UNIT 4 STRESS DISTRIBUTION IN SOILS

PART-A

SHORT QUESTIONS WITH SOLUTIONS

Q1. What is an isobar? What is a pressure bulb?

(Nov.-15, Set-2, Q5(b) | Nov.-15, Set-3, Q5(b) | Nov.-15, Set-4, Q5(b))

OR

What is an Isobar?

Oct./Nov.-16, Set-1, Q1(d)

OR

Define stress isobar.

Oct./Nov.-17, Set-1, Q1(d)

Answer :

A stress line below the ground surface, which connects all the points at which the vertical pressure is equal is called as an Stress Isobar. It is a curved surface, which resembles like a bulb, due to the vertical pressure at all the points below the ground surface in the horizontal plane. Hence it is also known as pressure bulb.

Q2. What are differences between Boussinesq's and Westergaard theories?

Answer :

(Model Paper-II, Q1(d) | Nov.-15, Set-1, Q1(d))

	Boussinesq's Theory		Westergaard's Theory
1.	In Boussinesq's theory, the self weight of the soil is neglected and soil is assumed to be isotropic, homogeneous and semi-finite medium.	1.	In Westergaard's theory, the soil is assumed to be isotropic and homogeneous and self weight of the soil is not neglected.
2.	It is assumed that initially the soil is unstressed.	2.	It is assumed that the soil is assumed to be placed or reinforced horizontally.
3.	Stresses are symmetrically distributed with respect to z-axis.	3.	It is assumed that deformation of the material is prevented by membranes.
4.	Difference in volume due to the application of loads on soil is ignored.	4.	The volume differences due to loads are taken into account.
5.	It is assumed that stress continuity exists in the medium.	5.	Horizontal sheets of negligible thickness are used to prevent the mass being subjected to lateral movements of soil.

Q3. List the assumptions of Boussinesq's theory.

(Model Paper-IV, Q1(d) | Oct./Nov.-16, Set-2, Q1(d))

Answer :

The assumptions made in Boussinesq theory are as follows,

(i) The self weight of the soil is neglected.

(ii) It is assumed that soil is homogeneous, elastic isotropic and semi-infinite medium.

(iii) The soil medium obey's Hooke's law.

(iv) Difference in volume due to the application of loads on the soil is ignored.

(v) It is assumed that stress continuity exists in the medium.

(vi) It is assumed that initially the soil is unstressed.

(vii) Stresses are symmetrically distributed with respect to Z-axis.

Q4. What are the assumptions of westergaard's theory?

Model Paper-III, Q1(d)

Answer :

The assumptions made in Westergaard's theory are as follows,

(i) It is assumed that the soil is isotropic and homogeneous.

(ii) It is assumed that the soil consists of weightless and linearly elastic material.

(iii) The materials of soil is assumed to be placed or reinforced horizontally with membranes of low thickness spaced closely to each other.

(iv) It is assumed that deformation of the material is prevented by the membranes.

Q5. Discuss about importance of pressure bulb.

Model Paper-I, Q1(d)

Answer :

Importance of pressure bulb in soil engineering,

(a) These are used to determine the effect of load on vertical stresses at different points.

(b) For a given system of load, a number of isobars can be drawn of various intensities of vertical pressure. But only one isobar among them is responsible for the settlement of structure in the soil.

Q6. What is the increase in vertical stress at a point 5 m below a point load of 100 kN, using Boussinesq's theory?

Oct./Nov.-16, Set-4, Q1(d)

Answer :

Given that,

Distance (vertical) of point below that load, $z = 5$ m

Horizontal distance of point from the load, $r = 0$

Intensity of load, $P = 100$ kN

$$\text{Vertical pressure, } \sigma_z = \frac{3P}{2\pi z^2}\left[\frac{1}{1+\left(\frac{r}{z}\right)^2}\right]^{5/2} \frac{3\times100}{2\times\pi\times5^2}\left[\frac{1}{1+\left(\frac{0}{5}\right)^2}\right]^{5/2} = \frac{3\times100}{2\times\pi\times25}$$

$$= \frac{3\times4}{2\pi}$$

$$\therefore \quad \sigma_z = 1.910 \text{ kN/m}^2$$

Q7. What is the use of New mark's influence chart?

(Nov.-15, Set-2, Q1(d) | Nov.-15, Set-3, Q1(d))

Answer :

For answer refer Unit-IV, Q33.

Q8. What is 2 : 1 stress distribution method?

Answer : Nov-15, Set-4, Q1(d)

For answer refer Unit-IV, Q34.

Q9. When is New mark's influence chart applicable? What are the differences between Boussinesq's and Westergaard's theories.

Answer : Oct./Nov.-16, Set-3, Q1(d)

For answer refer Unit-IV, Q33, First Line and Q2.

Q10. Determine the vertical pressure distribution on horizontal plane.

Answer : Oct./Nov.-17, Set-2, Q1(d)

For answer refer Unit-IV, Q30.

Q11. Determine the vertical pressure distribution along a vertical line.

Answer : Oct./Nov.-17, Set-4, Q1(d)

For answer refer Unit-IV, Q31.

PART-B
ESSAY QUESTIONS WITH SOLUTIONS

4.1 STRESSES INDUCED BY APPLIED LOADS – BOUSSINESQ'S AND WESTERGAARDS THEORIES FOR POINT LOADS AND AREA OF DIFFERENT SHAPES

Q12. Write a short note on geostatic stresses.

Answer :

Geostatic Stresses

The stresses which are induced in sedimentary deposits where the surface of ground is horizontal and the nature of soil does not vary along the horizontal direction are termed as geostatic stresses. The stresses in a soil mass are generally caused due to the applied external loads and self weight of the soil. The stress patterns generated due to the both external load and self weight are very complex but in sedimentary deposits a simple pattern of stress is induced by the self weight.

The soil deposit in which geostatic stresses are induced do not have shear stresses in both horizontal and vertical planes. The vertical geostatic stress at any depth is evaluated by considering the weight of soil above that depth and it is given by the equation,

$$\boxed{\sigma_V = \gamma Z} \quad \ldots (1)$$

Where,

σ_V - vertical geostatic stress

γ - weight per unit volume of soil

Z - depth under consideration.

The unit weight of soil should be constant throughout the depth. A linear variation of geostatic stress is observed with respect to the depth of soil layer. The following are the two cases of calculating geostatic stress depending upon the conditions along the depth of soil layer.

Case (i)

If the weight of soil changes along the depth,

$$\sigma_V = \int_o^z \gamma \, dZ$$

Case (ii)

If the soil is available in different strata and the unit weights of each strata is different,

$$\sigma_V = \gamma_1 Z_1 + \gamma_2 Z_2 + \gamma_3 Z_3 + \ldots\ldots$$

$$= \Sigma \gamma \, \Delta Z$$

Q13. Using Boussinesq's expression, derive the expression for vertical stress at depth 'z' under the center of circular area of radius 'r' loaded uniformly with a load 'q' at the surface of the mass of soil.

Model Paper-I, Q5(a)

OR

Derive an equation for determining the stress intensity at a given on the axis of loading due to the uniformly loaded circular area.

Nov.-15, Set-2, Q5(a)

OR

Derive an expression of vertical stress for normal load aver a circular area.

Oct./Nov.-17, Set-1, Q5(b)

nswer :

The vertical stresses under a circular area is shown below,

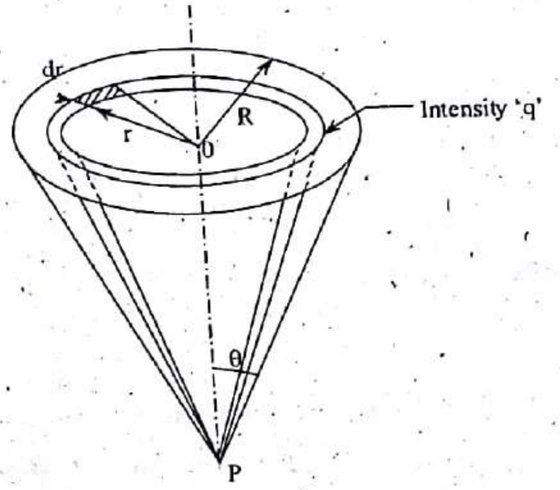


Figure: Uniformly Distributed Circular Load

Consider a circular area (dA) of radius 'r' loaded uniformly with a load 'q' at the surface mass of the soil as shown in figure. If the area of cross-section (dA) is divided into small elemental areas, then the point load on each small area is given by 'qdA'. Therefore, the vertical stress 'P' at a depth 'z' on vertical axis of the cross-section is given by,

$$\Delta\sigma_z = \frac{3qdA}{2\pi}\frac{z^3}{\left[r^2+z^2\right]^{5/2}}$$

Integrating the above equation to get the vertical stress due to entire load,

$$\int\Delta\sigma_z = \frac{3q}{2\pi}\int_0^R \frac{z^3\,dA}{\left[r^2+z^2\right]^{5/2}}$$

$$\sigma_z = \frac{3q}{2\pi}\int_0^R \frac{z^3(2\pi r dr)}{\left[r^2+z^2\right]^{5/2}} = 3qz^3\int_0^R \frac{r.dr}{\left[r^2+z^2\right]^{5/2}}$$

Substitute $r^2 + z^2 = m^2$, Therefore, $rdr = mdm$.

Applying limits from $r = 0$ to $m = z$ and

$r = R$ to $m = (R^2 + z^2)^{1/2}$

$$\sigma_z = 3qz^3\int_z^{(R^2+Z^2)^{1/2}} \frac{mdm}{\left[m^2\right]^{5/2}} \quad [\therefore rdr = mdm]$$

$$= 3qz^3\int_z^{(R^2+Z^2)^{1/2}} \frac{mdm}{m^5}$$

$$= 3qz^3\int_z^{(R^2+Z^2)^{1/2}} \frac{dm}{m^4}$$

$$= 3qz^3\left[\frac{M^{-4+1}}{-4+1}\right]^{(R^2+z^2)^{1/2}}$$

$$= \frac{-3qz^3}{3}\left[\frac{1}{m^3}\right]_z^{(R^2+z^2)^{1/2}}$$

$$= -qz^3\left[\frac{1}{(R^2+z^2)^{3\times 1/2}} - \frac{1}{z^3}\right] = -qz^3 \times \frac{1}{z^3}\left[\left[\frac{1}{\left[\frac{R^2}{z^2}+1\right]}\right]^{3/2} - 1\right] = q\left[1 - \frac{1}{\left[\frac{R^2}{z^2}+1\right]}\right]^{3/2}$$

Hence, $\sigma_z = qI_f$

Where; I_f – Boussinesq influence factor for uniformly distributed circular load $= \left[1 - \left[\frac{1}{\left[\frac{R^2}{z^2}+1\right]}\right]^{3/2}\right]$

The influence factor I_f can be written in terms of θ suspended at the point 'p' with the external point load.

Q14. Derive an expression for uniform load on rectangular area based on Boussinesq's Theory.

Answer :

Model Paper-III, Q5(a)

A rectangular shaped foundation is shown in figure. Consider a small elemental area of rectangular section of size $db \times dl$. The uniformly distributed load acting over the elemental area can be assumed as concentrated load acting at the centre.

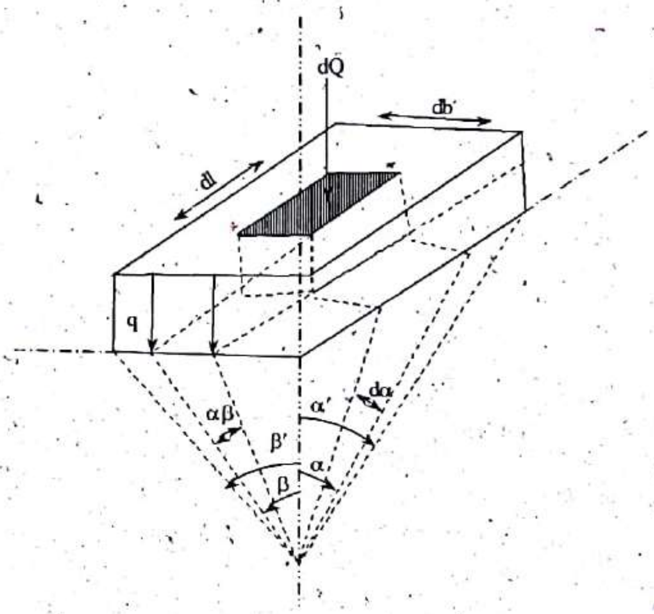


Figure: Vertical Stress at the Corner of a Uniformly Loaded Rectangular Area

Where,

$$Q = qA \quad \text{... (1)}$$

$$Q = q \times lb$$

$$dQ = qdl \times db \quad \text{... (2)}$$

Due to the load (dQ) on the small elemental area, vertical stress σ_z is given by,

$$d\sigma_z = \frac{3dQ \times z^3}{2\pi[z^2+r^2]^{5/2}} \quad \text{... (3)}$$

The dimensions of the elemental section (dl, db, r) are expressed in terms of angles α,β. Equation 3 is then integrated between the limits α,β to obtain the total stress over the entire area.

$$\int_{\alpha=0}^{\alpha=\alpha'}\int_{\beta=0}^{\beta=\beta'} d\sigma_z = \int_{\alpha=0}^{\alpha=\alpha'}\int_{\beta=0}^{\beta=\beta'} \frac{1}{2\pi}\left[\frac{3z^3}{(z^2+r^2)^{5/2}}\right].dQ$$

The solution of the above equation is given as,

$$\sigma_z = \frac{q}{4\pi}\left[\left[\frac{2mn\sqrt{m^2+n^2+1}}{m^2+n^2+m^2n^2+1}\right]\times\left[\frac{m^2+n^2+2}{m^2+n^2+1}\right]+\tan^{-1}\left[\frac{2mn\sqrt{m^2+n^2+1}}{m^2+n^2-m^2n^2+1}\right]\right]$$

Where, $m = \frac{b}{z}, n = \frac{l}{z}$

$$\therefore \quad \sigma_z = I_q \quad \text{... (4)}$$

Where, I – dimensionless factor.

The total vertical stress below the uniformly loaded rectangular area at a depth of 'z' from the surface is given by,

$$\sigma_z = q[I_1 + I_2 + I_3 + I_4] \quad \text{... (5)}$$

The total vertical stress outside the uniformly distributed rectangular area at a depth 'z' is given by,

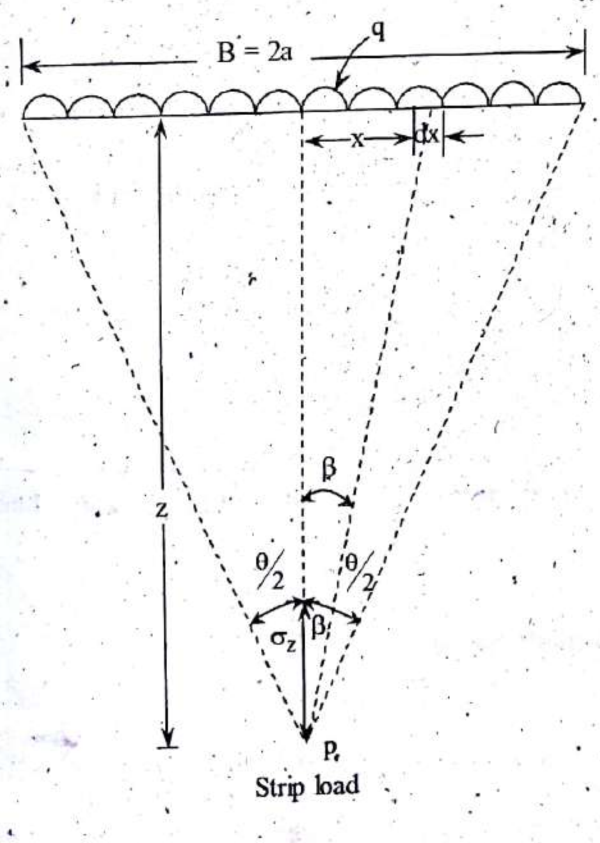
$$\sigma_z = q[I_1 - I_2 - I_3 + I_4]$$

Where, I_1, I_2, I_3, I_4 are the influence values for smaller rectangular areas cutout from the large rectangular area.

Q15. Explain how the vertical pressure determined due to strip load?

Answer : Oct./Nov.-17, Set-1, Q5(a)

Expression for Vertical Pressure Under Strip Load



Figure

Let 'B' be the width of the infinite strip which is uniformly loaded on the strip with an intensity of 'q' per unit area.

Let us find out the vertical pressure at a point 'p' which is at a depth 'z' on the vertical axis passing through the centre of the strip.

Consider width of strip load as 'dx' at a distance 'x' from the centre.

The load intensity of the elementary line along the elementary strip of width 'dx' is given as $qxdx$.

Due to the load of elementary line, the vertical pressure at point 'p' is given by,

$$\Delta\sigma_z = \frac{2(q.dx)}{\pi z}\times\frac{1}{\left[1+\left(\frac{x}{z}\right)^2\right]^2}$$

The total vertical pressure due to the entire strip load is given by,

$$\sigma_z = \frac{2q}{\pi z}\int_{-B/2}^{+B/2}\frac{dx}{\left[1+\left(\frac{x}{z}\right)^2\right]^2} \quad \text{... (1)}$$

Let,

$$\frac{x}{z} = \tan\beta$$

$$\therefore \quad dx = z.\sec^2\beta.d\beta$$

Substitute this in equation (1),

$$\therefore \quad \sigma_z = \frac{2q}{z\pi}\times 2\int_0^{\theta/2}\frac{z\times\sec^2\beta}{(1+\tan^2\beta)^2}.d\beta$$

$$\sigma_z = \frac{4q}{\pi}\int_0^{\theta/2}\frac{\sec^2\beta}{(1+\tan^2\beta)^2}.d\beta$$

$$\sigma_z = \frac{4q}{\pi}\int_0^{\theta/2}\cos^2\beta.d\beta$$

$$\sigma_z = \frac{4q}{\pi}\int_0^{\theta/2}\left(\frac{1+\cos\beta}{2}\right)d\beta$$

$$\sigma_z = \frac{4q}{2\pi}\int_0^{\theta/2}(1.d\beta+\cos\beta.d\beta)$$

$$\sigma_z = \frac{4q}{2\pi}[(\beta) + (\sin\beta)]_0^{\theta/2}$$

$$\sigma_z = \frac{4q}{2\pi}\left[\frac{\theta}{2} + \sin\frac{\theta}{2}\right]$$

$$\sigma_z = \frac{4q}{2\pi}\left[\frac{\theta + \sin\theta}{2}\right]$$

$$\sigma_z = \frac{4q}{4\pi}(\theta + \sin\theta)$$

$$\sigma_z = \frac{q}{\pi}(\theta + \sin\theta)$$

This is the vertical pressure under strip load.

Q16. Derive an expression for uniform load on rectangular area based on Westergaard's theory.

Answer :

Westergard made the assumptions that the soil mass consists of horizontal sheets of negligible thickness with infinite strain rigidity which does not allow the occurence of lateral strain and vertical stresses below the corner of cross-sectional area of uniformly loaded rectangular section.

The vertical stress σ_z obtained by Westergaard is given by,

$$\sigma_z = \frac{q}{2\pi}\left[\cot^{-1}\left[\left[\frac{1-2\nu}{2-2\nu}\right]\left[\frac{1}{m^2}+\frac{1}{n^2}\right]+\left[\frac{1-2\nu}{2-2\nu}\right]\times\frac{1}{m^2n^2}\right]^{1/2}\right]$$

If $\gamma = 0$, ($V \rightarrow$ poisson's ratio), we get.

$$\sigma_z = \frac{q}{2\pi}\left[\cot^{-1}\left(\frac{1}{2}\times\left(\frac{1}{m^2}+\frac{1}{n^2}\right)+\left(\frac{1}{2}\right)^2\frac{1}{m^2n^2}\right)^{1/2}\right]$$

$$\sigma_z = \frac{q}{2\pi}\left[\cot^{-1}\left[\frac{1}{2}\times\left(\frac{1}{m^2}+\frac{1}{n^2}\right)+\frac{1}{4}\times\frac{1}{m^2n^2}\right]^{1/2}\right]$$

$$\therefore \quad \sigma_z = q.I_{\sigma w}$$

$I_{\sigma w}$ = Influence coefficient for vertical stress at the end of the uniformly loaded rectangular area from westergaard's theory.

Q17. A concentrated load of 22.5 kN acts on the surface of a homogeneous soil mass of large extent. Find the stress intensity at a depth of 15 meters,

(i) Directly under the load and

(ii) At a horizontal distance of 7.5 meters. Use Boussinesq's equations.

Answer :

Given that,

Concentrated load, $Q = 22.5$ kN

Depth, $Z = 15$ m

From the Boussinesq's theory,

$$\sigma_z = \frac{Q}{Z^2} \times \frac{\left(\frac{3}{2\pi}\right)}{\left[1+\left(\frac{r}{z}\right)^2\right]^{\frac{5}{2}}} \quad \ldots (1)$$

Case (i)

Directly under the load,

$r = 0$

$\frac{r}{Z} = 0$

$Z = 15$ m, $Q = 22.5$ kN

By substituting the above values in equation (1) we get;

$$= \frac{22.5}{15^2} \times \frac{\frac{3}{2\pi}}{\left[1+\left(\frac{0}{15}\right)^2\right]^{\frac{5}{2}}}$$

$$= \frac{22.5}{15^2} \times \frac{3}{2\pi(1+0)^{\frac{5}{2}}}$$

$= 0.1 \times 0.4775$

$= 0.004775$ kN/m^2

$\therefore Z = 47.75$ N/m^2

Case (ii)

At a horizontal distance of 7.5 meters.

$r = 7.5$

$z = 15$ m

$$\sigma_z = \frac{Q}{z^2} \times \frac{\left(\frac{3}{2\pi}\right)}{\left[1+\left(\frac{r}{z}\right)^2\right]^{\frac{5}{2}}}$$

$$= \frac{22.5}{15^2} \times \frac{\left(\frac{3}{2\pi}\right)}{\left(1+\left[\frac{7.5}{15}\right]^2\right)^{\frac{5}{2}}}$$

$$= 0.1 \times \frac{0.4775}{(1+0.25)^{\frac{5}{2}}}$$

$= 0.1 \times 0.2733$

$= 0.02733$

$= 27.33 \times 10^{-3}$ kN/m^2

$\boxed{\therefore \sigma_z = 27.33 \text{ N/m}^2}$

Q18. Two point loads P and Q act on the ground surface 8m apart. The magnitude of P is 100 kN and that of Q is 80 kN. Point A is at a depth of 6 m directly below P and point B is at a depth of 5 m directly below Q. Point C is between P and Q and it is at a distance of 4m from P. Point C lies at a depth of 3 m below the ground surface. Calculate the increase in vertical stresses at A, B and C due to the point loads.

Answer :

Given that,

Spacing between loads = 8 m

Load acting at P = 100 kN

Load acting at Q = 80 kN

Increase in vertical stress,

$$\sigma_x = \frac{3P}{2\pi z^2}\left[\frac{1}{1+(r_1/z)^2}\right]^{5/2} + \frac{3Q}{2\pi z^2}\left[\frac{1}{1+(r_2/z)^2}\right]^{5/2}$$

(i) Increase in vertical stress at a depth of 6 m below 100 kN load

Given that,

P = 100 kN, Q = 80 kN, $r_1 = 0$, r_2 = 8 m and Z = 6 m

$$\sigma_z = \frac{3\times 100}{2\pi\times 6^2}\left[\frac{1}{1+(0/6)^2}\right]^{5/2} + \frac{3\times 80}{2\pi\times 6^2}\left[\frac{1}{1+(8/6)^2}\right]^{5/2} = 1.409 \text{ kN/m}^2$$

(ii) Increase in vertical stress at a depth of 6 m below 80 kN load

Given that,

P = 100 kN, Q = 80 kN, $r_1 = 0$, r_2 = 0 m and Z = 5 m

$$\sigma_z = \frac{3\times 100}{2\pi\times 5^2}\left[\frac{1}{1+(8/5)^2}\right]^{5/2} + \frac{3\times 80}{2\pi\times 5^2}\left[\frac{1}{1+(0/5)^2}\right]^{5/2} = 1.608 \text{ kN/m}^2$$

(iii) Increase in vertical stress at a depth of 3 m below the midpoint of the loads

Given that,

P = 100 kN, Q = 80 kN, $r_1 = 4$, r_2 = 4 m and Z = 3 m

$$\sigma_z = \frac{3\times 100}{2\pi\times 3^2}\left[\frac{1}{1+(4/3)^2}\right]^{5/2} + \frac{3\times 80}{2\pi\times 3^2}\left[\frac{1}{1+(4/3)^2}\right]^{5/2} = 0.743 \text{ kN/m}^2$$

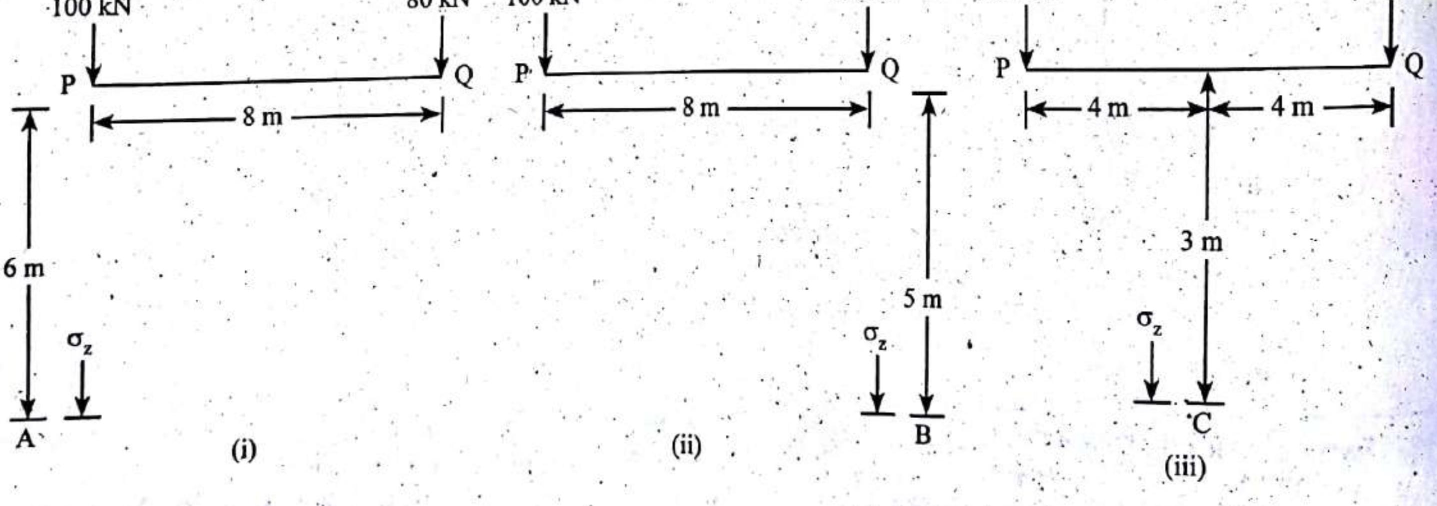


Figure: Vertical Stresses Due to Concentrated Loads

Q19. A three-legged tower forms an equilateral triangle of side 4m in plan. If the total weight of the tower is 450 kN and is equally carried by all the legs, compute the vertical stress increase caused in the soil by the tower at a depth of 4m directly below one of the legs and also at the same depth below the centroid of the triangle.

Answer : **Oct./Nov.-16, Set-4, Q5(b)**

Given that,

Depth of vertical stress = 4 m

Total weight of the tower = 450 kN

Load on each leg, $Q = 450/3 = 150$ kN

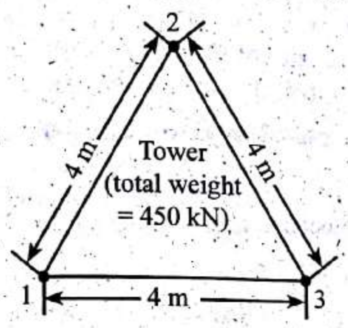


Figure: Plan of a Three-legged Tower

Increase in vertical stress below one of the legs = σ_z

$$\sigma_z = \frac{3Q}{2\pi z^2} + 2\left[\frac{3Q}{2\pi z^2}\left\{\frac{1}{1+(r/z)^2}\right\}^{5/2}\right]$$

$$= \frac{3\times150}{2\pi\times4^2} + 2\left[\frac{3\times150}{2\pi\times4^2}\left\{\frac{1}{1+(4/4)^2}\right\}^{5/2}\right]$$

$$= 6.06 \text{ kN/m}^2$$

Increase in vertical stress below centroid of triangle,

$$r = \frac{2}{3} \text{ (length of median of triangle)}$$

$$= \frac{2}{3}\left(\frac{4\sqrt{3}}{2}\right)$$

$$= 2.309 \text{ m}$$

$$\sigma_z = \frac{3Q}{2\pi z^2} + 2\left[\frac{3Q}{2\pi z^2}\left\{\frac{1}{1+(r/z)^2}\right\}^{5/2}\right]$$

$$= \frac{3\times150}{2\pi\times4^2} + 2\left[\frac{3\times150}{2\pi\times4^2}\left\{\frac{1}{1+\left(\frac{2.309}{4}\right)^2}\right\}^{5/2}\right]$$

$$= 8.840 \text{ kN/m}^2$$

Q20. A concentrated load of 300 kN acts on the surface of a homogeneous soil mass of large extent. Find the stress intensity at a depth of 2 m,

(i) Directly under the load

(ii) At a horizontal distance of 3.0 m away from the point of application of load. Use Boussinesqs equation.

April/May-13, Set-2, Q5

Answer :

Load, $Q = 300$ kN

Depth, $Z = 2$ m

From the Boussinesq's equation theory,

$$\boxed{\sigma_z = \frac{Q}{z^2}\times\frac{\left(\frac{3}{2\pi}\right)}{\left[1+\left(\frac{r}{z}\right)^2\right]^{\frac{5}{2}}}} \quad \text{... (1)}$$

(i) Directly Under the Load

Directly under the load, $r = 0$

$$\frac{r}{z} = 0$$

$Z = 2$ m, $Q = 300$ kN

By substituting the above values in equation (1), we get,

$$= \frac{300}{2^2}\times\frac{\left(\frac{3}{2\pi}\right)}{\left[\left(1+\frac{0}{15}\right)^2\right]^{\frac{5}{2}}}$$

$$= \frac{300}{2^2}\times\frac{3}{2\pi(1+0)^{\frac{5}{2}}} = 35.810 \text{ N/m}^2$$

$\therefore$ $Z = 35.810$ N/m^2.

(ii) At a Horizontal Distance

At a horizontal distance 3 mts

$r = 3$

$Z = 2$ m

$$\sigma_z = \frac{Q}{z^2}\times\frac{\left(\frac{3}{2\pi}\right)}{\left[1+\left(\frac{r}{z}\right)^2\right]^{\frac{5}{2}}}$$

$$= \frac{300}{2^2}\times\frac{\left(\frac{3}{2\pi}\right)}{\left[1+\left(\frac{3}{2}\right)^2\right]^{\frac{5}{2}}}$$

$$= 75\times\frac{0.48}{(1+2.25)^{\frac{5}{2}}}$$

$$= 1.9$$

$$= 19\times10^{-3} \text{ kN/m}^2$$

$\therefore$ $\sigma_z = 19$ N/m^2

Q21. A circular area on the surface of an elastic mass of great extent carries a uniformly distributed load of 120 kN/m². The radius of the circle is 3 m. Compute the intensity of vertical pressure at point 5 metres beneath the centre of the circle using boussinesq's method.

Answer :

Given that,

Radius of circle, $r = 3$ m

Uniformly distributed load $= q = 120$ kN/m²

$Z = 5$ m

According to Boussinesq's equation,

$$\sigma_z = q\left[1-\frac{1}{\left(1+\left[\frac{r}{z}\right]^2\right)^{3/2}}\right] \quad \ldots (1)$$

By substituting the values of r, z and q in equation (1), we get,

$$\sigma_z = 120\left[1-\frac{1}{\left[1+\left(\frac{3}{5}\right)^2\right]^{3/2}}\right]$$

$$= 120\left[1-\frac{1}{\left(1+\frac{9}{25}\right)^{3/2}}\right]$$

$$= 120\left[1-\frac{1}{\left(\frac{34}{25}\right)^{3/2}}\right]$$

$$= 120\left[1-\frac{1}{(1.36)^{3/2}}\right]$$

$$= 120\,(1-0.630)$$

$$= 120 \times 0.369$$

$$\therefore \sigma_z = 44.33 \text{ kN/m}^2$$

Q22. A water tank is required to be constructed with a circular foundation having a diameter of 16 m founded at a depth of 2 m below the ground surface. The estimated distributed load on the foundation is 325 kN/m². Assuming that the subsoil extends to a great depth and is isotropic and homogeneous, determine the stresses at points,

(i) z = 8 m, r = 0

(ii) z = 8 m, r = 8 m

(iii) z = 16 m, r = 0 and

(iv) z = 16 m, r = 8 m

Where r is the radial distance from the central axis and 'z' is the depth below the foundation level. Neglect the effect of the depth of the foundation on the stresses.

(May/June-14, Set-3, Q5(b) | Model Paper-II, Q5(b))

Answer :

Given that,

Diameter = 16 m

Depth = 2 m

Distributed load (q) = 325 kN/m²

$$\text{Radius } R_D = \frac{\text{Diameter}}{2} = \frac{16}{2} = 8 \text{ m}$$

(i) When z = 8 m, r = 0

$$\frac{z}{R_0} = \frac{8}{8} = 1, \quad \frac{r}{R_0} = \frac{0}{8} = 0$$

$I_z = 0.7$ (From figure: Influence diagram for vertical normal stress)

$$\therefore \sigma_z = I_z \times q$$

$$= 0.7 \times 325 = 227.5 \text{ kN/m}^2$$

(ii) When z = 8 m, r = 8 m

$$\frac{z}{R_0} = \frac{8}{8} = 1, \quad \frac{r}{R_0} = \frac{0}{8} = 1.0$$

$I_z = 0.33$ (From figure: Influence diagram for vertical normal stress)

$$\sigma_z = I_z \times q$$

$$= 0.33 \times 325$$

$$= 107.25 \text{ kN/m}^2$$

(iii) When z = 16 m, r = 0

$$\frac{z}{R_0} = \frac{16}{8} = 2, \quad \frac{r}{R_0} = \frac{0}{8} = 0$$

$I_z = 0.3$ (From figure influence diagram for vertical normal stress)

$$\sigma_z = I_z \times q$$

$$= 0.3 \times 325$$

$= 97.5$ kN/m²

(iv) When z = 16 m, r = 8

$$\frac{z}{R_0} = \frac{16}{8} =, \frac{r}{R_0} = \frac{8}{8} = 1$$

$I_z = 0.2$ (From figure influence diagram for vertical normal stress)

$$\sigma_z = I_z \times q$$

$$= 0.2 \times 325$$

$$= 65.0 \text{ kN/m}^2$$

Q23. (i) A long strip footing of width 2 m transmits a pressure of 200 kPa to the underlying soil. Using 2 : 1 dispersion method, compute the approximate value of the vertical stress at a depth of 5m below the footing.

(ii) A line load of 100 kN/m run extends to a long distance. Determine the intensity of vertical stress at a point 2 m below the surface at a distance of 2 m perpendicular to the line load. Use Boussinesq's theory.

Answer :- Oct./Nov.-16, Set-1, Q5(b)

(i) Given data,

Width of footing, $B = 2$ m

Pressure, $q = 200$ kPa

Depth below the footing, $Z = 5$ m

Using 2:1 dispersion method, the approximate value of the vertical stress is given as,

$$\sigma_z = \frac{q \times (B \times 1)}{(B+Z) \times 1}$$ [Assuming unit length of footing]

$$= \frac{200 \times (2 \times 1)}{(2+5) \times 1}$$

$$= 57.143 \text{ kPa}$$

(ii) Given that,

Line load, $Q = 100$ kN/m

Depth, $(Z) = 2$ m

Intensity of vertical stress at a distance 2 m perpendicular to line,

$$\sigma_z = \frac{2Q}{\pi Z} \times \frac{1}{\left[1+\left(\frac{d}{Z}\right)^2\right]^2}$$

$$= \frac{2 \times 100}{\pi \times 2} \times \frac{1}{\left[1+\left(\frac{2}{2}\right)^2\right]^2}$$

$$= \frac{100}{\pi} \times \frac{1}{4}$$

$$\sigma_{z_{at\, d=2\,m}} = 7.96 \text{ kN/m}$$

Q24. A line load of 90 km/m run extends to a long distance. Determine the intensity of vertical stress at a point 1.5 m below the surface,

(i) Directly under the line load and

(ii) At a distance 1 m perpendicular to the line. Use boussinesq's theory.

Answer : April/May-13, Set-3, Q5

Given that,

Line load, $Q = 90$ km/m

Depth, $(Z) = 1.5$ m

(i) Intensity of vertical stress at point 1.5 and distance will be zero.

$$d = 0$$

So,

$$\sigma_Z = \frac{2Q}{\pi Z}$$

$$= \frac{2 \times 90}{\pi \times 1.5}$$

$$\sigma_{Z at d=0} = 38.197 \text{ km/m}$$

(ii) Vertical stress at distance 1 m perpendicular to line,

$$\sigma_Z = \frac{2Q}{\pi Z} \cdot \frac{1}{\left[1+\left(\frac{d}{Z}\right)^2\right]^2}$$

$$= \frac{2 \times 90}{\pi \times 1.5} \cdot \frac{1}{\left[1+\left(\frac{1}{1.5}\right)^2\right]^2}$$

$$\sigma_{Z at\, d=1m} = 18.30 \text{ km/m}$$

Q25. A rigid footing of 3 m diameter carries a column load of 1500 kN at foundation level. Compute the increase in stress due to the column load at a radial distance of 5 m and vertical downward from foundation level of 2 m.

Answer : May/June-14, Set-2, Q5(b)

Given that,

Diameter of column = 3 m

Column load = 1500 kN

Radial distance = 5 m

Depth, $Z = 2$ m

According to Boussineq's equation,

$$\sigma_z = q\left[1 - \frac{1}{\left[\left(1+\frac{r}{z}\right)^2\right]^{3/2}}\right]$$

$$= 1500\left[1 - \frac{1}{\left[1+\left(\frac{5}{2}\right)^2\right]^{3/2}}\right]$$

$$= 1500\left[1 - \frac{1}{\left(1+\frac{25}{4}\right)^{3/2}}\right]$$

$$= 1500\left[1 - \frac{1}{\left(\frac{29}{4}\right)^{3/2}}\right]$$

$$= 1500\left[1 - \frac{1}{(7.25)^{3/2}}\right]$$

$$= 1500\,(1 - 0.051)$$

$$\sigma_z = 1423 \text{ kN/m}^2$$

Q26. Write a short note on westergaard's analysis.

Answer : Oct./Nov.-17, Set-2, Q5(a)

For answer refer Unit-IV, Q2, Topic: Westergaard's Theory.

4.2 NEWMARK'S INFLUENCE CHART – 2:1 STRESS DISTRIBUTION METHOD

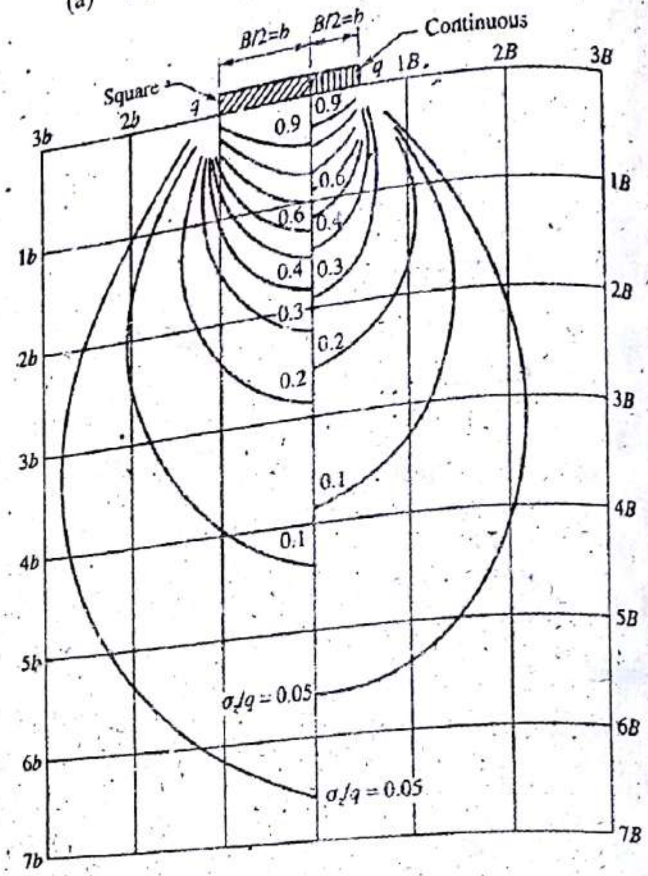
Q27. Mention the different pressure isobars based on either Boussinesq or Westergaard's equation and also provide their sketches.

Answer : Model Paper-IV, Q5(a)

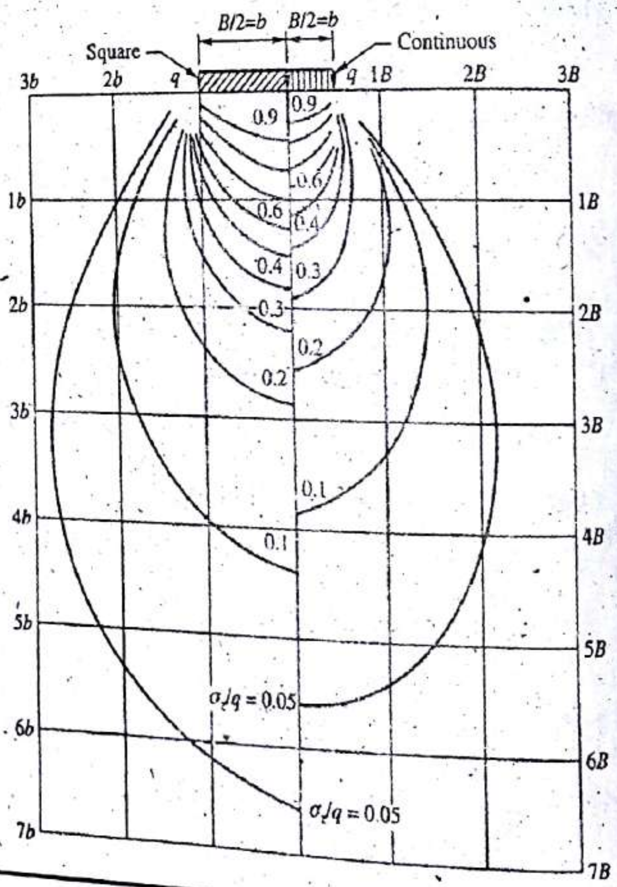
The different pressure isobars for square, rectangular and circular footings used to determine the vertical pressure (σ_z) based on Boussinesq's or Westergaard's equation are as follows,

1. Boussinesq's isobars for

(a) Square and continuous footings

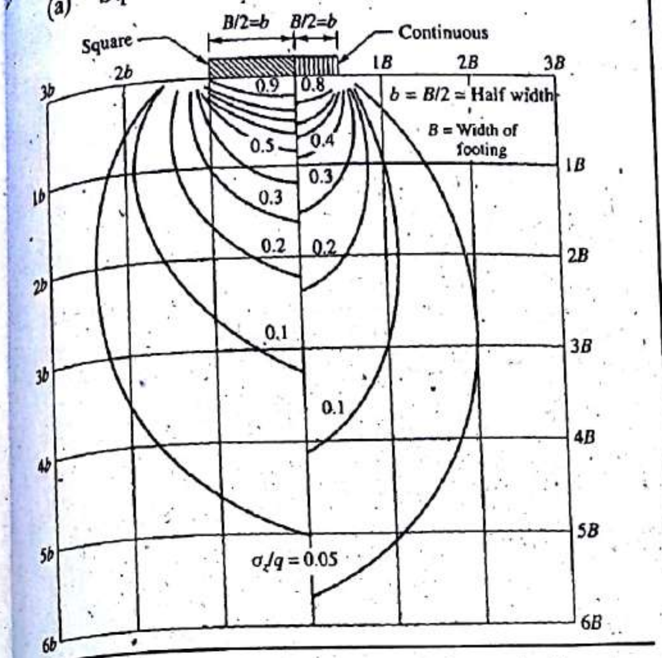


(b) Circular footings



Westergard Isobars for,

(a) Square and continuous Footings.



Q28. Derive Boussinesq's formula for point loads.

Answer : Model Paper-I, Q5(b)

Boussinesq made the following assumptions for point loads,

(a) The soil mass possess the properties such as elasticity, isotropic, homogeneous and semi-infinite.

(b) The point load acts on the surface of soil mass.

(c) Soil is weightless.

The vertical stress 'σ_z' at point 'P' is obtained based on the Boussinesq's formula as,

$$\sigma_z = \frac{3Q}{2\pi z^2} \frac{1}{\left[1+\left(\frac{r}{z}\right)^2\right]^{5/2}}$$

$$\sigma_z = \frac{Q}{z^2} I_b$$

Where,

r – Horizontal distance between the point load 'P' below the surface and vertical axis through the point load Q.

z – Vertical distance from the surface to the point load P.

$$I_b = \frac{3}{2\pi\sqrt{\left(1+\left(\frac{r}{z}\right)^2\right)^5}}$$

The various values of I_b and $\frac{r}{z}$ are obtained from the below figure as shown.

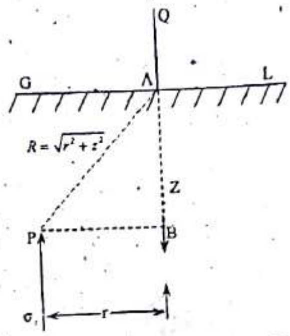


Figure: Vertical Pressure below the Earth Surface

Q29. Derive Westergaard's formula for Point loads.

Answer : Model Paper-II, Q5(a)

According to the assumptions of Westergaard, the elastic soil mass is laterally reinforced by number of closely spaced, horizontal sheets of negligible thickness of infinite rigidity which allows only vertical movement and prevents the mass being subjected to lateral movements of soil grains.

The Westergaad's formula for the vertical stress σ_z for surface point load Q is given by,

$$\sigma_z = \frac{\theta}{2\pi z^2} \frac{\sqrt{\frac{(1-2\mu)}{(2-2\mu)}}}{\left[\frac{(1-2\mu)}{(2-2\mu)+(r/z)^2}\right]^{3/2}}$$

Where, μ = Poisson's ratio,

If μ is considered to be '0' for practical purpose then,

$$\sigma_z = \frac{Q\left(\frac{1}{2}\right)^{1/2}}{2\pi z^2\left[\frac{1}{2+\left(\frac{r}{z}\right)^2}\right]^{3/2}}$$

$$\sigma_z = \frac{Q}{2\sqrt{2}\pi z^2 \times \frac{1}{2^{3/2}}\left[\frac{1}{1+2\left(\frac{r}{z}\right)^2}\right]^{3/2}}$$

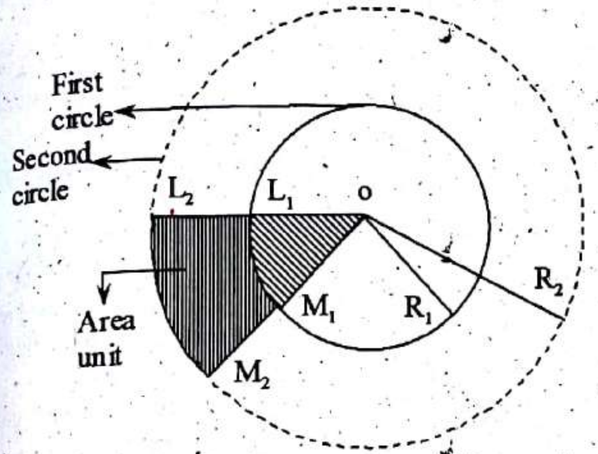
$$= \frac{Q}{2^{3/2}\pi z^2 \times \frac{1}{2^{3/2}}\left[\frac{1}{1+2\left(\frac{r}{z}\right)^2}\right]^{3/2}}$$

Answer :

Newmarks' Influence Chart

In the determination of the vertical stress, Newmark's influence Chart is the accurate method. It comprises of radiating lines and number of circles are made so, that the effect of each unit area at the centre of the concentric circles is similar, that is, the vertical stress caused by each unit area is equal at the midpoint of the chart.

A circle of radius 'R_1' is divided into 20 sections. If σ_z is the vertical pressure at a height 'z' below the centre of area, the intensity of loading is 'w' then the pressure exerted at each part as OL_1m_1 which is equal to $\frac{\sigma_z}{z_0}$ at the centre.



The vertical pressure at depth z due to OL_1m_1 is given by,

$$\frac{\sigma_z}{20} = \frac{w}{20}\left[1-\left\{\frac{1}{1+\left(\frac{R_1}{Z}\right)^2}\right\}^{3/2}\right] = i_{fw} \quad \text{... (1)}$$

Where,

$$\text{Influence value} = i_f = \frac{1}{20}\left[1-\left\{\frac{1}{1+\left(\frac{R_1}{Z}\right)^2}\right\}^{3/2}\right]$$

By fixing the value of i_f as any arbitrary value for example 0.005, then,

$$\frac{w}{20}\left[1-\left\{\frac{1}{1+\left(\frac{R_1}{Z}\right)^2}\right\}^{3/2}\right] = 0.005\text{ w} \quad \text{... (2)}$$

By assuming $z = 5$, and substituting in equation (2)

We get,

$$\frac{w}{20}\left[1-\left\{\frac{1}{1+\left(\frac{R_1}{5}\right)^2}\right\}^{3/2}\right] = 0.005\,w$$

$$\frac{1}{20}\left[1-\left\{\frac{1}{1+\left(\frac{R_1^2}{25}\right)}\right\}^{3/2}\right] = 0.005$$

$$0.05\left[1-\left\{\frac{25}{25+R_1^2}\right\}^{3/2}\right] = 0.005$$

$$0.05\left[1-\left\{\frac{125}{\left(25+R_1^2\right)^{3/2}}\right\}\right] = 0.005$$

$$(25+R_1^2)^{3/2} - 125 = \frac{0.005}{0.05}(25+R_1^2)^{3/2}$$

$$(25+R_1^2)^{3/2} = \frac{1}{10}(25+R_1^2)^{3/2} + 125$$

$$(25+R_1^2)^{3/2} - \frac{1}{10}(25+R_1^2)^{3/2} = 125$$

$$(25+R_1^2)^{3/2}\left\{1-\frac{1}{10}\right\} = 125$$

$$(25+R_1^2)^{3/2}\{0.90\} = 125$$

$$R_1^2 = 26.82 - 25$$

$$R_1^2 = 1.82$$

$$R_1 = 1.35\text{ cm}$$

Therefore, if a circle with radius $R_1 = 1.35$ cm is drawn and is separated into 20 equal parts, each having 0.005 w intensity of pressure at a height of 5 cm.

Let us assume 'R_2' as radius of second concentric circle. The twenty radial lines are extended by which the area within two concentric circles is again separated into 20 equal parts. In this, $L_1L_2m_1m_2$ is one such part. Due to this equal parts, the intensity of vertical pressure will be 0.005 w. Hence the total pressure due to equal parts OL_1m_1 and $L_1L_2m_1m_2$ at height $Z = 5$ cm below the centre is $2 \times 0.005\,w$.

The vertical pressure at depth z due to OL_2M_2 is given by,

$$\frac{w}{20}\left[1-\left\{\frac{1}{1+\left(\frac{R_2}{z}\right)^2}\right\}^{3/2}\right] = 2\times 0.005 \quad \text{... (3)}$$

By substituting $z = 5$ in equation 3 we get,

$$\frac{w}{20}\left[1-\left\{\frac{1}{1+\left(\frac{R_2}{5}\right)^2}\right\}^{3/2}\right] = 2 \times 0.005$$

$$\frac{1}{20}\left[1-\left\{\frac{1}{1+\left(\frac{R_2^2}{25}\right)}\right\}^{3/2}\right] = 2 \times 0.005$$

$$\frac{1}{20}\left[1-\left\{\frac{125}{(25+R_2^2)^{3/2}}\right\}\right] = 2 \times 0.005$$

$$(25 + R_2^2)^{3/2} - 125 = \frac{0.010}{0.05}(25 + R_2^2)^{3/2}$$

$$(25 + R_2^2)^{3/2} = \frac{1}{5}(25 + R_2^2)^{3/2} + 125$$

$$(25 + R_2^2)^{3/2} - \frac{1}{5}(25 + R_2^2)^{3/2} = 125$$

$$(25 + R_2^2)^{3/2} \cdot \left\{1-\frac{1}{5}\right\} = 125$$

$$(25 + R_2^2)^{3/2} - \{0.80\} = 125$$

$$R_2^2 = 29 - 25$$

$$R_2^2 = 4$$

$$R_2 = 2 \text{ cm}$$

Therefore, the circles from 3 to 9 is calculated based on the above procedure. The radius of 10th circle is given by the equation,

$$\frac{w}{20}\left[1-\left\{\frac{1}{1+\left(\frac{R_{10}}{z}\right)^2}\right\}^{3/2}\right] = 10 \times 0.005 \text{ w} = \frac{w}{20} \text{ w}$$

Therefore, from the above equation R_{10} = infinity.

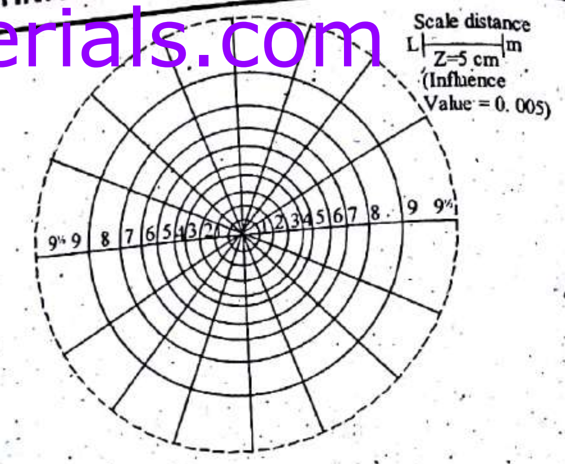


Figure: Newmarks Influence Chart

Q33. Explain the use of Newmark's influence chart in vertical stress estimation in case of irregularly loaded areas.

Answer :

Newmark's influence chart is used to evaluate the vertical stresses for uniformly loaded area of any shape.

Explanation of the use of Newmark's Influence Chart is as follows,

(i) A point 'O' below any uniformly loaded area is considered, on which vertical stress is to be determined.

(ii) On a trace paper, the plan of the uniformly loaded area is traced with a suitable scale. Such that the length is equal to the depth of the point 'O'.

(iii) For example, if depth (d) is of 5 m, then the scale taken to draw the plan must be 1 cm = 1m (if the line OP = 5m) where OP is the vertical line drawn to represent depth (d).

(iv) The traced plan is then kept on Newmark's influence chart such that the point 'O' should coincide with the centre of the circles.

(v) The vertical stress (σ) at a depth (d) at the specified point 'O' is given by,

$$\sigma_z = I \times n \times q$$

Where, I – Influence coefficient

n – Number of small area units covered by p...

q – Load Intensity.

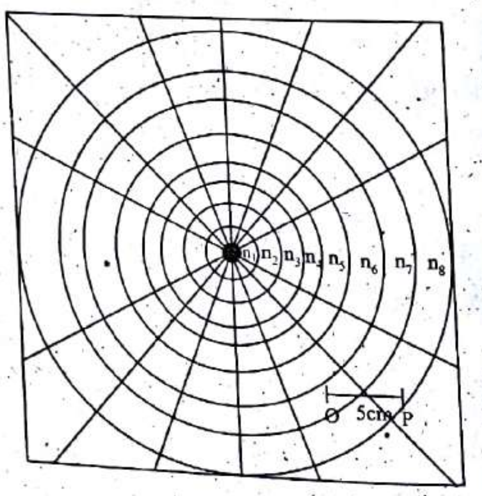


Figure: Newmarks Influence Chart

Q34. Explain 2:1 stress distribution method.

Nov.-15, Set-1, Q5(b)

OR

Write a note on 2:1 stress distribution method.

Oct./Nov.-16, Set-2, Q5(a) | Oct./Nov.-16, Set-4, Q5(a)

Answer :

The 2:1 stress distribution method assumes that the stresses are uniformly distributed over the areas below the foundation at an angle of 2(vertical) to 1 (horizontal) from the loaded edge points as shown in figure below. This method gives an average vertical stress (σ_z) at any depth (z) i. e.,

$$\sigma_z = \frac{\varphi}{(b+z)(L+z)}$$

The value of maximum stress σ_m is different form the average stress σ_z below the loaded area at same depth.

$$\left(\frac{\sigma_m}{\sigma_z}\right)_{\max} = 1.6\, at\left(\frac{z}{b}\right) = 0\text{–}5 \text{ where b = half width.}$$

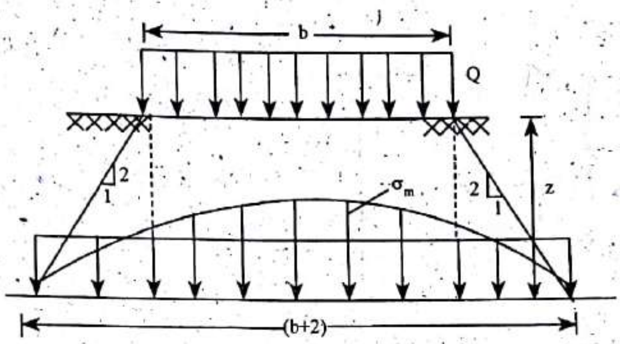


Figure: σ_m 2:1 Method

Q35. A ring foundation of 10 m external diameter and 9 m internal diameter carries a uniformly distributed load of 150 kPa. Determine the vertical stress due to the load at a depth of 6 m below the centre of the foundation.

Answer :

Oct./Nov.-16, Set-2, Q5(b)

Given that,

Outer diamter of ring = 10 m

Inner diamter of ring = 9 m

q = Intensity of loading = 150 kPa

z = Depth of vertical stress = 6 m

a_0 = Outer radius of ring = $\frac{10}{2} = 5$ m

a_I = Inner radius of ring = $\frac{9}{2}$

= 4.5 m.

The vertical stress,

$$\sigma_x = q\left[1-\left\{\frac{1}{1+(a_0/z)^2}\right\}^{\frac{3}{2}}\right] - q\left[1-\left\{\frac{1}{1+(a_i/z)^2}\right\}^{\frac{3}{2}}\right]$$

$$= 150\left[1-\left\{\frac{1}{1+(5/6)^2}\right\}^{\frac{3}{2}}\right] - 150\left[1-\left\{\frac{1}{1+(4.5/6)^2}\right\}^{\frac{3}{2}}\right]$$

$$= 150\,[1-0.454] - 150\,[1-0.512] = 8.7 \text{ kN/m}^2$$

Q36. A Light house is constructed on a circular ring type foundation. The intensity of loading on the foundation is 200 kPa. The outer diameter of the ring foundation is 16m and the internal diameter is 12 m. Determine the vertical pressure directly below the centre of the foundation at depths of 4 m and 6 m.

Answer :

Given that,

The intensity of loading on the foundation = 200 kPa

The outer diameter of the ring foundation = 16 m

The internal diameter of the ring foundation = 12 m

Case (i)

(At Depth, Z_1 = 4 m)

Outer diameter of the Ring

$$D_o = 16 \text{ m}$$

$$r_o = \frac{D_o}{2} = \frac{16}{2} = 8 \text{ m}$$

For $\frac{r_o}{Z_1} = 2$, then $\frac{\sigma_z}{q} = \left[1 - \frac{1}{\left[1+\left(\frac{r_o}{z_1}\right)^2\right]^{3/2}}\right]$

$$\sigma_z = q\left[1 - \frac{1}{\left[1+(2)^2\right]^{3/2}}\right]$$

$$\sigma_z = q\left[1 - \frac{1}{(5)^{3/2}}\right]$$

$$\sigma_z = \left[1 - \frac{1}{11.18}\right]$$

$$\sigma_z = 0.91 \text{ q}$$

Internal diameter of the ring,

$$D_i = 12 \text{ m}$$

$$r_i = \frac{D_i}{2} = \frac{12}{2} = 6\text{m}$$

$$\frac{r_i}{z_i} = \frac{6}{4} = 1.5$$

For $\frac{r_i}{z_i} = 1.5$, then $\frac{\sigma_z}{q} = \left[1 - \frac{1}{\left[1+\left(\frac{r_i}{z_i}\right)^2\right]^{3/2}}\right]$

$$\sigma_z = q\left[1-\frac{1}{\left[1+(1.5)^2\right]^{3/2}}\right]$$

$$= q\left[1-\frac{1}{(3.25)^{3/2}}\right]$$

$$= q\left[1-\frac{1}{5.859}\right]$$

$$\therefore \sigma_z = 0.82\, q$$

The Vertical Pressure is,

$$\sigma_z = 0.91\, q - 0.82\, q$$

$$= 0.09\, q$$

$$= 0.09 \times 200$$

$$= 18 \text{ kN/m}^2$$

Case (ii)

(At Depth Z_2 = 6m)

$$\frac{r_o}{z_2} = \frac{8}{6} = 1.333$$

$$\text{for } \frac{r_o}{z_2} = 1.333,\ \frac{\sigma_z}{q} = \left[1-\frac{1}{\left[1+\left(\frac{r_0}{z_2}\right)^2\right]^{3/2}}\right]$$

$$\sigma_z = q\left[1-\frac{1}{\left[1+(1.333)^2\right]^{3/2}}\right]$$

$$= q\left[1-\frac{1}{[1+1.776]^{3/2}}\right]$$

$$= q\,[1-0.216]$$

$$\sigma_z = 0.78\, q$$

$$\frac{r_i}{z_2} = \frac{6}{6} = 1.$$

$$\text{For } \frac{r_i}{z_2} = 1, \text{ then } \frac{\sigma_z}{q} = \left[1-\frac{1}{\left[1+\left(\frac{r_i}{z_2}\right)^2\right]^{3/2}}\right]$$

$$\sigma_z = q\left[1-\frac{1}{\left[1+(1)^2\right]^{3/2}}\right] = q\left[1-\frac{1}{(2)^{3/2}}\right]$$

$$= q\,[1-0.353]$$

$$= 0.646\, q$$

The vertical pressure is,

$$\sigma_z = 0.78\, q - 0.646\, q$$

$$= 0.134\, q$$

$$= 0.134 \times 200$$

$$= 26.80 \text{ kN/m}^2$$

Q37. A reinforced concrete water tank of size 6 m × 6 m and resting on ground surface carries a uniformly distributed load of 200 kN/m². Estimate the maximum vertical pressure at a depth of 12 m vertically below the centre of the base. Use equivalent point load method.

Answer :

Given that,

w = Uniformly distributed load

= 200 kN/m²

Depth = 12 m

Dividing the loaded area into four equal squares of 6 m size.

The load from each small square may be taken to act through its centre.

$$Q = 200 \times 12 = 2400$$

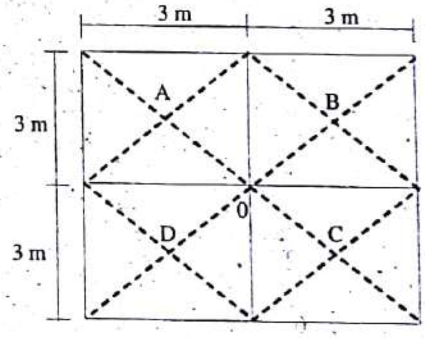


Figure: Loaded Area

The radial distance r to 0 for each of the loads are,

$$= \sqrt{3^2+3^2} = \sqrt{9+9} = \sqrt{18} = 3\sqrt{2}$$

$$\frac{r}{z} = \frac{3\sqrt{2}}{6} = \frac{1}{\sqrt{2}}$$

By symmetry, the stress σ_z at 0 to 12 m depth is four times that caused by single load.

$$\sigma_z = \frac{4 \times 2400}{6 \times 6} \times \frac{(3/_{2\pi})}{\left[1+\left(1/\sqrt{2}\right)^2\right]^{5/2}}$$

$$\sigma_z = 266.67 \times \frac{(0.477)}{2.756} = 46.154 \text{ kN/m}^2$$

Q38. An Annular circular raft has outer and inner diameters as 24 m and 18 m respectively. If the load intensity on the raft is 250 kpa, estimate the increase in vertical stress at a depth 2 m and 4 m from the ground surface and exactly below the centre of the raft.

Answer :

Given that,

Outer diameter $D_o = 24$ m,

Outer radius, $R_1 = \frac{24}{2} = 12$ m

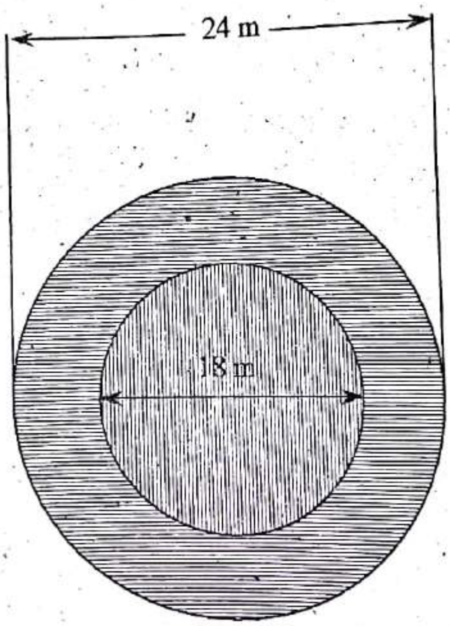
Inner diameter $D_i = 18$ m,

Inner radius, $R_2 = \frac{18}{2} = 9$ m

Depth, $Z_1 = 2$ m

$Z_2 = 4$ m

Load Intensity, $q = 250$ kN/m²



Figure

We know that,

$$\sigma_{z_1} = q\left[\frac{1}{\left\{1+\left(\frac{R_2}{z_1}\right)^2\right\}^{3/2}} - \frac{1}{\left\{1+\left(\frac{R_1}{z_1}\right)^2\right\}^{3/2}}\right]$$

For Depth $Z_1 = 2m$

$$\sigma_{z_1} = 250\left[\frac{1}{\left\{1+\left(\frac{9}{2}\right)^2\right\}^{3/2}} - \frac{1}{\left\{1+\left(\frac{12}{2}\right)^2\right\}^{3/2}}\right]$$

$$\sigma_{z_1} = 250\left[\frac{1}{(21.25)^{3/2}} - \frac{1}{(37)^{3/2}}\right]$$

$$\sigma_{z_1} = 250[0.0102 - 0.0044]$$

$$\sigma_{z_1} = 250\ [0.0058]$$

$$\therefore \sigma_{z_1} = 1.45\ \text{kN/m}^2$$

For depth $Z_2 = 4m$

$$\sigma_{z_2} = 250\left[\frac{1}{\left\{1+\left(\frac{9}{4}\right)^2\right\}^{3/2}} - \frac{1}{\left\{1+\left(\frac{12}{4}\right)^3\right\}^{3/2}}\right]$$

$$\sigma_{z_2} = 250\left[\frac{1}{(6.06)^{3/2}} - \frac{1}{(10)^{3/2}}\right]$$

$$\sigma_{z_2} = 8.84\ \text{kN/m}^2$$

$\therefore$ Therefore the depth of 2 m is 1 45 kN/m² and at depth of 4 m is 8.84 kN/m²

Q.39 Briefly explain the procedure to find vertical stresses caused by a flexible hexagonal shaped foundation with all sides 3 m long carrying a uniform load of 500 kPa, at depths of 1.5, 3 and 4.5 m beneath a corner of the foundation by using Newmark's influence chart.

Answer : **May/June-14, Set-1, Q5(b)**

Given that,

Uniform load, $q = 500$ kPa

Foundation sides = 3 m

Depths = 1.5, 3, 4.5 m

At depth 1.5,

Foundation plan is drawn on the newmarks influence chart at the distance of 1.5,

$\therefore$ Number of area unit occupied by plan is, $n = 313$

$\therefore$ Vertical stress $(\sigma_z) = I \times n \times q$

$= 0.0015 \times 313 \times 500$

$= 235$ kN/m³

At depth 3 m,

Foundation plan is drawn at a distance of 3 m

$\therefore$ Number of area unit occupied by plan is, $n = 174$

$\therefore$ Vertical stress $(\sigma_z) = I \times n \times q$

$= 0.003 \times 174 \times 500$

$= 261 \text{ kN/m}^2$

At depth 4.5 m,

Foundation plan is drawn on new marks influence chart at a distance of 4.5 m,

$\therefore$ Number of area unit occupied by plan is, $n = 71$

$\therefore$ Vertical stress $(\sigma_z) = I \times n \times q$

$= 0.0045 \times 71 \times 500$

$= 159.75 \text{ kN/m}^2$

Q40. A raft of the size 4 m × 4 m carries a uniform load of 180 kN/m². Using the point load approximation with four equivalent point loads. Calculate the stress increment at a point in the soil which is 4 m below the centre of the loaded area.

Model Paper-IV, Q5(b)

Answer :

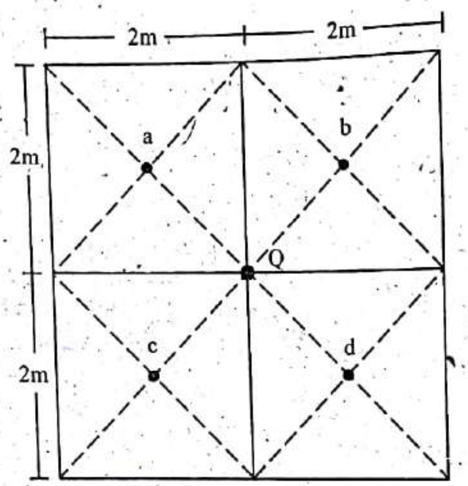
Given that,

Size of the raft = 4 m × 4 m

Uniform load, $q = 180 \text{ kN/m}^2$

Depth below the centre of the loaded area, $Z = 4$ m

Figure indicates the loaded area, divided into four equal squares of 2 m size.



Figure

The load may act at the centre of each square. Therefore, the each point loads (Q_1, Q_2, Q_3, Q_4) at points a,b,c and d are,

$Q = 180 \times 4$

$= 720$ kN

Radial distance for each of the point loads,

$r = 2\sqrt{2}$ m

$\therefore \frac{r}{z} = 2\frac{\sqrt{2}}{4}$

$\frac{r}{z} = \frac{1}{2\sqrt{2}}$

$$\sigma_z = \frac{Q}{z^2} \times \frac{(3/2\pi)}{\left[1+\left(r/z\right)^2\right]^{5/2}}$$

Stress (σ_z) at 0 to 4 m depth is four times that is caused by one load.

$$\therefore \quad \sigma_z = \frac{4\times720}{4^2} \times \frac{\left(\frac{3}{2\pi}\right)}{\left[1+\left(\frac{1}{2\sqrt{2}}\right)^2\right]^{5/2}}$$

$$= 180 \times 0.356$$

$$\sigma_z = 64.08 \text{ kN/m}^2$$

Q41. A rectangular area of 2 m × 4 m carries a uniformly distributed load of 180 kN/m² at ground surface. Find the vertical pressure at 3 m below the centre and corner of the loaded area. Solve the problem by,

(a) Dividing the rectangle into four equivalent rectangles

(b) 2 : 1 method.

Answer :

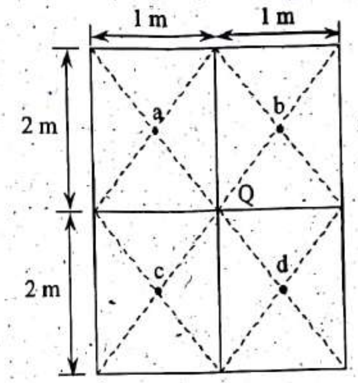
Given that,

Rectangular area = 2 m × 4 m

Uniform load, q = 180 kN/m²

Depth below the centre of the loaded area, z = 3 m

(a) Dividing the Rectangle into Four Equivalent Rectangle



Figure

The load may act at the centre of each rectangle therefore, the each point load (Q_1, Q_2, Q_3, Q_4) at points ***a***, ***b***, ***c***, and ***d*** is,

$$Q = 180 \times 4$$

$$= 720 \text{ kN}$$

Radial distance for each of the point loads,

$$r = \frac{\sqrt{5}}{2} \text{ m}$$

$$\boxed{\frac{r}{z} = \frac{\sqrt{5}}{6}}$$

$$\sigma_z = \frac{Q}{z^2} \times \left[\frac{\frac{3}{2\pi}}{\left[1+\left(\frac{r}{z}\right)^2\right]^{\frac{5}{2}}}\right]$$

$$= \frac{4\times720}{32} \times \left[\frac{\frac{3}{2\pi}}{\left[1+\left(\frac{\sqrt{5}}{6}\right)^2\right]^{\frac{5}{2}}}\right]$$

$$\sigma_z = 110.37 \text{ kN/m}^2$$

b) **2:1 Method**

$$\sigma_z = \frac{q(B \times L)}{(B+Z)(L+Z)}$$

$$q = \frac{Q}{2} = \frac{180}{2} = 90\ kN$$

$$\sigma_z = \frac{90 \times (4 \times 2)}{(4+3)(2+3)}$$

$$\sigma_z = 20.57\ \text{kN/m}^2$$

Q42. Explain Newmark's influence chart preparation and usage.

Nov.-15, Set-1, Q5(a) Nov.-15, Set-3, Q5

Answer :

For answer refer Unit-IV, Q33, Q32.