum_{k=0}^{D-1} H(e^{\frac{j\omega - 2\pi k}{D}}) \times X(e^{\frac{j\omega I - 2\pi I}{D}})$$

⇒ The freq response of the lowpass filter, ie; $H(e^{j\omega})$ by using this filter we can avoid the filter components

∴ The o/p sequence finally $\boxed{Y(e^{j\omega}) = \frac{I}{D} X(e^{j\omega I/D})}$

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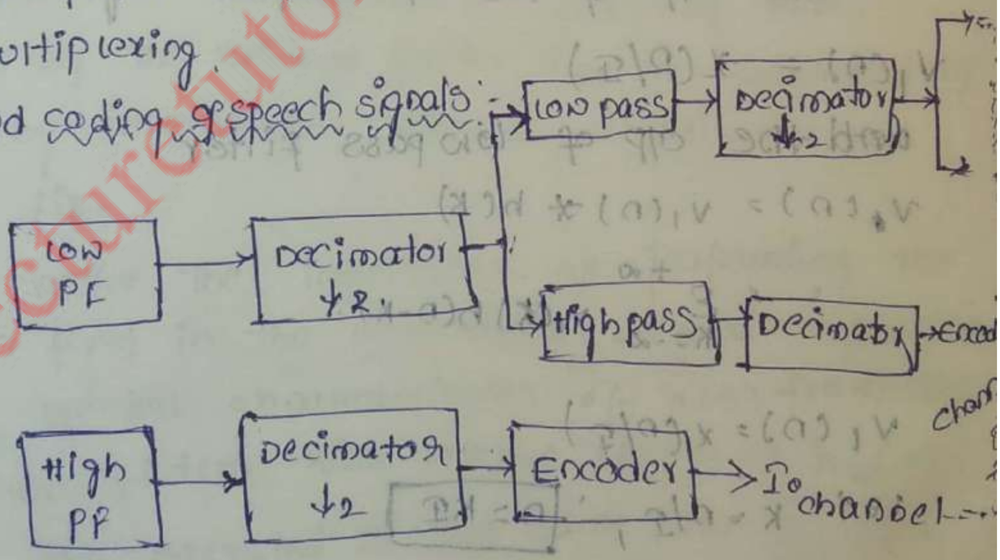
Applications of MRDSP:-

1) Sub band coding of speech signals.
2) Digital filter Banks.
3) Transmultiplexing

1) Sub band coding of speech signals:-



2) Digital Filter Banks:-

A variety of techniques have been develope to efficiently represent speech signals in Dital form for either transmission (or) storage.

Since most of the speech signal contain low frequencies

we can encode the low frequency band with more bits then the high frequency band sub-band coding is a method where the speech signal is subdivided into several frequency band and each band is digitally encoded saperately.
An example of frequency sub-division is shown in fig
let us assume that the speech signal is sampled at a rate fs samples/sec.
The First Frequency sub-division splits the signal spectrum into two equal segments ie; lowpass signal and highpass signal.
The 2nd frequency sub-division splits the low-pass signal from the 1st stage into two equal bands ie; low-pass signal and high pass signal.
Finally the 3rd Frequency sub-divisions splits the low pass signal from the 2nd stage into two equal band widths signals.

2) Digital Filter Banks:-

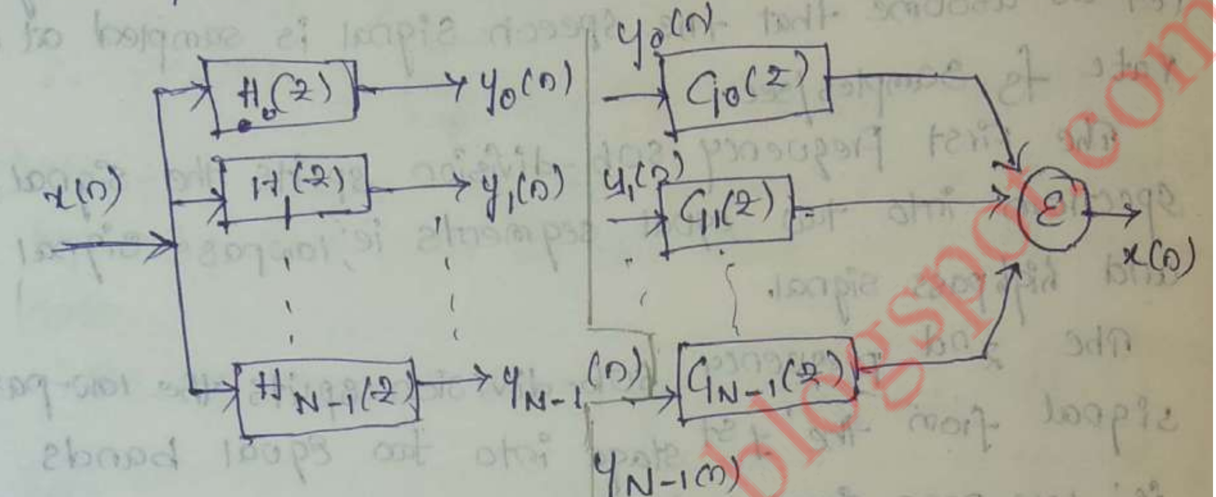
These are used for performing spectrum analysis and signal synthesis.
Filter bands are generally categorised into 2 types
1) Analysis Filter Banks
2) synthesis Filter Banks.

1) Analysis Filter Banks:-

It consists of set of filters with system functions $H_K(z)$. Arranged in a parallel bank as shown in fig.
The Frequency Response characteristics of this filter bank split the signal into a corresponding no. of sub-banks.
And the other band a synthesis filter band

consists of set of filters with system functions "$G_K(z)$".

Arrange as shown in fig. with corresponding i/p's "$y_K(n)$". The o/p's of the filters are summed to form the synthesized signal.



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3) Transmultiplexing :

The application of Digital Filter bands is in the Designed & implementation of Digital Transmultiplexers which are devices for converting b/w TDM signals and FDM signals.

In a transmultiplexer for TDM to FDM conversion the i/p signal x(n) is a time DM signal consisting of "L" signals which are saperated by a switch each of these 'L' signals is modulated and a different carrier frequency to obtain an FDM signal for transmission.

In a transmultiplexer for FDM / TDM conversion the composed signal is saperated by filtering into the "L" signal components which are TDM signals.

→ FDM signal

DM signal