Heat Transfer

→ Exchange of heat
→ Heat flow order
→ Substance expansion
→ Change of state
→ Weight of a Substance

DMM-II → 24
OR → 24
mcmt → 19
Dom → 23
TE-2 → 24

38 – 24 TE2
36 – 23 Mom
37 – 21 mcmt

→ Whenever heat exchange takes place, amount of heat is consumed means heat lost by hot body is equal to heat gain by cold body. This is a universal truth. The Amount of heat flows from hotter body to cooler body

→ A substance expands on heating

→ Change of state of a system from solid to liquid or liquid to gas without raise in Temperature happen only when certain amount of heat is supplied

→ The weight of a system can't be change by heating or cooling.

→ open system – matter in and out
→ isolated system – There is no heat transmission

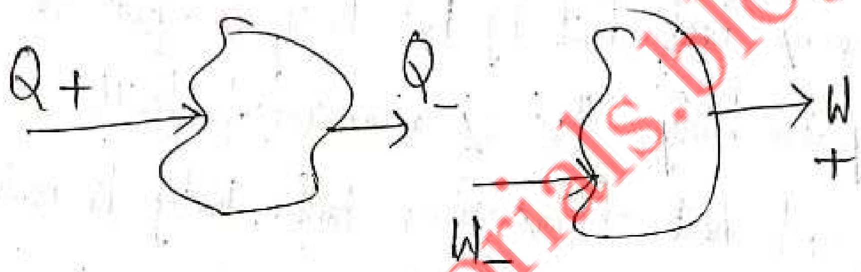
Zeroth law of thermodynamics

It gives the importance of Temperature and explains the Thermal equilibrium ~~at any~~ [[same Temperature] among all bodies]

Heat → ~~high~~ low graded energy

Work → high " "

Sign convention:-



Heat is a low graded energy where as work is a high graded energy

point function:- Temperature, volume, ~~Temp~~ pressure

dT dv dp

path function:- heat work enthalaphy entropy

Q W h s

Intensive property – independent man of a system eg:- Temp, Pres

Extencive property – dependent man of a system eg:- volume, specific volume

Work done $W = -\int_{V_1}^{V_2} p\,dv$ on the piston

$= + \int_{V_1}^{V_2} p\,dv$ by the piston

Modes of Heat Transfer:– There are 3 modes of heat transfer

1 Conduction
2 Convention
3 Radiation

Sun – earth rays	–	8 ½ min
velocity of light	–	3×10^8 m/sec
velocity of sound	–	330 m/s

~~1. Conduction:-~~

1. Radiation – Transfer of heat energy by the electromagnetic waves [without Medium], eg:- Infrared rays. uv light

2. Conduction – (conductor means which holds e^- / flows through electrons)

Transfer of heat energy Takes place between the particles of the objects in direct contact

3. Convention — Transfer of heat energy takes place by moment of fluid

State :- A unique condition of a system is called as state

process :- change in state of a system under steady condition is called as process

- heat and work are related to process

Similarities b/w heat and work :-

- Both are ~~point~~ path function
- Both are boundary phenomonona. i.e They are identified at the boundary while ~~process~~ crossing it.

Quasistatic process :-

- this process is succusation of equilibrium state and feature is infinite slowness
- it is a Reversible process

Temperature :- It describes the Thermal status of a system

- which is used to measure the heat and Temp is propotional to stored Molecular energey (or) kinetic Molecular energey.
- It is Measured in absolute state

pressure

pressure is Measured in bar N/m^2 (or) $= 10^5 N/m^2$ and

absolute pressure = gauge pressure + atomsphere Pressure.

Energy :- capacity to do work.

It is Measured in Joules.

Work :- It is high graded energy is Measured in N-m (or) dyne.

Heat - Heat energy is created by the Movement of particles (atoms) That produces heat

o Temp Incre ⇐ heat Increa

because as Temp ↑ atoms Move faster [have more K.E]

conduction - occurs most in solids and liquids

Conductor :- That Transfer Thermal energy very well

silver, copper, gold, alluminium, Mg, Cu, Ni, platinum, Iron

Insulator :- oil, fur, silk, plastic, paper, Wax, glass, glass, pure Water

Basic Governing laws of heat transfer:-

1) First law of thermodynamics :-

When a system undergoes a thermodynamic cycle a net heat flow supplied to the system from surroundings is equal to net work done by system on surroundings

→ It is applicable to reversible as well as irreversible process

$$\oint dQ = \oint dw$$

→ This law is required for non cyclic process and it explains the concept of internal energy

→ For a closed system the energy transported across the system boundary + heat generated inside the system is equal to change in internal energy of the system

→ For a open system the net energy transported through a control volume + net energy generated inside a control volume is equal to change in internal energy through a control volume

2) Second law of thermodynamics :-

It explains the direction of heat transfer takes place from one reservoir to another reservoir at lower temperature naturally, but not in opposite direction without resistance

(3) Law of Conservation of Mass:

Mass neither be created nor destroyed total Mass remains constant.

→ This law is used to determine parameters of heat flow.

4) Newtons law of Motion:-

This law is used to determine fluid flow parameters

5) Rate equations :-

These equations are made applicable depending upon the mode of heat transfer.

6) Radiation - It is the process of transfer of heat energy by electromagnetic waves without any medium or in medium

properties :-

→ It doesn't require the presence of material medium for its Transmission.

→ Radient heat can be reflected from the surfaces and obey's the ordinary laws of reflection

→ It's travels with velocity of light.

→ ~~light~~ like light it shows interference, diffraction and polarization

→ Its follows inverse square law

Note - The wave length of heat ~~radiations~~ waves is longer than that of light waves hence they are ... to the eye

Heat Transfer by Conduction -

Transfer of heat energy Takes place

Fourier laws of heat Conduction :-

Rate of heat conduction is directly proportional to the Area Measured normal to the direction of heat flow and to the Temperature gradient in that direction

(or)

Rate of heat flow through homogeneous solid is directly proportional to area of section at right angles to the direction of heat flow and to the change in Temperature with respect to length of a path of in the direction of heat flow

$$\boxed{Q \propto A \frac{dt}{dx}}$$

K - Thermal Conductivity of a material

$\frac{dt}{dx}$ - Temp gradient

$$Q = -KA\frac{dt}{dx}$$

Where Q - Heat flow through a body per unit time.

A - Surfaces area of faces of block

dt - Change in Temp

dx - thickness

$Q = W$ always

$W = K \cdot m^{2} \cdot \frac{k}{m}$

$W = K \cdot \frac{}{m \cdot k}$

' - ' sign indicates that heat flows in decreasing Temp. and Temp gradient is always -ve $\therefore$ Q = +ve

Assumptions :-

→ Fourier laws is based on following Assumption.

- Conduction Takes place under steady state conditions
- Heat flow is unidirection.
- Material is homogenous and isotropic. (Temp is all direction and const)
- Temperature gradient is const and Temp is linear

Features :-

- This is applicable to all Types of Matter
- It defines Thermal Conductivity.
- This is derived by experimental evidence but not by first law of Thermodynamics.
- This a vector expression indicating heat flow rate is in the direction of decreasing Temp.

Thermal Conductivity of Materials :-

$Q =$ Watts , $A = m^2$

$W = m^2 \cdot K , \frac{°K}{m}$

$$\boxed{K = \frac{W}{m°K}}$$

$Q = KA\frac{dt}{dx}$

$$K = \frac{Q}{A} \cdot \frac{dx}{dt}$$

when $Q = 1, A = 1 ; \frac{dx}{dt} = 1$

∴ the value of $K = 1$

The ability of a Material to conduct heat is called Thermal conductivity.

- Thermal conductivity of a Material depends on Material structure.
- Moisture content
- density of a Material
- pressure and Temperature

Silver – 410

Copper – 385

Alluminium – 225

Cast iron – 55 to 65

steel – 20 to 40

Concrete – 1.20.

Window glass – 0.75, ambestas – 0.17

Water – 0.55 to 0.7

Freon – 0.0083

- The ratio of Thermal conductivity and electrical conductivity is same for all metals at the same Temp, and this ratio is directly propotional to absolute Temp is given as $\frac{K}{\sigma} \propto T$

K – Thermal conductivity

σ – electric conductivity of a metal at Temp T.

T – Temp

① Calculate the rate of heat Transfer per unit area Through a copper plate 45 mm thick. Where one face maintained at 350°c and other face at 50°c. Take thermal conductivity of copper as 370 W/m°c

Sol → Given $dx = 45mm$ – Thickness of copper plate

Temperature difference $dt = t_2 - t_1$

$= 50 - 350 = -300°C$

$$Q = -A\,k\,\frac{dt}{dx}$$

$$\frac{Q}{A} = \frac{-[-300]}{0.045} \times 370$$

heat Transfer per unit Area $\frac{Q}{A} = 2.46 \times 10^6$ W/m²

② A plain wall is 150 mm thickness and its wall area 4.5 m² if its conductivity is 9.35 $\frac{W}{m°c}$ and surface Temperatures are steady at 150°c and 45°c determine (i) heat flow across of plain wall. (ii) Temp gradient in flow direction

Sol → Given Thickness = 150mm = 0.15m

Wall area $A = 4.5\,m^2$

$K = 9.35$ W/m°c

Temp $T_1 = 150°c$, $T_2 = 45°c$

According to Fourier's law $Q = \frac{t_2 - t_1}{...}$

(i) $$Q = -kA\frac{dt}{dx}$$

$$= -9.35 \times 4.5 \times \left[\frac{-105}{0.15}\right]$$

$$Q = 29452.5 \frac{W}{m^\circ C}$$

(ii) Temp gradient

$$Q = -kA\left(\frac{dt}{dx}\right)$$

$$-\left[\frac{29452.5}{9.35 \times 4.5}\right] = \frac{dt}{dx}$$

$$\frac{dt}{dx} = -700 \frac{^\circ C}{m}$$

③. Following data related to an oven thickness of the side wall 82.5mm. Thermal conductivity of wall insulation 0.044 W/m°c. The Temp of inside wall 175°c and energy distributed by the electrical coil ~~1~~ 40.5 W. determine the ~~Are~~ a of wall surface along to the heat flow so that Temp of outer side wall = 75°c

Sol →

Given thickness $dx = 82.5\ mm = 0.825\ m$

Thermal conductivity $K = 0.044\ W/m^\circ C$

$t_1 = 175^\circ c$, $t_2 = 75^\circ C$

$Q = 40.5 W$

We know That $Q = -K \cdot A \frac{dt}{dx}$

$$A = \frac{Q}{-k\left[\frac{dx}{dt}\right]} = \frac{Q \cdot dx}{-k \cdot dt}$$

$$= \frac{40.5}{-[0.044]\left[\frac{0.85}{-100}\right]}$$

$$\boxed{A = 0.75 \, m^2}$$

(4). The inner surface of a plane brick wall is at 40°c and outer surface is at 20°c calculate the rate of heat Transfer per m² of the Surface Area of wall which is 250 mm Thickness The Thermal conductivity of brick is 0.52 W/m°c

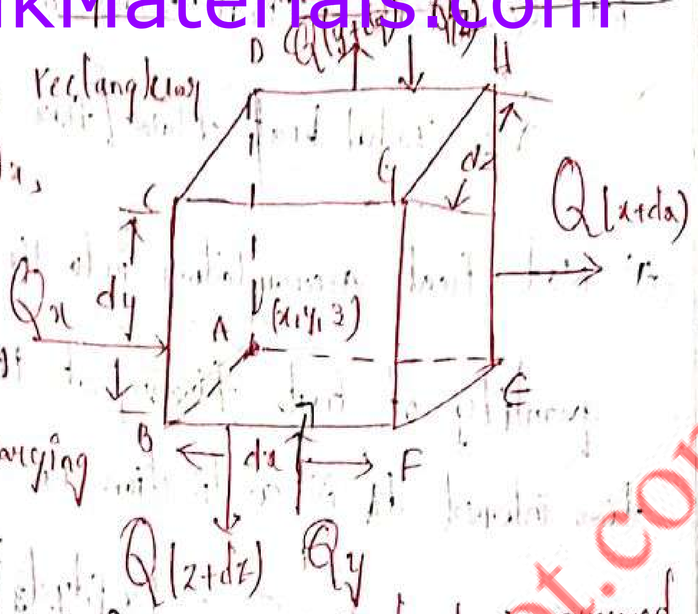
(5). A plain wall of 100 mm thickness and area 3 m² has steady surface Temp of 170°c and 100°c determine

(i) rate of heat flow across the plain wall

(ii) The Temp ~~reading~~ gradient to the flow direction.

General heat conduction equation for cartesian coordinate system

Consider an infinitesimal rectangular parallelopiped having dx, dy, dz sides w.r.t to x, y, z axis in a medium in which temperature is varying with location and time.



let 't' = Temperature at left face ABCD and it is assumed as uniformly distributed over the entire face

(ii) $\frac{dt}{dx}$ = Temp change in the direction of 'x', then

$\left(\frac{\partial t}{\partial x}\right) \cdot dx$ = change of Temp in x direction for a small arbitary element of a face.

Total Temp from the face EFGH = ~~heat~~ $t + \left(\frac{\partial t}{\partial x}\right) dx$

let k_x, k_y, k_z are the thermal conductivities for the element in a time interval $d\tau$.

Q_g - heat generated per unit volume per unit time.

Energy balance equation:

(A) Net heat accumulated in the element due to conduction of heat from all direction + Heat generated in the element (B) = Energy stored in the element.

let Q = Rate of heat flow in a direction

Q' = Total heat flow / flux in particular direction

(A) Net heat Accumulation into the element from all directions

quantity of heat flowing at the left face ABCD in a time interval $d\tau$ in x-direction is

$$Q'_x = -K_x [dy.dz] \frac{\partial t}{\partial x} . d\tau \longrightarrow (i)$$

During the same time interval quantity of heat flow from right face EFGH is $Q'_{(x+dx)} = Q'_{(x)} + \frac{\partial}{\partial x}[Q'_x] . dx \longrightarrow (ii)$

Now, (i) – (ii)

$$d(Q'_x) = Q'_x - \left[Q'_x + \frac{\partial}{\partial x}(Q'_x) \cdot dx\right]$$

$$= -\frac{\partial}{\partial x}[Q'_x] \, dx$$

$$= -\frac{\partial}{\partial x}\left[-K_x \, dy \, dz\right] \frac{\partial t}{\partial x} \, d\tau \, dx$$

$$\boxed{dQ'_x = \frac{\partial}{\partial x}\left[K_x \frac{\partial t}{\partial x}\right] . dx \, dy \, dz \, . d\tau}$$

Similary $\boxed{dQ'_y = \frac{\partial}{\partial y}\left[K_y \frac{\partial t}{\partial y}\right] dx \, dy \, dz \, d\tau}$

$$\boxed{dQ'_z = \frac{\partial}{\partial z}\left[K_z \frac{\partial t}{\partial z}\right] dx \, dy \, dz \, d\tau}$$

Add all directions

Net heat Accumulation

$$A = \left[\frac{\partial}{\partial x}\left(k_x \frac{\partial t}{\partial x}\right) + \frac{\partial}{\partial y}\left(k_y \frac{\partial t}{\partial y}\right) + \left(\frac{\partial}{\partial z} k_z \frac{dt}{dz}\right)\right] dx\,dy\,dz\,d\tau$$

Heat Generated inside the element (B)

$$\boxed{Q_g'' = q_g \cdot dx\,dy\,dz \cdot d\tau}$$

Energey stored in the element (C)

Due to heat accumulation and heat generation in a element there is a increase in thermal energy.

$$Q = m \cdot c \cdot \Delta t$$

c – specific heat

m – mass

$$Q = \rho \cdot v \cdot c \, \partial t$$

$$= \rho \cdot dx\,dy\,dz \cdot c \, \frac{\partial t}{\partial \tau} \, d\tau$$

(C) $$Q = \rho \cdot c \cdot dx\,dy\,dz\,d\tau \, \frac{\partial t}{\partial \tau}$$

$$A + B = C$$

$$\left[\frac{\partial}{\partial x}\left(k_x \frac{\partial t}{\partial x}\right) + \frac{\partial}{\partial y}\left(k_y \frac{\partial t}{\partial y}\right) + \frac{\partial}{\partial z}\left[k_z \frac{\partial t}{\partial z}\right)\right] dx\,dy\,dz\,d\tau + q_g\,dx\,dy\,dz \cdot d\tau$$

$$= \rho \cdot c \cdot \cancel{dx\,dy\,dz\,d\tau} \, \frac{\partial t}{\partial \tau}$$

$$\boxed{k_x \frac{\partial^2 t}{\partial x^2} + k_y \frac{\partial^2 t}{\partial y^2} + k_z \frac{\partial^2 t}{\partial z^2} + q_g = \rho \cdot c \cdot \frac{\partial t}{\partial \tau}}$$

In case of homogeneous and isotropic → ($k_x = k_y = k_z$, ρ, c = same)

$k_x = k_y = k_z = k$

$$k_x \frac{\partial^2 t}{\partial x^2} + k_y \frac{\partial^2 t}{\partial y^2} + k_z \frac{\partial^2 t}{\partial z^2} + q_g = \rho \cdot c \cdot \frac{\partial t}{\partial \tau}$$

$$k \cdot \frac{\partial^2 t}{\partial x^2} + \frac{\partial^2 t}{\partial y^2} \cdot k + \frac{\partial^2 t}{\partial z^2} \cdot k + q_g = \rho c \frac{\partial t}{\partial \tau}$$

$$k \left[\frac{\partial^2 t}{\partial x^2} + \frac{\partial^2 t}{\partial y^2} + \frac{\partial^2 t}{\partial z^2} + \frac{q_g}{k} \right] = \frac{\rho c}{k} \cdot \frac{\partial t}{\partial \tau}$$

$$\nabla - \left(\frac{\partial \theta}{\partial x} + \frac{\partial \theta}{\partial y} + \frac{\partial \theta}{\partial z} \right)$$

$$\Rightarrow \boxed{t \cdot \nabla^2 + \frac{q_g}{k} = \frac{1}{\alpha} \cdot \frac{\partial t}{\partial \tau}}$$

Thermal diffusivity $(\alpha) = \frac{\text{Thermal conductivity}}{\text{thermal capacity}} = \frac{k}{\rho \cdot c}$

Conditions :-

i) When no internal ~~generation~~ source and heat generation is at present.

$$\boxed{\frac{\partial^2 t}{\partial x^2} + \frac{\partial^2 t}{\partial y^2} + \frac{\partial^2 t}{\partial z^2} = \frac{1}{\alpha} \frac{\partial t}{\partial \tau}} \rightarrow \text{fourier eqn}$$

(ii) When Temp doesn't depend upon time (mean steady state)

$$\nabla^2 t + \frac{q_g}{k} = \frac{1}{\alpha} (0)$$

$$\boxed{\nabla^2 t + \frac{q_g}{k} = 0} \rightarrow \text{poisson's eqn}$$

(iii) when heat generation are [illegible] for steady state

$$\boxed{\nabla^2 t = 0} \rightarrow \text{laplace eqn.}$$

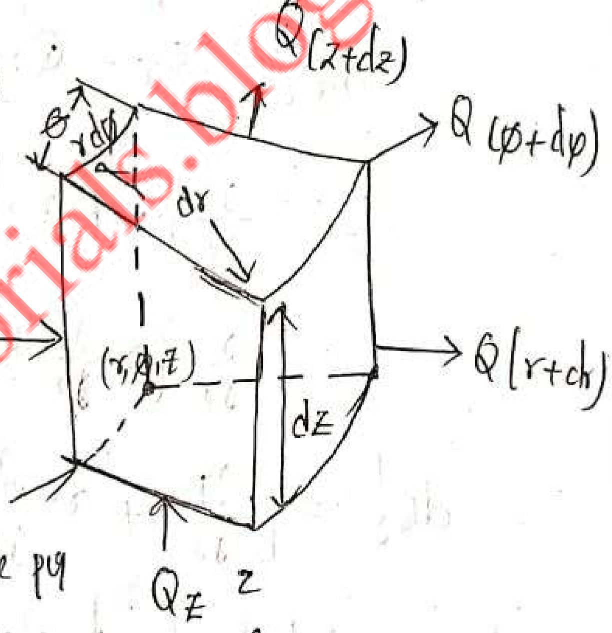
(iv) steady state of, 1D heat

$$\boxed{\frac{d^2 t}{dx^2} + \frac{q_g}{k} = 0}$$

General heat condition in cylindrical Co-ordinate :-

Consider an elemental volume with coordinate axis r, ϕ, z for a 3-dimensional conduction

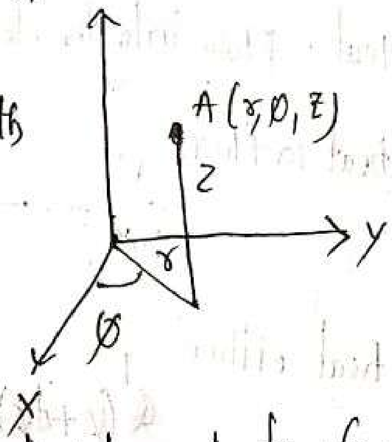
Analysis



volume of element $= r\,d\phi \cdot dr\,dz$

let q_g = heat flow per unit volume per unit time

let us Assume k, ρ, c doesn't alter with position (change)



Energy balance equation

net heat accumulated (A) in the element due to conduction from all coordinate directions + heat generated in element (B)

= Energy stored in the element.

(A) Net heat accumulated in the element.

Net heat flowing along radial direction (r-ϕ) plane [radial direc..

$$Q'_r = -k\,(r\,d\phi.\,dz)\cdot\frac{\partial t}{\partial r}\cdot d\tau \longrightarrow (i)$$

heat influx

heat efflux (outgoing)

$$Q'_{(r+dr)} = Q'_r + \frac{\partial}{\partial r}(Q'_r)\cdot dr \longrightarrow (ii)$$

(i) - (ii)

$$dQ'_r = \cancel{Q'_r} - \cancel{Q'_r} - \frac{\partial}{\partial r}(Q'_r)\cdot dr$$

$$= -\frac{\partial}{\partial r}\left(-k\cdot r\cdot d\phi\,dz\right)\frac{\partial t}{\partial r}\cdot d\tau\cdot dr$$

$$= \frac{\partial}{\partial r}\left(r\cdot\frac{\partial t}{\partial r}\right) k\cdot d\phi\,dr\,dz\cdot d\tau$$

(1)

$$dQ'_r = \left[\frac{\partial t}{\partial r} + r\cdot\frac{\partial^2 t}{\partial r^2}\right] k\cdot d\phi\,dr\,dz\cdot d\tau \longrightarrow (*)$$

Heat flow into the element along Tangential direction (r-z) plane

heat influx

$$Q'_\phi = -k\,dr.\,dz\,\frac{\partial t}{r.d\phi}\cdot d\tau \longrightarrow (iii)$$

heat efflux

$$Q'_{(\phi+d\phi)} = Q'_\phi + \frac{\partial}{r.\partial\phi}(Q'_\phi)\cdot r.d\phi \rightarrow (iv)$$

(iii) - (iv)

$$dQ'_\phi = \frac{\partial}{r.\partial\phi}\,k.dr\,dz\,\frac{\partial t}{r.\partial\phi}\cdot r.d\phi.d\tau$$

$$dQ'_\phi = \frac{1}{r}\frac{\partial}{\partial\phi}\left(\frac{\partial t}{\partial\phi}\right) k . d\phi . dr . dz \, d\tau \longrightarrow ②$$

Heat flow into the element along axial direction $(r-\phi)$ plane.

Heat influx.

$$Q'_z = -k \cdot r \cdot d\phi \, dr \cdot \frac{\partial t}{\partial z} \cdot d\tau \longrightarrow (v)$$

Heat efflux

$$Q'_{(z+dz)} = Q'_z + \frac{\partial}{\partial z}(Q'_z) . dz \longrightarrow (vi)$$

(v)-(vi)

$$dQ'_z = k \frac{\partial}{\partial z}\left(\frac{\partial t}{\partial z}\right) \cdot r \cdot d\phi . dr . dz . d\tau$$

$$dQ'_z = \frac{\partial^2 t}{\partial z^2} . r . k . d\phi . dr . dz . d\tau \longrightarrow ③.$$

①+②+③

$$dQ'_r + dQ'_\phi + dQ'_z = \left[\frac{\partial t}{\partial r} + r . \frac{\partial^2 t}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial\phi}\frac{\partial t}{\partial\phi} + r . \frac{\partial^2 t}{\partial z^2}\right] . k . d\phi \, dr \, dz . d\tau .$$

Heat generated in the element (B)

$$Q'_g = q_g \, r . d\phi . dr . dz . d\tau$$

Energy stored in the element (C)

$$Q = \rho . (r . d\phi . dr \, dz) . c . \frac{\partial t}{\partial \tau} d\tau$$

$$\rightarrow \left[\frac{\partial t}{\partial r} + r\cdot\frac{\partial^2 t}{\partial r^2} + \frac{1}{r}\frac{\partial^2 t}{\partial \phi^2} + r\cdot\frac{\partial^2 t}{\partial z^2}\right] k\cdot \cancel{d\phi\, dr\, dz}\, d\tau$$

$$+ q_g\cdot r\cdot d\phi\cdot \cancel{dr\, dz}\, d\tau = \rho\cdot r\, \cancel{d\phi\, dr}\, dz\, c\cdot\frac{\partial t}{\partial \tau}\, d\tau$$

$$\left(\frac{\partial t}{\partial r} + r\frac{\partial^2 t}{\partial r^2}\right) + \frac{1}{r}\frac{\partial^2 t}{\partial \phi^2} + \left(r\cdot\frac{\partial^2 t}{\partial z^2}\right] + r\cdot\frac{q_g}{k} = \frac{\rho c\cdot r\cdot}{k}\frac{\partial t}{\partial \tau}$$

$$\boxed{\frac{\partial t}{\partial r} + r\frac{\partial^2 t}{\partial r^2} + \frac{1}{r}\frac{\partial^2 t}{\partial \phi^2} + r\frac{\partial^2 t}{\partial z^2} + r\frac{q_g}{k} = \frac{1}{\alpha} r\cdot\frac{\partial t}{\partial \tau}}$$

Note:-

→ Incase there are no heat sources present and the heat flow is steady and 1-Dimensional.

$$\frac{\partial^2 t}{\partial r^2}\cdot r + \frac{\partial t}{\partial r} = 0.$$

General heat conduction in spherical coordinates:-

Consider an elemental volume of spherical coordinates $[r, \phi, \theta]$ for a 3-D heat conduction analysis

let q_g - heat flow ~~vol~~ per unit volume per unit time

let us assume that k, ρ, c don't alter with position.

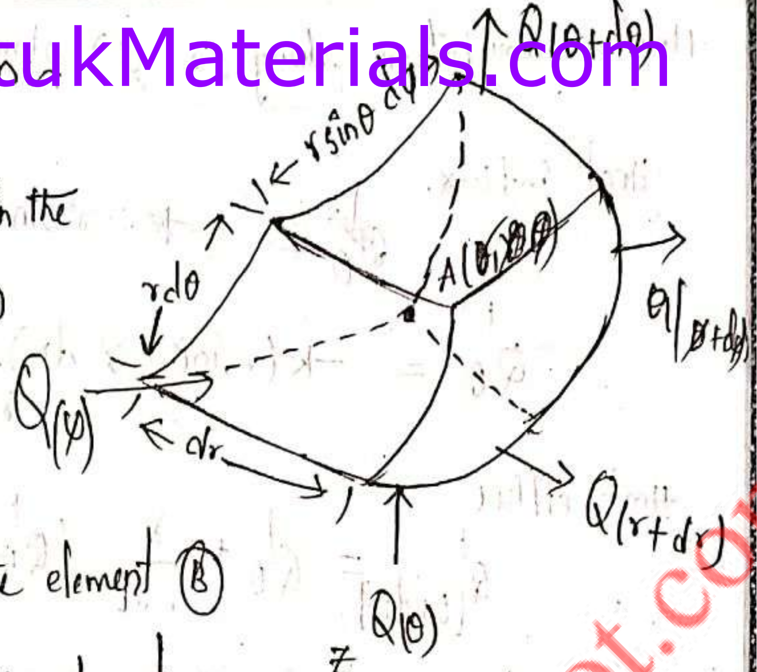
volume of element $= dr\cdot r\sin\theta\, d\phi\cdot r\cdot d\theta$

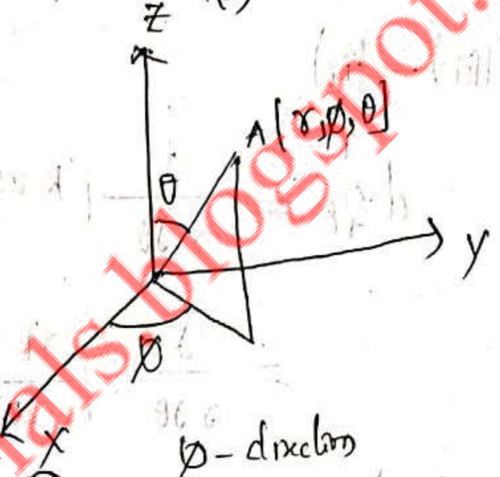
Energy balance equation

Net heat accumulation in the element due to conduction of heat form all coordinate directions (A)

+ Heat generated in the element (B)

= Energy stored in the element.





Net heat Accumulation in element (A)

ϕ – direction

Heat accumulation in the element along $(r-\theta)$ plane.

Heat influx. $Q'_{(\phi)} = -k(r.d\theta.dr)\dfrac{\partial t}{r\sin\theta\,\partial\phi}\,.dT \longrightarrow$ (i)

Heat efflux $Q'_{(\phi+d\phi)} = Q'_{\phi} + \dfrac{\partial}{r\sin\theta\,\partial\phi}(Q'_{\phi})\,.r\sin\theta\,d\phi \longrightarrow$ (ii)

(i) – (ii)

$$dQ'_{(\phi)} = \frac{\partial}{r\sin\theta\,\partial\phi}\left(k.r\,d\theta\,dr.r\sin\theta\,d\phi\,\frac{\partial t}{r\sin\theta\,\partial\phi}\right)dT$$

$$dQ'_{(\phi)} = k.r.d\theta.dr\,r\sin\theta\,d\phi\,.\frac{\partial}{r^2\sin^2\theta\,\partial\phi}\left(\frac{\partial t}{\partial\phi}\right).dT \longrightarrow (1)$$

Heat influx.

$$\cancel{Q'_\theta = -k \cdot r\sin\theta \frac{\partial t}{r\sin}}$$

$$Q'_\theta = -k\,(r\sin\theta\, d\phi\, dr)\frac{\partial t}{r\cdot\partial\theta}\cdot d\tau \longrightarrow \text{(iii)}$$

Heat efflux

$$Q'_{[\theta+d\theta]} = Q'_\theta + \frac{\partial}{r\partial\theta}(Q'_\theta)\, r\cdot d\theta \longrightarrow \text{(iv)}$$

(iii) − (iv)

$$dQ'_\theta = \frac{\partial}{r\,\partial\theta}\left[k\, r\sin\theta\, d\phi\, dr\,\frac{\partial t}{r\cdot\partial\theta}\, r\,d\theta\right] d\tau$$

$$= \frac{\partial}{r\partial\theta}\left[\frac{\partial t}{r\cdot\partial\theta}\right] k\cdot r\sin\theta\, .d\phi\, dr\, .r\,d\theta\cdot d\tau$$

$$dQ'_\theta = k\cdot r\cdot d\theta\cdot dr\; r\sin\theta\cdot d\phi\cdot\frac{1}{r^2}\;\frac{\partial^2 t}{\partial\theta^2}\cdot d\tau \longrightarrow (2)$$

<u>Heat accumulation along (θ−φ) plane :-</u> r-direction

Heat influx.

$$Q'_r = -k\cdot[r\sin\theta\, d\phi\, .r\cdot d\theta]\,\frac{\partial t}{\partial r}\; d\tau \longrightarrow \text{(v)}$$

Heat efflux

$$Q'_{(r+dr)} = Q'_r + \frac{\partial}{\partial r}(Q'_r)\cdot dr \longrightarrow \text{(vi)}.$$

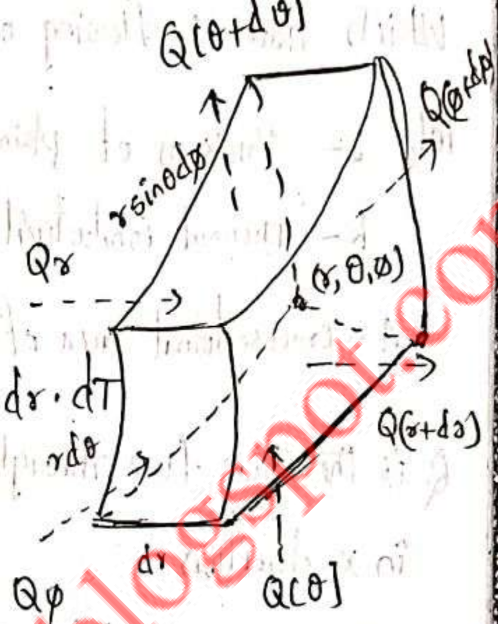
(v)−(vi)

$$dQ'_r = \frac{\partial}{\partial r}\left(\frac{\partial t}{\partial r}\right) k\cdot r\sin\theta\, d\phi\; r\,d\theta\; dr\; d\tau$$

$$dQ'_{(r)} = k\cdot r\,d\theta\cdot dr\; r\sin\theta\, d\phi\,\frac{\partial^2 t}{\partial r^2}\, .d\tau \longrightarrow (3)$$

Net heat accumulation (1)+(2)+(3)

$$\left[\frac{\partial}{r^2\sin^2\theta\,\partial\phi}\left[\frac{\partial t}{\partial\phi}\right]+\frac{1}{r^2}\cdot\frac{\partial^2 t}{\partial\theta^2}+\frac{\partial^2 t}{\partial r^2}\right]K\cdot r\sin\theta\cdot d\phi\, d\theta\, r\cdot dr\, dT$$



Heat generated in The element (B)

$$Q'_g = q_g\cdot r\sin\theta\, d\phi\cdot r d\theta\cdot dr\cdot dT$$

Energy stored in the element (C)

$$Q = \rho\times dr\, r\sin\theta\, d\phi\, r\cdot d\theta\cdot c\cdot\frac{\partial t}{\partial T}\cdot dT$$

A + B = C.

$$\left[\frac{\partial}{r^2\sin^2\theta\,\partial\phi}\left[\frac{\partial t}{\partial\phi}\right]+\frac{1}{r^2}\frac{\partial^2 t}{\partial\theta^2}+\frac{\partial^2 t}{\partial r^2}\right]K\cdot \cancel{r\sin\theta}\cdot\cancel{d\phi}\cdot\cancel{d\theta}\cdot\cancel{r}\cdot\cancel{dr}\,\cancel{dT}$$

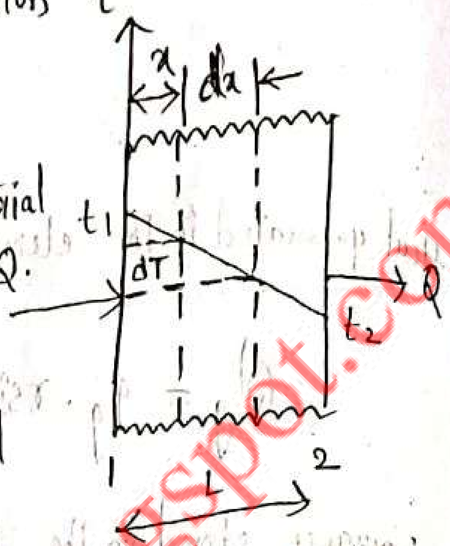
$$+\ q_g\,\cancel{r\sin\theta}\cdot\cancel{d\phi}\,\cancel{d\theta}\cdot\cancel{r}\cdot\cancel{dr}\cdot\cancel{dT} = \rho\times \cancel{dr}\,\cancel{r}\cdot\cancel{r\sin\theta}\,\cancel{d\phi}\cdot\cancel{d\theta}\cdot c\,\frac{\partial t}{\partial T}\cdot\cancel{dT}$$

$$\frac{1}{r^2\sin^2\theta}\frac{\partial^2 t}{\partial\phi^2}+\frac{1}{r^2}\frac{\partial^2 t}{\partial\theta^2}+\frac{\partial^2 t}{\partial r^2}+\frac{q_g}{k}=\frac{\rho\cdot c}{k}\cdot\frac{\partial t}{\partial T}$$

$$\boxed{\frac{1}{r^2\sin^2\theta}\frac{\partial^2 t}{\partial\phi^2}+\frac{1}{r^2}\frac{\partial^2 t}{\partial\theta^2}+\frac{\partial^2 t}{\partial r^2}+\frac{q_g}{k}=\frac{1}{\alpha}\frac{\partial t}{\partial T}}$$

Conduction of heat through a plain Wall :- "Uniform Thermal conductivity"

Consider a plain wall of homogeneous material through which heat is flowing only x-direction



let L – thickness of plain wall

K – Thermal conductivity of the material

A – cross sectional area of wall

Q is the heat flow through a plain wall in x-direction

t_1, t_2 are temperatures maintained at the two faces 1, 2 of plain wall.

We know that

General heat conduction in cartesian form eqn.

$$\frac{\partial^2 t}{\partial x^2}+\frac{\partial^2 t}{\partial y^2}+\frac{\partial^2 t}{\partial z^2}+\frac{q_g}{k}=\frac{1}{\alpha}\frac{\partial t}{\partial T}$$

let us apply the following conditions

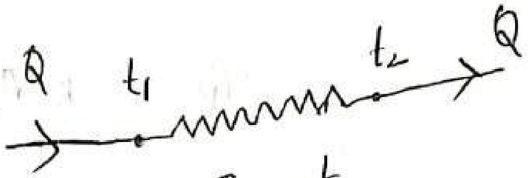
steady state ; 1-D with no internal heat source

$$\frac{\partial t}{\partial T}=0 \qquad \frac{\partial^2 t}{\partial y^2}+\frac{\partial^2 t}{\partial z^2}=0 \qquad q_g=0$$

$$\Rightarrow \boxed{\frac{\partial^2 t}{\partial x^2}=0} \quad (or) \quad \frac{d^2 t}{dx^2}=0 \longrightarrow (i)$$

Integrating eq (i) twice.

$$\int \frac{d^2 t}{dx^2} = 0$$



$$R_{th} = \frac{L}{KA}$$

$$\Rightarrow \frac{L \cdot L}{K \cdot A \cdot L} \Rightarrow \frac{L^2}{KV}$$

$$\frac{dt}{dx} = C$$

$$dt = C_1\, dx$$

$$t = C_1 x + C_2$$

Apply boundary conditions at $x = 0$; $t = t_1$

$x = L$; $t = t_2$

$$t = C_1 x + C_2$$

$$t_1 = C_1(0) + C_2$$

$$\boxed{t_1 = C_2}$$

$$\boxed{t_2 = C_1 L + C_2}$$

$$t_2 = C_1 L + t_1$$

$$\boxed{C_1 = \frac{t_2 - t_1}{L}}$$

sub C_1, C_2 in eq

$$t = C_1 x + C_2$$

$$t = \left[\frac{t_2 - t_1}{L}\right] x + t_1 \longrightarrow \text{(ii)}$$

According to Fourier laws we know that $Q = -KA\frac{dt}{dx}$

eq (ii) represents that Temp distribution across the wall is linear. and it is independent of thermal conductivity

$$\cancel{Q = -KA} \quad Q = -KA\frac{dt}{dx}$$

$$\frac{dt}{dx} = \frac{d}{dx}\left[\left[\frac{t_2 - t_1}{L}\right] x + t_1\right]$$

$$\frac{dt}{dx} = \frac{t_2 - t_1}{L}$$

Sub. Fourier law

$$Q = -K \cdot A \left[\frac{t_2 - t_1}{L}\right]$$

$$\therefore \quad Q = K \cdot A \frac{[t_1 - t_2]}{L}$$

$$= \frac{(t_1 - t_2)}{\frac{L}{KA}}$$

(Thermal resistivity Through Conduction $R_{Th} = \frac{L}{KA}$)

$$\boxed{Q = \frac{t_1 - t_2}{R_{Th}}}$$

Note:- let us find out condition when instead of space the weight is maintained for selection of insulation of plain wall.

i.e. $W = \rho \times V$

$$W = \rho \times A \times L$$

We know that $L = KA \cdot R_{th}$

$$W = \rho \times A \times KA\, R_{th}$$

$$W = \rho \times A^2 \times K \times R_{th(cond)}$$

It's stipulates that for a specified thermal resistance the lighest insulation will be 1 which has the smallest product of 'K' & 'ρ'

~~Variable thermal Conductivity~~

Variable thermal Conductivity:-

The dependence of Temperature with thermal conductivity varies linearly i.e $k = k_0 [1+\beta t]$

where k - thermal conductivity

k_0 - Thermal conductivity at $0^\circ C$

β - Temperature coefficient with thermal conductivity

t - Temperature

the value of β becomes '+ve' for non metals and insulators

the value of β becomes '-ve' for metalic conductors

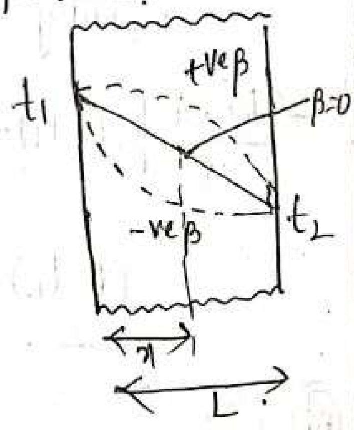
let Temperature varies with thermal conductivity with the eqn.

$$k = k_0 [1+\beta t]$$

from the fourier's equation $Q = -k \cdot A \cdot \frac{dt}{dx}$

$$Q = -k_0 [1+\beta t] A \frac{dt}{dx}$$

$$\frac{Q}{A} dx = -k_0 [1+\beta t] dt$$



Integrating on both sides

$$\frac{Q}{A} \int_0^L dx = -k_0 \int_{t_1}^{t_2} (1+\beta t)\, dt$$

$$\frac{Q}{A} [x]_0^L = -k_0 \left[t + \frac{\beta t^2}{2} \right]_{t_1}^{t_2}$$

$$\frac{Q}{A} L = -k_0 \left[\left(t_2 + \frac{\beta t_2^2}{2}\right) - \left(t_1 + \frac{\beta t_1^2}{2}\right) \right]$$

$$= -k_0 \left[[t_2 - t_1] + \frac{\beta}{2} [t_2^2 - t_1^2] \right] \longrightarrow (i)$$

$$= k_0 \left[(t_1 - t_2) + \frac{\beta}{2} \cdot (t_1^2 - t_2^2) \right]$$

$= k_0 (t_1 - t_2) \left[1 + \frac{\beta}{2}(t_1 + t_2)\right]$

$$\frac{Q}{A} L = k_0 [t_1 - t_2] [1 + t_m \beta] \qquad t_m = \frac{t_1 + t_2}{2}$$

$$Q = k_0 [1 + \beta t_m] \cdot A \cdot \frac{(t_1 - t_2)}{L}$$

$$\boxed{Q = k_m \cdot A \left[\frac{t_1 - t_2}{L}\right]} \longrightarrow (iv)$$

Where k_m — Mean Thermal conductivity. varies linearly with Temp difference

t_1, t_2 — Temp. of surface of wall.

Further 't' is a Temperature at a distance 'x' from left Surface.

$$\therefore \frac{Q \cdot x}{A} = -k_0 \left[(t - t_1) + \frac{\beta}{2}(t^2 - t_1^2)\right] \longrightarrow (ii)$$

Equating (i) and (ii)

$$\frac{\cancel{-}k_0 \cdot \cancel{A}}{x}\left[(t - t_1) + \frac{\beta}{2}[t^2 - t_1^2]\right] = \frac{\cancel{-}k_0 \cancel{A}}{L}\left[(t_2 - t_1) + \frac{\beta}{2}(t_2^2 - t_1^2)\right]$$

$$\frac{(t - t_1) + \frac{\beta}{2}(t^2 - t_1^2)}{x} = \frac{(t_2 - t_1) + \frac{\beta}{2}(t_2^2 - t_1^2)}{L}$$

$$(iii) \quad \boxed{\therefore t = \frac{1}{\beta}\left[(1 + \beta t_1)^2 - \{(1 + \beta t_1)^2 - (1 + \beta t_2)\}\frac{x}{L}\right]^{1/2} - \frac{1}{\beta}}$$

eqn. (iii) represents Temp. variation in Terms of 't_1' & 't_2'

Temp variation in terms of heat flux

let the Temp varies with thermal conductivity linearly

$$k = k_0 [1+\beta t]$$

We know that the fourier eqn $Q = -kA \frac{dt}{dx}$.

$$Q = -k_0 [1+\beta t] A \frac{dt}{dx}$$

$$Q \cdot dx = -k_0 \cdot A [1+\beta t] \, dt$$

Integrating on both sides

$$Q \cdot \int_0^x dx = -k_0 A \int [1+\beta t] \, dt$$

$$Q \cdot x = -k_0 A \left[t + \beta \frac{t^2}{2}\right] + c$$

Apply boundary conditions $x=0$, $t=t_1$

$$Q \cdot (0) = -k_0 A \left[t_1 + \beta \frac{t_1^2}{2}\right] + c$$

$$c = k_0 A \left[t_1 + \beta \frac{t_1^2}{2}\right]$$

Sub 'c' in above eqn;

$$\frac{Q \cdot x}{k_0 \cdot A} = -\left[t + \beta \frac{t^2}{2}\right] + k_0 A \left[t_1 + \beta \frac{t_1^2}{2}\right]$$

$$\left[\frac{\beta t^2}{2} + t\right] - k_0 A \left[t_1 + \beta \frac{t_1^2}{2}\right] + \frac{Q \cdot x}{k_0 \cdot A} = 0.$$

$$t = \frac{-1 + \sqrt{1 - 4\left(\frac{\beta}{2}\right)\left[\frac{Q \cdot x}{k_0 A} - k_0 \cdot A \left(t_1 + \beta \frac{t_1^2}{2}\right)\right]}}{\cancel{2}\left(\frac{\beta}{\cancel{2}}\right)}$$

$$\boxed{t = \frac{-1}{\beta} + \left[\frac{1}{\beta^2} - \frac{2}{\beta}\left\{\frac{Q\cdot x}{k_0 A} - k_0 A\left(t_1 + \beta \frac{t_1^2}{2}\right)\right\}\right]^{1/2}}$$

If Temp doesn't varies linearly with ~~temperature~~ thermal Conductivity

$$\therefore K = k_0 f(t)$$

$$Q = -k \cdot A \frac{dt}{dx}$$

$$Q = -k_0 f(t) \cdot A \frac{dt}{dx}$$

$$\frac{Q \cdot dx}{A} = -k_0 f(t)\, dt$$

$$\cancel{\frac{Q}{A}[x]_0^L = -k_0}$$

$$\frac{Q}{A}\int_0^L dx = -k_0 \int_{t_1}^{t_2} f(t)\, dt \longrightarrow (i)$$

We know that Mean thermal conductivity

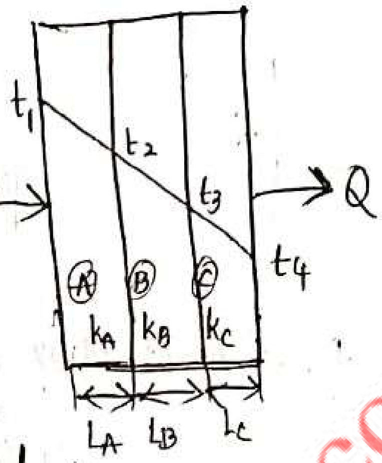
$$Q = k_m \cdot A\left[\frac{t_1 - t_2}{L}\right] \longrightarrow (iv)$$

equating (i) & (iv)

$$\frac{k_m \cdot \cancel{A} \cdot \left[\frac{t_1 - t_2}{L}\right]}{\cancel{A}} \int_0^L dx = -k_0 \int_{t_1}^{t_2} f(t)\, dt$$

$$\boxed{k_m = \frac{-k_0}{(t_1 - t_2)} \cdot \int_{t_1}^{t_2} f(t)\, dt}$$

Heat conduction Through Composite walls:-



Consider heat transmitted through a composite wall consisting of no. of slabs.

let L_A, L_B, L_C are the thickness of slab A, slab B, slab C.

k_A, k_B, k_C are the thermal conductivites of slabs.

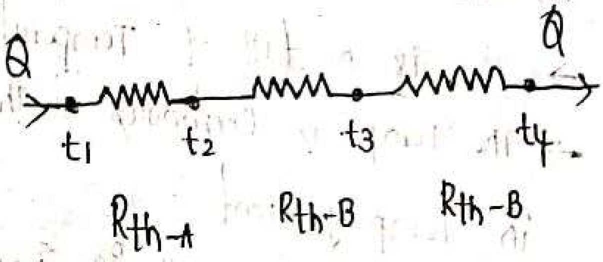
t_1, t_4 are the Temperatures of wall surfaces. and t_2, t_3 are the Temperatures at interfaces.

Since the heat flow ~~through~~ per unit time through a single slab is given by

$$Q = \frac{k_m A (t_1 - t_2)}{L}$$

let us assume that there is no temperature drop at the interfaces. i.e Perfect contact b/w the layers.

$$Q = \frac{k_A \cdot A \; (t_1 - t_2)}{L_A} = \frac{k_B \cdot A \cdot (t_2 - t_3)}{L_B} = \frac{k_C \cdot A \cdot [t_3 - t_4]}{L_C}$$



$\rightarrow R_{th-A} = \frac{L_A}{k_A \cdot A}$

$\rightarrow R_{th-B} = \frac{L_B}{k_B \cdot A}$

$\rightarrow R_{th-C} = \frac{L_C}{k_C \cdot A}$

$$t_1 - t_2 = \frac{L_A \cdot Q}{k_A \cdot A}$$

$$t_2 - t_3 = \frac{L_B \cdot Q}{k_B \cdot A}$$

$$t_3 - t_4 = \frac{L_C \cdot Q}{k_C \cdot A}$$

Adding (3).

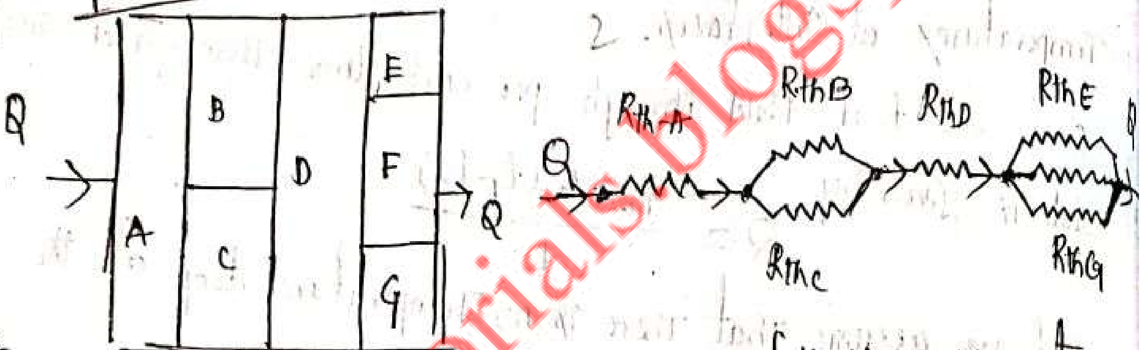
$$t_1 - t_4 = Q\left[\frac{L_A}{k_A \cdot A} + \frac{L_B}{k_B \cdot A} + \frac{L_C}{k_C \cdot A}\right] = Q[R_{th-A} + R_{th-B} + R_{th-C}]$$

$$\boxed{Q = \frac{[t_1 - t_4]}{[R_{th-A} + R_{th-B} + R_{th-C}]}}$$

For 'n' no. of slabs

$$\boxed{Q = \frac{t_1 - t_{n+1}}{\sum_1^n \frac{L}{k \cdot A}}} \text{ (or) } = \frac{t_1 - t_{n+1}}{\sum_1^n R_{th}}$$

eg:- parallel.



Note:-

To solve probleams of composite wall the following assumptions to be follow.

→ the contact of adjacent layers is perfect

→ there is no fall of Temperature at the interface.

→ The Temp is continous at the interfaces although there is a ~~discontinuity~~ dis continuty in Temp gradient.

In real system it is impossible due to surface roughness and void spaces [filled with air] therefore which no heat transfer Takes place

∴ there is a small area

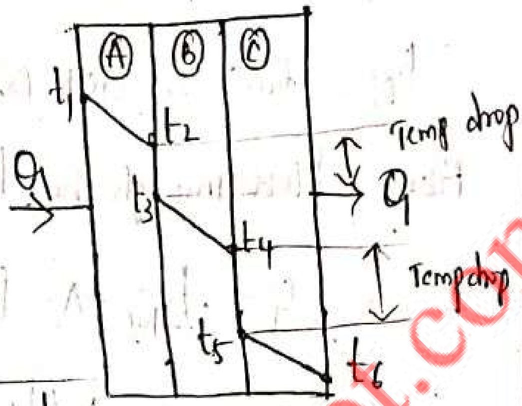
Thermal

Overall heat Transfer coefficient :-

thermal constant resistence $R_c = \dfrac{t_2 - t_3}{Q \cdot A}$

$$R_{th(AB)} = \frac{t_2 - t_3}{Q \cdot A}$$

$$R_{th(BC)} = \frac{t_4 - t_5}{Q \cdot A}$$



Factors effecting thermal constant resistence :

→ Micro hardnes of the Material.

→ Contact pressure

→ Surface roughness

→ thermal conductivity of the Materials

→ slope.

Overall Heat Transfer coefficient :-

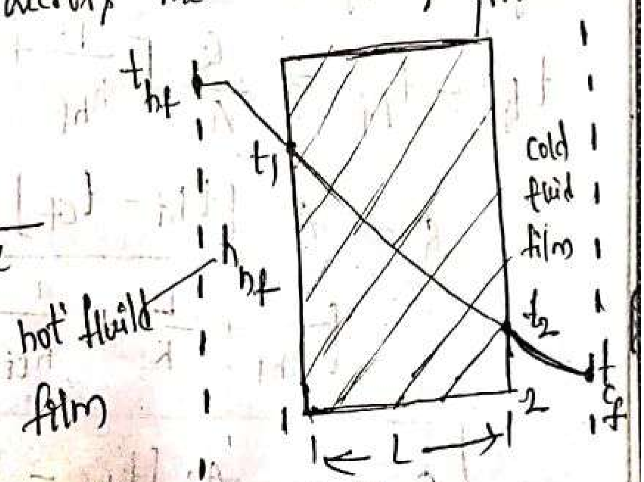
While solving the probleams of fluids to fluid heat Transfer accross the metal boundary we adopt overall heat transfer coefficient which gives heat flow per unit area per unit time per degree Temp difference across the bulk fluids.

L - thicknes of Metal Wall

k - Thermal conductivity of the wall Material

t_1 - Temp at Surface 1

t_2 - " " 2



t_{hf} – Temp of hot fluid

t_{cf} – Temp of cold fluid.

h_{hf} – heat Transfer coefficient from hot fluid to metal surface

h_{cf} – heat Transfer coefficient from metal surface to cold fluid

Heat flow Through the fluid and the Metal surface

$$Q = h_{hf} \cdot A \cdot [t_{hf} - t_1] \longrightarrow (i)$$

$$Q = \frac{K \cdot A \cdot (t_1 - t_2)}{L} \longrightarrow (ii)$$

$$Q = h_{cf} \cdot A \cdot [t_2 - t_{cf}] \longrightarrow (iii)$$

$$(t_{hf} - t_1) = \frac{Q}{h_{hf} \cdot A} \longrightarrow (iv)$$

$$(t_1 - t_2) = \frac{Q \cdot L}{K \cdot A} \longrightarrow (v)$$

$$[t_2 - t_{cf}] = \frac{Q}{h_{cf} \cdot A} \longrightarrow (vi)$$

by adding (iv) (v) (vi)

$$t_{hf} - t_{cf} = \frac{Q}{A}\left[\frac{1}{h_{hf}} + \frac{L}{k} + \frac{1}{h_{cf}}\right]$$

$$Q = \frac{A \cdot [t_{hf} - t_{cf}]}{\left[\frac{1}{h_{hf}} + \frac{L}{k} + \frac{1}{h_{cf}}\right]}$$

$$\boxed{U = \frac{1}{\frac{1}{h_{hf}} + \frac{L}{k} + \frac{1}{h_{cf}}}}$$

$$\boxed{Q \cdot U = A \cdot [t_{hf} - t_{cf}]}$$

U – overall heat Transfer Coefficient

$$Q = A(t_{hf} - h_{cf}) \cdot U$$

Effects of various parameters on Thermal conductivity on solids

→ chemical composition [pure metals having high thermal conductivity (k)]

Impurities or alloying elements reduce the Thermal conductivity

→ metal forming [forging, drawing and bending are heat Treatment of cause considerable variation in Thermal conductivity

eg:- conductivity (k) of hardened steel is lower than that of annealed steel.

→ Temperature raise The value of 'k' for most metals decreases with Temp raise.

→ Non metalic solids, have 'k' much lower Than that of metals

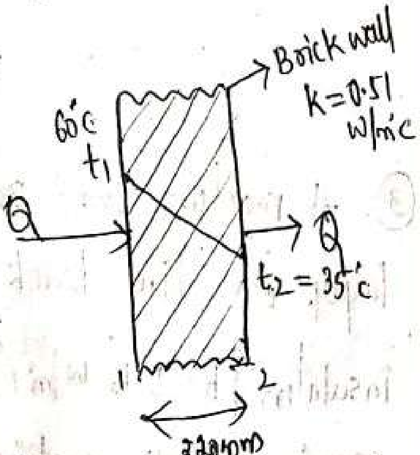
→ presence of air thermal conductivity is reduced due to presence of air filled ~~force~~ cavities

→ Damperness 'k' is higher Than That of ~~base~~ dry material

→ Thermal conductivity of insulating powder or asbestos increases with density growth

problems

1) The inner surface of a plain brick wall is at 60°C and the outer surface is at 35°C. Calculate the rate of heat Transfer per area of surface of the wall, which is 220 mm thickness. The Thermal conductivity of the wall is i.e 0.51 W/m°C



Sol Given $k = 0.51\ W/m°c$

$t_1 = 60°c$, $t_2 = 35°c$

$L = 220mm = 0.22m$

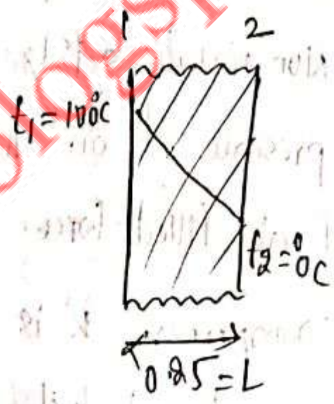
rate of heat flow $q = \frac{Q}{A} = ?$

$$\frac{Q}{A} = \frac{K(t_1 - t_2)}{L}$$

$$q = \frac{0.51\,[60-35]}{0.22}$$

$$\boxed{q = 57.95 \ W/m^2}$$

(2). Consider a slab of Thickness 0.25 m one surface is kept at 100°C and another surface at 0°C determine net flux across the slab if the slab is made of pure copper thermal conductivity of copper is 387.6 W/m°C



Heat flux $\frac{Q}{A} = ?$

$t_1 = 100°C$

$t_2 = 0°C$

$L = 0.25 m$

$K = 387.6 \ W/m°C$

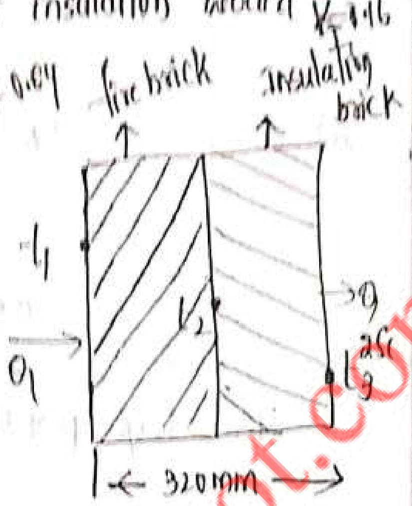
We know that fourier eqn $Q = -K \cdot A \frac{dt}{dx}$

$$\frac{Q}{A} = \frac{-387.6 \times (0-100)}{0.25}$$

$$\boxed{\frac{Q}{A} = 155.04 \times 10^3 \ W/m^2}$$

(3). A reactors wall 320 mm thickness is made up of an inner layer of fire brick [$K = 0.84$ W/m°C] covered with a layer of insulation [$K = 0.16$ W/m°C] the reactor operates at a temp of 1325°C and the ambient Temp is 25°C determine the thickness of ~~fire~~ firebrick and insulation which gives minimum heat loss.

calculate the heat loss. Assuming that the insulating material has a maximum temp of 1200°C. If the calculated heat loss is not acceptable then state whether addition of another layer of insulation would provide a satisfactory soln.



Sol Given $t_1 = 1325°C$

$t_2 = 1200°C$

$t_3 = 25°C$

$L = 320mm = 0.32m$

$L_A = ?$ $L_B = ?$

(i) $\because L = L_A + L_B \rightarrow L_A = 0.32 - L_B$ $t_2 = 1200°C$

W.K.T $Q = \dfrac{t_1 - t_3}{\frac{1}{A}\left[L_A/K_A + L_B/K_B\right]} = \dfrac{t_1 - t_2}{\frac{L_A}{K_A \cdot A}} = \dfrac{t_2 - t_3}{\frac{L_B}{K_B \cdot A}}$

$$\Rightarrow \frac{[t_1 - t_3]}{\left[\frac{L_A}{K_A \cdot A} + \frac{L_B}{K_B \cdot A}\right]} = \frac{[t_1 - t_2]}{\frac{L_A}{K_A \cdot A}}$$

$$\frac{1325 - 25}{\frac{L_A}{0.84} + \frac{L_B}{0.16}} = \frac{1325 - 1200}{\frac{L_A}{0.84}}$$

$$\frac{1300}{L_A + L_B} = \frac{945}{L_A}$$

$$\frac{1300}{(0.32 - L_B) + L_B} = \frac{945}{0.32 - L_B}$$

$$945\left[-L_B^2 + 0.32 L_B\right] = 1310\left[0.32 - L_B\right]$$

$$-945 L_B^2 + 302.4 L_B - 1300 L_B = 416$$

$$945 L_B^2 + 997.6 L_B - 416 = 0$$

$$\boxed{\begin{array}{c} L_A = 0.114\text{ m} \\ \hline L_B = 0.21\text{ m} \end{array}}$$

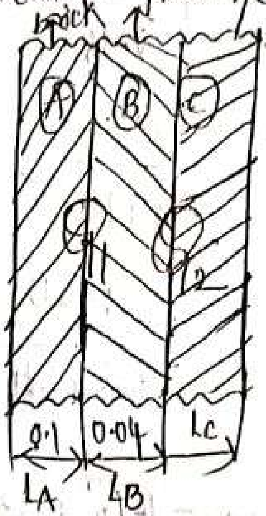
ii) $$\frac{Q}{A} = \frac{k_A(t_1 - t_2)}{L_A}$$

$$\frac{Q}{A} = 954.5$$

Heat loss per unit area $= \underline{\underline{954.5}}$ W/m^2

If another layer of insulating material is added the heat loss from the wall will reduce. Consequently the temp drop across the firebrick will drop and the interface Temp T_2 will raise.

③. An exterior wall of a house may be approximated by 0.1 m clay layer of common brick [$K = 0.7$ W/m°C] followed by a 0.04 m layer of gypsum plaster whose [$K = 0.48$ W/m°C]. What wall thickness of loosely packed rock wool insulation [$K = 0.065$ W/m°C] should be added to reduce the heat loss or gain through the wall by 80%.



Sol Given $k_A = 0.7$ W/m°C $L_A = 0.1$ m

$k_B = 0.48$ W/m°C $L_B = 0.04$ m

$k_C = 0.065$ W/m°C $L_C = x$.

rate of heat flow

$$Q_1 = \frac{\Delta t}{\frac{L_A}{K_A \cdot A} + \frac{L_B}{K_B \cdot A}}$$

$$\Rightarrow \frac{\Delta t}{\frac{0.1}{0.7(A)} + \frac{0.04}{0.48(A)}} = \frac{A \cdot \Delta t}{4.42}$$

$$Q_2 = \frac{A \cdot \Delta t}{\frac{L_A}{K_A} + \frac{L_B}{K_B} + \frac{L_C}{K_C}}$$

$$Q_2 = \frac{A \cdot \Delta t}{4.42 + \frac{L_C}{0.065}}$$

We know that $Q_2 = 0.2 \times Q_1$

$$Q_2 = 0.\cancel{8} \times \frac{A \cdot \Delta t}{4.42}$$

$$\boxed{Q_2 = 0.45}$$

$\cancel{0.45}$

$$\frac{\cancel{A \cdot \Delta t}}{4.42 + \frac{L_C}{0.065}} = \left[\frac{\cancel{A \cdot \Delta t}}{4.42}\right] \times 0.2$$

$$\boxed{L_C = 0.062}$$

④ A Wall of a furnance is Made up of inside layer of silica brick 120mm thick covered with a layer of magneside brick 240 mm Thick the Temp at the inside Temp of silica brick wall and outside surface of a magneside brick wall are 725°c and 110°c respectively. the contact Thermal resistence b/w The two Walls at The interface is 0.0035 °c/W per unit Wall area. if the thermal conductivities of silica and magneside brick Walls are 1.7 W/m°c and 5.8 W/m°c. calculate The rate of heat loss per unit area of the Walls.

(ii) Temp drop at The interface.

Sol Given $K_A = 1.7\ W/m^\circ C$

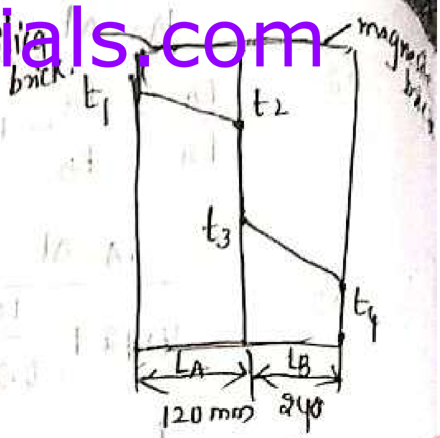
$K_B = 5.8\ W/m^\circ C$

$L_A = 120mm = 0.12m$

$L_B = 240mm = 0.24m$

$t_1 = 725^\circ C$

$t_4 = 110^\circ C$ $\qquad R_{th} = 0.0035\ ^\circ C/W$



rate of heat flow $q = \frac{Q}{A} = \frac{t_1 - t_4}{\frac{L_A}{K_A} + 0.0035 + \frac{L_B}{K_B}}$

$$= \frac{725 - 110}{\frac{0.12}{1.7} + 0.0035 + \frac{0.24}{5.8}}$$

$$\boxed{q = 5326.17\ W/m^2}$$

$$q = \frac{t_1 - t_2}{\frac{L_A}{K_A} + \cancel{0.0035}} = \frac{t_3 - t_4}{\cancel{0.0035} + \frac{L_B}{K_B}}$$

$$5326.17 = \frac{725 - t_2}{\frac{0.12}{1.7} \cancel{+0.0035}}$$

$$5326.17 = \frac{-330.39 + t_3}{\cancel{0.0035+} \frac{0.24}{5.8}}$$

$$\boxed{t_2 = \cancel{330.39}\ 349.03^\circ C}$$

$$\boxed{t_3 = 330.39^\circ C}$$

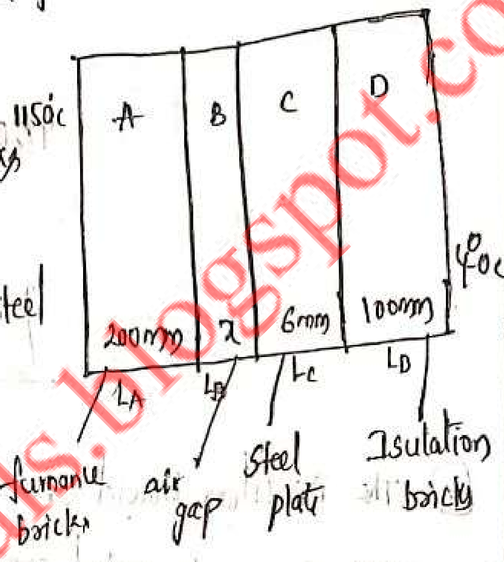
$$\boxed{t_2 - t_3 = 18.63^\circ C}$$

Temp drop at the interface $= 18.63^\circ C$

(5) A. Furnace wall Consists of 200mm layers of refractory bricks, 6mm layer of steel plate and a 100mm layer of insulation (Bricks) The Maxi. Temp of wall is 1150°c on the furnace side and the Maximum Temp. is 40°c on the outermost side of the wall an accurate energy balance over the furnace shows that the heat loss from the wall is 400 mm W/m² it is known that there is a thin layer of air b/w the layers of refractory bricks and the steel plate Thermal Conductivity of 3 layers are 1.52, 45, and 0.138 W/m°c

Find How may mm of insulation bricks is the air layer equalent.

What is the Temp of outer surface of steel plate?



Sol →

Given $k_A = 1.52$

$k_B = k_D$

$k_C = 45\ w/m°c$

$k_D = 0.138\ W/m°c$.

$L_A = 200mm = 0.2\ m$

let $L_B = x$

$L_C = 6mm = 0.006m$

$L_D = 100mm = 0.1\ m$

$$\frac{Q}{A} = \frac{\Delta t}{\frac{L_A}{K_A} + \frac{L_B}{K_B} + \frac{L_C}{K_C} + \frac{L_D}{K_D}}$$

$$400 = \frac{1150 - 40}{\frac{0.2}{1.52} + \frac{(x/1000)}{0.138} + \frac{0.006}{45} + \frac{0.1}{0.138}}$$

$\Rightarrow \boxed{x = 2.928}$

$L_B = 2.928\ mm$

⑥ ~~A square plate heater~~

outer surface of steel plate $t_s = ?$

$$Q = \frac{\Delta t}{\frac{L_D}{K_D}}$$

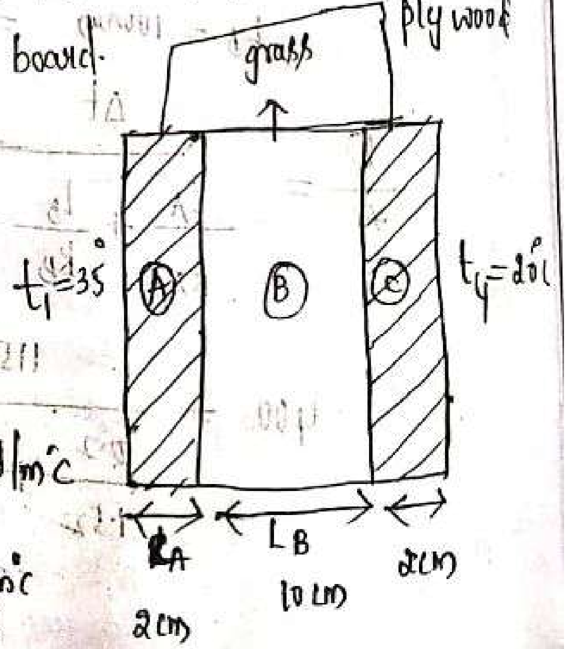
$$Q = \frac{t_s - t_4}{\frac{L_D}{K_D}}$$

$$480 = \frac{t_s - 40}{\frac{0.1}{0.138}}$$

$$\boxed{t_s = 329.85^\circ c}$$

⑥ The insulation board for air condition purpose on made of 3 layers middle being of packet grass 10 cm thick [$K = 0.02\ W/m^\circ c$] sides are made of 2cm thickness [$K = 0.12\ W/m^\circ c$] they are glude with each others. determine the heat flow per m^2 area. if one surface is $35^\circ c$ and other is $20^\circ c$ neglect the glude resistence

Instead of glude it these 3 pieces are bolted by four ~~board~~ steel bolts of 1 cm dia at the corner $1m^2$ area of the board then find the heat flow per $1m^2$ area of the combined board.



sol Given $t_1 = 35^\circ c$

$t_4 = 20^\circ c$

$L_A = 2cm = 0.02m$

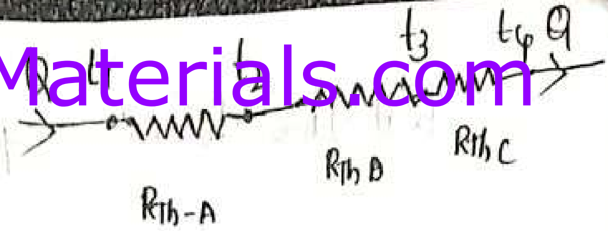
$L_B = 0.1m$

$L_C = 0.02m$

$K_A = K_C = 0.12\ W/m^\circ c$

$K_B = 0.02\ W/m^\circ c$

$$q = \frac{Q}{A} = \frac{t_1 - t_4}{R_{Th\text{-}A} + R_{Th\text{-}B} + R_{Th\text{-}C}}$$

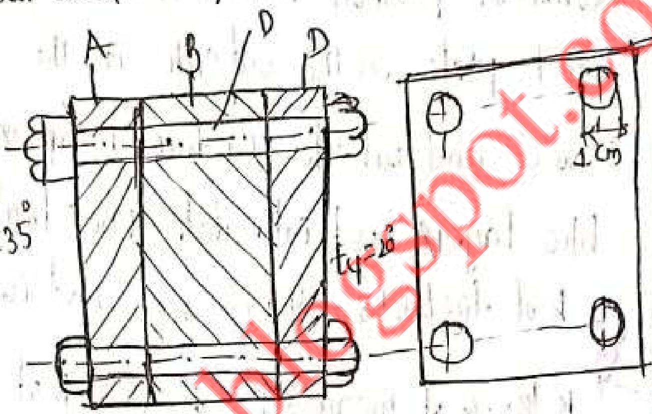


$$q = \frac{35 - 20}{\frac{0.02}{0.12} + \frac{0.1}{0.02} + \frac{0.02}{0.12}} = 2.8\ W/m^2$$

(ii) 3 boards are bolted four steel bolts each bolt having 1cm dia

Area of bolt $A = \frac{\pi}{4}(0.01)^2$

$= 7.85 \times 10^{-5}\ m^2$

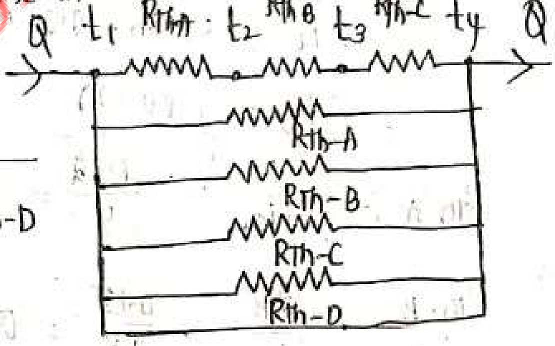


dia of bolt = 1cm = 0.01m

No. of steel bolts used = 4

Given thermal conductivity of four bolts $K_D = 40$

Equivalent Thermal resistence of the thermal circuit for the system.



$$\frac{1}{(R_{Th})_{equ.}} = \frac{1}{R_{Th\text{-}A} + R_{Th\text{-}B} + R_{Th\text{-}C}} + \frac{4}{R_{Th\text{-}D}}$$

$$R_{Th\text{-}A} = \frac{L_A}{K_A} = \frac{0.02}{0.12} = 0.16$$

$$R_{Th\text{-}B} = \frac{L_B}{K_B} = \frac{0.1}{0.02} = 5$$

$$R_{Th\text{-}C} = \frac{L_C}{K_C} = \frac{0.02}{0.12} = 0.16$$

$$R_{Th\text{-}D} = \frac{[L_A + L_B + L_C]}{K_D \cdot A_D} = \frac{[0.02 + 0.1 + 0.02]}{40 \times 7.85 \times 10^{-5}} = 44.58\ °C/W$$

$$(R_{Th})_{equi} = \frac{1}{0.167 + 5 + 0.16} + \frac{4}{44.58} = 0.276 = \frac{1}{0.276} = 3.6\ °C/W$$

heat flow for m^2 $q = \frac{\Delta t}{(R_{th})_{eqvi}}$

$$q = \frac{35-20}{3.6} = 4.16\ W/m^2$$

(7) A furnace wall is composed of 220mm of fire bricks, 150mm of common brick, 50mm of 85% magnesia and 3mm of steel plate on the outside if the inside ~~Tem~~ surface Temp is 1500°c and outside surface Temp is 90°c estimate the Temperatures b/w layers and calculate the heat loss in KJ/hr m^2.

Sol → k of fire bricks — 4 KJ/m-h°c, k of common bricks — 2.8 KJ/m-h°c

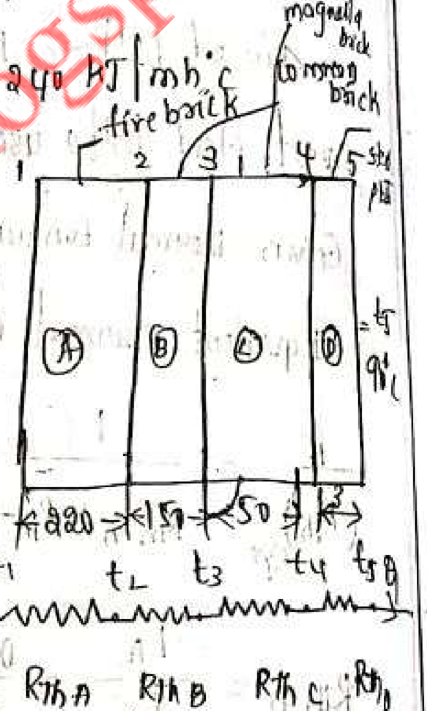
K (85% of magnesia) — 0.24, k of steel — 240 KJ/mh°c

Sol → Given $L_A = 220mm - 0.22m$

$L_B = 150\ mm - 0.15\ m$

$L_C = 50\ mm - 0.05\ m$

$L_D = 0.003\ m$



$$R_{Th-A} = \frac{L_A}{K_A} = \frac{0.22}{4} = 0.055\ °c/w$$

$$R_{Th-B} = \frac{L_B}{K_B} = \frac{0.15}{2.8} = 0.05\ °c/w$$

$$R_{Th-C} = \frac{L_C}{K_C} = \frac{0.05}{0.24} = 0.2\ °c/w$$

$$R_{Th-D} = \frac{L_D}{K_C} = \frac{0.003}{240} = 1.25\times10^{-3}\ °c/w$$

$$\therefore (R_{Th})_{equivalent} = R_{Th\ A} + R_{Th-B} + R_{Th\ C} + R_{Th\ D}$$

$$= 0.055 + 0.05 + 0.2 + 1.25\times10^{-3}$$

$$[R_{Th}]_{eqvi} = 0.305\ °c/w$$

$$q = \frac{\Delta t}{[R_{th}]_{total}} = \frac{1500-90}{0.305}$$

$$= 4622.76\ kJ/h.m^2$$

Temp b/w layers.

$$q = \frac{t_4 - t_5}{[R_{th}]_D}$$

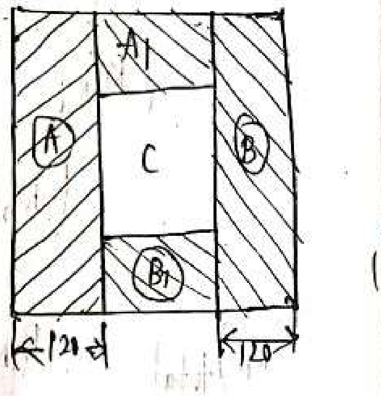
$$t_4 = q[R_{th}]_D + t_5 = 4622.76[1.25\times10^{-3}] + 90^\circ$$

$$\boxed{t_4 = 95.77\,^\circ C}$$

$$t_3 = q[R_{th}]_C + t_4 = 1020.32\,^\circ C$$

$$t_2 = q[R_{th}]_B + t_3 = 1251.46\,^\circ C$$

(8) Two slabs each 120mm thickness have thermal conductivites of 14.5 W/m°c and 210 W/m°c These are placed in contact, but due to roughness only 30% of area is in contact and the gap in the remaining area is 0.025mm thickness and is filled with air If the Temp of face of the hot surface is at 220°c and outside surface of the other slab is 30°c determine heat flow through The composite systems, The contact resistance and Temp drop incontact. [Assume The conductivity of air is 0.032 W/m°c] and That half of the contact due to either metal



Sol →

$L_A = L_B = 120\ mm = 0.12\ m$

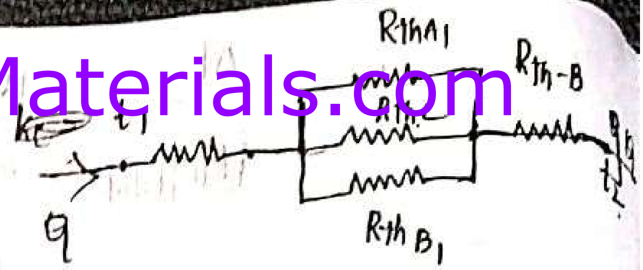
$L_{A_1} = L_{B_1} = L_C = 0.025mm = 0.025\times10^{-3}\ m$

$t_1 = 220^\circ c \quad k_A = 14.5\ W/m^\circ c$

$t_2 = 30^\circ c \quad k_B = 210\ W/m^\circ c$

$L_{A_1} = L_{B_1} = 0.025mm$

$k_{A_1} = 14.5$ W/mK



$k_{B_1} = 210$ W/m°C

$k_c = 0.032$ W/m°C

$$R_{th-A} = \frac{0.12}{14.5 \times 1} = 8.27 \times 10^{-3}$$

$$R_{th-A_1} = \frac{0.025 \times 10^{-3}}{14.5 \times 0.15} = 1.14 \times 10^{-4}$$

$$R_{th-B_1} = \frac{0.025 \times 10^{-3}}{210 \times 0.15} = 7.93 \times 10^{-6}$$

$$R_{th-C} = \frac{0.025 \times 10^{-3}}{0.032 \times 0.7} = 1.11 \times 10^{-4}$$

$$R_{th-B} = \frac{0.12}{210 \times 1} = 5.71 \times 10^{-4}$$

$$\frac{1}{(R_{th})_{eq}} = \frac{1}{R_{th\,A_1}} + \frac{1}{R_{th-C}} + \frac{1}{R_{th\,B_1}}$$

$$\frac{1}{R_{th\,eqv}} = \frac{1}{1.14 \times 10^{-4}} + \frac{1}{1.11 \times 10^{-4}} + \frac{1}{7.93 \times 10^{-6}}$$

$$\frac{1}{(R_{th})_{eq}} = 143388 4.34$$

$$(R_{th})_{eqv} = 6.950 \times 10^{-6}$$

$$R_{th})_{total} = R_{th A} + R_{th eq} + R_{th B}$$

$$= 8.27 \times 10^{-3} + 6.950 \times 10^{-6} + 5.71 \times 10^{-4} = \underline{8.84 \times 10^{-3}}$$

$$\text{Rth(total)}$$

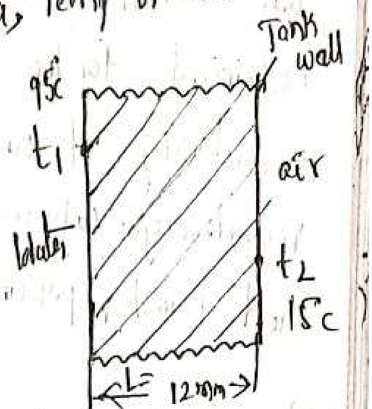
$$q = \frac{220-30}{8.84\times10^{-3}} = 27.33\times10^{6}$$

Temp drop $\cdot\ \Delta t = Q \times$ contact resistence

$$= 27.33\times10^{6} \times 6.950\times10^{-6}$$

$$\boxed{\Delta t = 189.99^{\circ}C}$$

Q. A mild steel Tank of wall thickness 12mm contains water at 95°c the thermal conductivity of mild steel is 50 W/m°c and the heat transfer coefficients for the inside and outside the Tank are 2850 and 10 W/m²°c if the atmospheric Temp is 15°c calculate rate of heat loss per m² for the Tank surface area, Temp of the outside surface of the Tank



Sol: Given $L = 12mm = 0.012m$

$t_{hf} = 95^{\circ}c$, $15^{\circ}c = t_{cf}$

$h_{hf} = 2850$, $h_{cf} = 10\ W/m^2{}^{\circ}c$

$k = 50\ W/m^{\circ}c$

$q = ?$

$$q = U\cdot A\,[t_{hf} - t_{cf}]$$

$$U = \frac{1}{\frac{1}{h_{hf}} + \frac{L}{k} + \frac{1}{h_{cf}}} \Rightarrow \frac{1}{U} = \frac{1}{h_{hf}} + \frac{L}{k} + \frac{1}{h_{cf}}$$

$$= \frac{1}{2850} + \frac{0.012}{50} + \frac{1}{10}$$

$$\frac{1}{U} = 0.100$$

$u = 9.941 \ W/m^2 {}^\circ C$

$$q = 9.941 \times 1 \times [95 - 15]$$

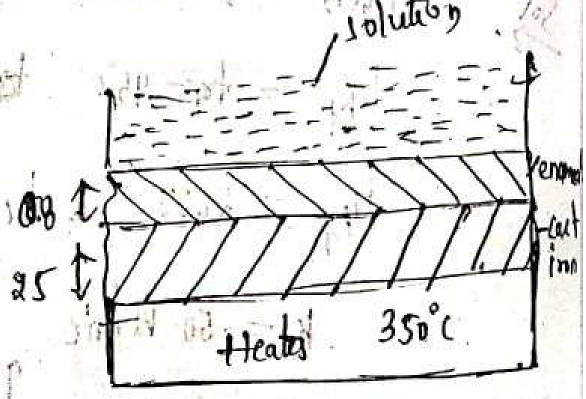
$$= 795.3$$

$$q = h_{cf} \times A \times [t_2 - t_{cf}]$$

$$795.3 = 10 \times 1 \times [t_2 - 15]$$

$$t_2 = 94.53^\circ C$$

(10) A electric hot plate is maintained at a Temp of $350^\circ C$ and is used to keep a soln ~~boily~~ at boiling at $95^\circ C$ the solution is contained in a cast iron vessel of thickness 25 mm which is enamel inside to a thickness of 0.8 mm. The heat transfer coefficient for the boiling soln is $5.5 \ kW/m^2K$ and the thermal conductivity of cast iron and enamel are 50 and $1.05 \ W/mK$ respectively. Calculate overall heat transfer coefficient, rate of heat transfer per unit area.



sol → Given $L_{cast\,iron} = 25 mm$
$= 0.025 m$

$L_{enamel} = 0.8 mm$
$= 0.0008 m$

$h_{bf} = 5.5 \times 10^3 \ W/m^2 K$

$K_{c.i} = 50$, $K_{enamel} = 1.05$

$$q = u \cdot A \left(t_{heater} - t_{soln}\right)$$

where $U = \dfrac{1}{\dfrac{L_{CI}}{K_{CI}} + \dfrac{L_F}{K_F} + \dfrac{1}{h_{gf}}}$

$$\frac{1}{U} = \frac{0.025}{50} + \frac{8\times10^{-4}}{1.05} + \frac{1}{5.5\times10^{3}}$$

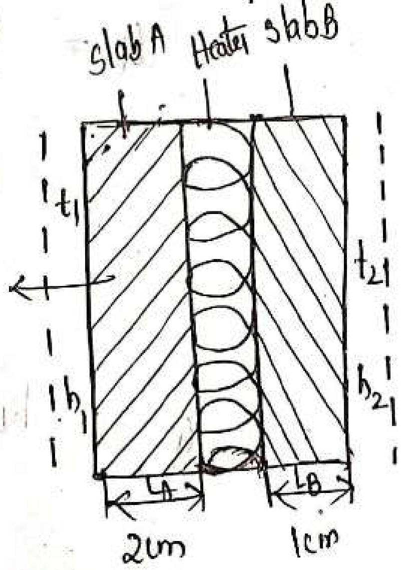
$$\frac{1}{U} = 1.44\times10^{-3}$$

$$\underline{U = 692.65}$$

$$\therefore Q = 692.65 \times 1 \times [350-95]$$

$$Q = \underline{\underline{176626.68}}$$

11) A square plate heater is insulated b/w two slabs of area (15cm×15) slab 'A' is 2cm thick [K = 50 W/m°C] and slab 'B' is 1 cm thick [k = 0.2 W/m°C] the outside heat transfer coefficient on side A and side B are 200 W/m²°c and 50 W/m²°c the Temp of surrounding air is 25°c if rating of heater is 1 KW find Maximum Temp in the System, outer surface Temp of two slabs.



Sol) Given $t_a = 25°c$

$t_1 = ?$

$h_1 = 200$ W/m²°c

$K_A = 50$ W/m°c

$h_2 = 50$ W/m²°c

$K_B = 0.2$ W/m°c

$L_A = 2cm = 0.02$ m , $A = 15\times15 = 225$ cm²

$L_B = 1 cm = 0.1 m$ $= 0.0225$ m²

$q = 1000$ W

Max. Temp of the heater (t_{max})

Steady flow

$q =$ heat flow through slab A + heat flow through slab B

$$1000 = \frac{t_{max} - t_a}{\frac{L_A}{K_A} + \frac{1}{h_1 A}} + \frac{t_{max} - t_a}{\frac{L_B}{K_B} + \frac{1}{h_2 A}}$$

$$1000 = \frac{t_{max} - 25^\circ}{\frac{0.02}{50} + \frac{1}{200 \times 2.25 \times 10^{-3}}} + \frac{t_{max} - 25}{\frac{0.01}{0.2} + \frac{1}{50 \times 2.25 \times 10^{-3}}}$$

$$1000 \cancel{t_{max}} = \frac{t_{max} - 25}{22.22} + \frac{t_{max} - 25}{88.93}$$

$$t_{max} = \cancel{202.} \; 202.91$$

out side Temp of slab B

$$Q = \frac{K_A A (t_{max} - t_1)}{L_A} = h_1 A (t_1 - t_a)$$

$$1000 = 200 \times 2.25 \times 10^{-3} [t_1 - 25]$$

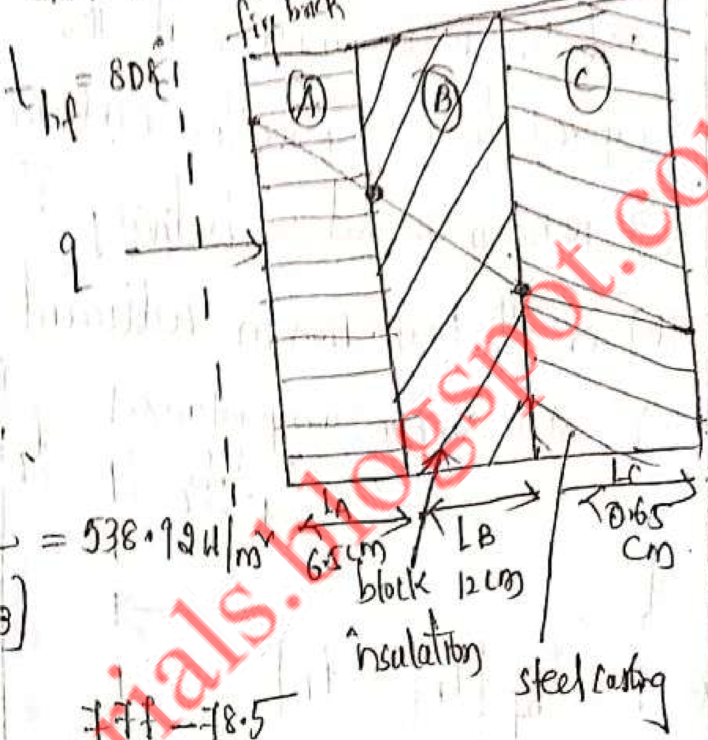
$$\boxed{t_1 = 185.21^\circ C}$$

$$Q = \frac{K_B \cdot A \, [t_{max} - t_a]}{L_B} = h_2 \cdot A \, (t_2 - t_a)$$

$$t_2 = \left[\frac{1000}{2.25 \times 10^{-3} \times 50}\right] + 25$$

$$\boxed{t_2 = 78.25\,C}$$

(1.3) A figure shows the temp distribution through a furnace wall consisting of fire brick and high temp block insulation and steel plate. Thermal conductivity of fire brick is 1.13 W/m°C determine (i) Rate of Heat per unit area of the furnace wall (ii) Thermal conductivities of block insulation and steel plate



sol → Given $q = q_A = q_B = q_C = q_{cf}$

$$q = q_A = \frac{t_1 - t_2}{\frac{L_A}{k_A}}$$

$$q = \frac{806 - 777}{(0.065/1.13)} = 538.92\ W/m^2$$

(ii) $$q = \frac{t_2 - t_3}{(L_B/k_B)} \Rightarrow \frac{777 - 78.5}{(0.12/k_B)}$$

$$\boxed{k_B = 538.92}$$

$$q = \frac{t_3 - t_4}{(L_C/k_C)} \Rightarrow 538.92 = \frac{78.5 - 78.4}{(0.0065/k_C)}$$

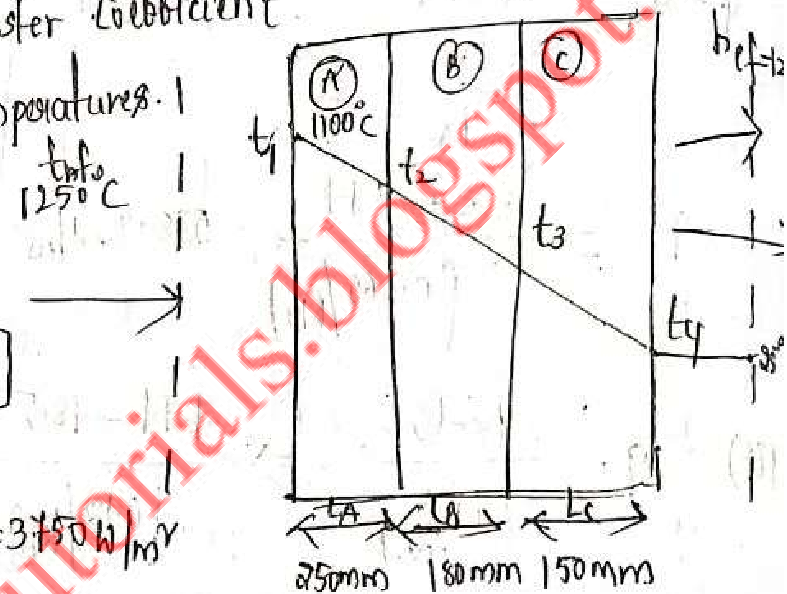
$$\boxed{k_C = 35.02\ W/m°C}$$

(14) A furnance wall is made of three layers of thickness 250 mm, 100 mm, 150 mm with thermal conductivity is 1.65 W/m°c, K W/m°c and 9.2 W/m°c respectively the inside is exposed to gases at 1250°c with a convective coefficient of 25 W/m²°c and the inside surface is at 1100°c, The outside surface is exposed to with convective coefficient of 12 W/m²°c determine

(a) unknown thermal conductivity k

(b) overall heat transfer coefficient

(c) All surface Temperatures.



Sol

(a) $q = h_{hf}[t_{hf} - t_1]$

$= 25 \times [1250 - 1100] = 3750\ W/m^2$

(b) $q = \frac{[\Delta t]_{overall}}{[R_{th}]_{total}}$

$$3750 = \frac{t_{hf} - t_{cf}}{R_{th\ convective\ hf} + R_{th-A} + R_{th-B} + R_{th-C} + R_{th-hcf}}$$

$$3750 = \frac{t_{hf} - t_{cf}}{\left[\frac{1}{h_{hf}} + \frac{L_A}{k_A} + \frac{L_B}{k_B} + \frac{L_C}{k_C} + \frac{1}{h_{cf}}\right]}$$

$$3750 = \frac{1250-25}{\left[\frac{1}{25}+\frac{0.25}{1.65}+\frac{0.1}{k_B}+\frac{0.15}{9.2}+\frac{1}{12}\right]}$$

$$\left[\frac{1}{25}+\frac{0.25}{1.65}+\frac{0.1}{k_B}+\frac{0.15}{9.2}+\frac{1}{12}\right] = 0.326$$

$$0.291 + \frac{0.1}{k_B} = 0.326$$

$$\boxed{k_B = 2.857 \ W/m^\circ C}$$

(ii) $$U = \frac{1}{[R_{th}]_{total}} = \frac{1}{0.32} = 3.102 \ W/m^2 {}^\circ C$$

$W/m^2 {}^\circ C$

(iii) $$q = \frac{t_1 - t_2}{(L_A/k_A)} \Rightarrow 3750 = \frac{1100 - t_2}{[0.25/1.65]}$$

$$\boxed{t_2 = 531.81^\circ C}$$

$$q = \frac{t_2 - t_3}{[L_B/k_B]} \Rightarrow 3750 = \frac{531.81 - t_3}{[0.18/2.85]}$$

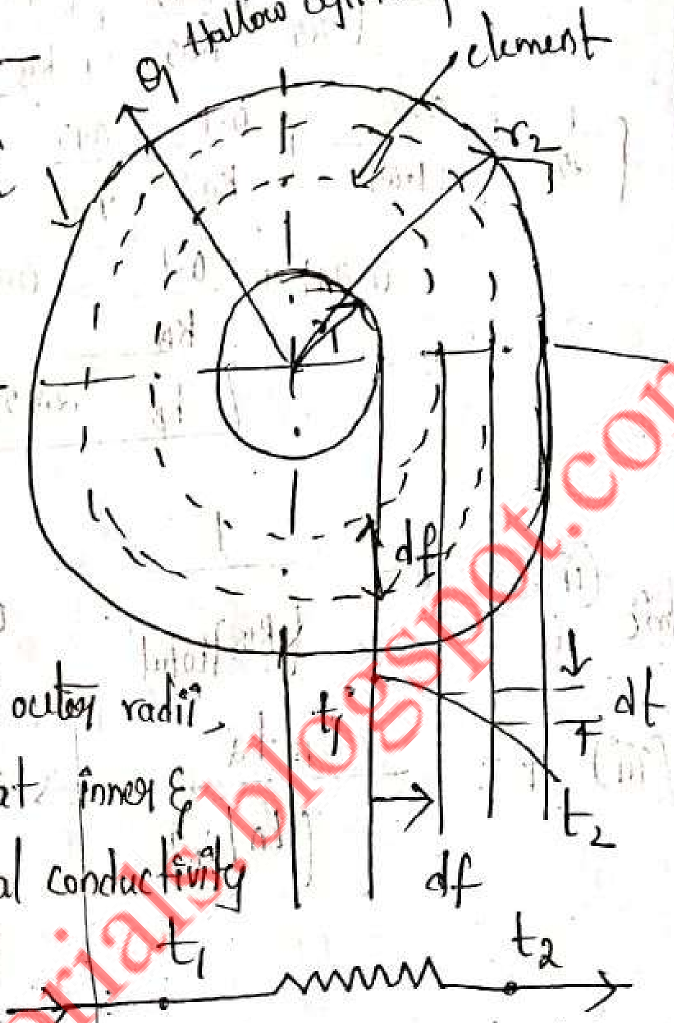
$$\boxed{t_3 = 294.96^\circ C}$$

$$q = \frac{t_3 - t_4}{\left[\frac{L_C}{k_C}\right]} \Rightarrow 3750 = \frac{294.96 - t_4}{\left[\frac{0.15}{9.2}\right]}$$

$$\boxed{t_4 = 233.81^\circ C}$$

Conduction of Heat through Hollow cylinder

(i) uniform conductivity :-



→ consider a Hallow length 'L' cylinder made of material of constant Thermal conductivity insulated at both ends

→ let r_1, r_2 are inner and outer radii, t_1, t_2 are the temp's at inner & outer surface, 'k' is thermal conductivity of a material

→ According to general heat conduction eqn. in cylindrical coordinates

$$R_{th} = \frac{\log[r_2/r_1]}{2\pi r L}$$

$$\left[\frac{\partial^2 t}{\partial r^2} + \frac{1}{r}\cdot\frac{\partial t}{\partial r} + \frac{1}{r^2} + \frac{\partial^2 t}{\partial z^2}\right] + \frac{q_g}{k} = \frac{1}{\alpha}\frac{\partial t}{\partial T}$$

→ For a steady state $\left(\frac{\partial t}{\partial T} = 0\right)$, unidirection $[t \neq \phi, z]$ with no internal heat generation

$$\frac{d^2 t}{dr^2} + \frac{1}{r}\frac{dt}{dr} = 0$$

$$\frac{1}{r}\frac{d}{dr}\left[r\cdot\frac{dt}{dr}\right] = 0$$

since $\frac{1}{r} \neq 0$; $\frac{d}{dr}\left[r.\frac{dt}{dr}\right]=0$

Integrating above eqn.

$$r.\frac{dt}{dr}=0$$

Again integrating above eqn.

$$dt = c.\frac{dr}{r}$$

$$t = c\log r + c_1 \longrightarrow (i)$$

Applying boundary conditions

At $r=r_1$, $t=t_1 \Rightarrow t_1 = c\log r_1 + c_1 \longrightarrow (ii)$

At $r=r_2$, $t=t_2 \Rightarrow t_2 = c\log r_2 + c_1 \longrightarrow (iii)$

$$t_1 - c\log r_1 - \cancel{c_1} = 0$$

$$t_2 - c\log r_2 - \cancel{c_1} = 0$$

$$- \quad + \quad +$$

$$(t_1 - t_2) + c[\log r_2 - \log r_1] = 0$$

$$c[\log r_2 - \log r_1] = -[t_1 - t_2]$$

$$\Rightarrow c = \frac{-(t_1-t_2)}{\log[r_2/r_1]}$$

sub 'c' in eqn (2)

$$\Rightarrow t_1 = c\log r_1 + c_1$$

$$t_1 = \frac{-(t_1-t_2)}{\log[r_2/r_1]} \times \log r_1 + c_1$$

$$c_1 = t_1 + \frac{(t_1-t_2)}{\log[r_2/r_1]}\log r_1$$

Sub c_2, c_1 in eqn ①

$$t = -\frac{(t_1-t_2)}{\log[r_2/r_1]}\log r + \left[t_1 + \frac{(t_1-t_2)}{\log[r_2/r_1]}\log r_1\right]$$

$$\Rightarrow t - t_1 = \frac{t_1-t_2}{\log[r_2/r_1]}[-\log r + \log r_1]$$

$$t - t_1 = \frac{t_1-t_2}{\log[r_2/r_1]}\log[r_1/r]$$

$$\boxed{\frac{t-t_1}{t_1-t_2} = \frac{\log(r_1/r)}{\log[r_2/r_1]}}$$

the above eqn is called as Temp distribution in a hallow cylinder

from the above eqn following conditions

i) Temp distribution is logarithmic

ii) Temp at any point in a cylinder can be expressed as function of radius

iii) The Temp profile is nearly for values of r_2/r_1

Destimation of heat conduction using Temp distribution.

W.K.T $Q = -KA\frac{dt}{dr}$

$$Q = -k\,2\pi r l\,\frac{d}{dr}(t)$$

$$t = t_1 + \frac{(t_1-t_2)}{\log(r_2/r_1)} - \frac{(t_1-t_2)}{\log(r_2/r_1)}\log r$$

$$Q = -2\pi K L \cdot x \frac{d}{dr}\left[t_1 + \frac{(t_1-t_2)}{\log(r_2/r_1)}\log r_1 - \frac{(t_1-t_2)}{\log(r_2/r_1)}\log r\right]$$

$$Q = -2\pi K L \cdot \cancel{r}\left[-\frac{(t_1-t_2)}{\log[r_2/r_1]}\frac{1}{\cancel{r}}\right]$$

$$\boxed{Q = 2\pi K L \frac{t_1-t_2}{\log(r_2/r_1)}} = \frac{t_1-t_2}{\left[\frac{\log(r_2/r_1)}{2\pi K L}\right]} = \frac{\Delta t}{R_{th}}$$

where $R_{th} = \dfrac{\log(r_2/r_1)}{2\pi K L}$

(ii) Variable Thermal conductivity :-

Temp variation interms of interface Temp's t_1, t_2 :-

W.K.T $\quad Q = -KA\dfrac{dt}{dr}$

$$K = k_0[1+\beta t]$$

$$Q = -k_0[1+\beta t]\,2\pi r L\frac{dt}{dr}$$

$$Q\frac{dr}{r} = -2\pi k_0 L\,[1+\beta t]\,dt$$

Integrating the above eqn from t_1 to t_2

$$Q\int_{r_1}^{r_2}\frac{dr}{r} = -2\pi k_0 L\int_{t_1}^{t_2}(1+\beta t)\,dt$$

$$Q[\log r]_{r_1}^{r_2} = -2\pi k_0 L\left[t+\beta\frac{t^2}{2}\right]_{t_1}^{t_2}$$

$$Q[\log r_2 - \log r_1] = -2\pi k_0 L\left[\left[t_2+\beta\frac{t_2^2}{2}\right]-\left(t_1+\beta\frac{t_1^2}{2}\right)\right]$$

$Q \log\left[\frac{r_2}{r_1}\right] = 2\pi k_0 L\left[(t_2-t_1)\left[1+\frac{\beta}{2}(t_2+t_1)\right]\right]$

$$Q = \frac{2\pi k_0 L}{\log(r_2/r_1)}\left[(t_2-t_1)\left[1+\frac{\beta}{2}\cdot(t_2+t_1)\right]\right] \rightarrow (i)$$

Integrating b/w r_1 and r

$$Q = \frac{2\pi k_0 L}{\log(r/r_1)}\left[(t_1-t)\left[1+\frac{\beta}{2}\cdot(t_1+t)\right]\right] \rightarrow (ii)$$

Equating eqn (i) & (ii)

$$\frac{2\pi k_0 L}{\log[r_2/r_1]}\left[(t_1-t_2)\left[1+\frac{\beta}{2}(t_1+t_2)\right]\right] = \frac{2\pi k_0 L}{\log[r/r_1]}\left[(t_1-t)\left[1+\frac{\beta}{2}\cdot(t_1+t)\right]\right]$$

$$\frac{1}{\log\left(\frac{r_2}{r_1}\right)}\left[(t_1-t_2)\left[1+\frac{\beta(t_1+t_2)}{2}\right]\right] = \frac{1}{\log\left(\frac{r}{r_1}\right)}\left[(t_1-t)\left(1+\frac{\beta}{2}(t_1+t)\right)\right]$$

$$\log(r/r_1)\left\{(t_1-t_2)\left(1+\frac{\beta(t_1+t_2)}{2}\right)\right] = \log\left(\frac{r_2}{r_1}\right)\left[(t_1-t)\left(1+\frac{\beta(t_1+t)}{2}\right)\right]$$

$$\frac{\log(r/r_1)}{\log(r_2/r_1)}\frac{(t_1-t_2)}{(t_1-t)}\left\{1+\frac{\beta(t_1+t_2)}{2}\right] = t_1-t+\frac{\beta}{2}(t_1^2-t^2)$$

$$\frac{\beta}{2}t^2+t-\frac{\beta}{2}t_1^2-t_1+\frac{\log(r/r_1)}{\log(r_2/r_1)}\left(\frac{t_1-t_2}{t_1-t}\right)\left(1+\frac{\beta(t_1+t_2)}{2}\right) = 0$$

$$t = \frac{-1 \pm \sqrt{1-4\left(\frac{\beta}{2}\right)\left(\frac{\log(r/r_1)}{\log(r_2/r_1)}\left[\frac{t_1-t_2}{t_1-t}\right]\left[1+\frac{\beta}{2}(t_1+t_2)\right]-t_1\right)}}{\cancel{2}\left(\frac{\beta}{2}\right)}$$

$$t = \frac{1}{\beta}\left[(1+\beta t_1)^2 - \frac{\log(r/r_1)}{\log(r_2/r_1)}\left[(1+\beta t_1)^2-(1+\beta t_2)^2\right]\right]^{1/2} - \frac{1}{\beta}$$

Temp variation interms of heat flux(Q) :-

W.K.T $Q = -kA\frac{dt}{dr}$

$k = k_0(1+\beta t)$

$Q = -k_0[1+\beta t]\frac{dt}{dr} 2\pi r L$

$Q \cdot \frac{dr}{r} = -2\pi k_0 L(1+\beta t)\,dt$

Integrating the above eqn.

$Q \int dr/r = -2\pi k_0 L \int (1+\beta t)\,dt$

$Q \log r = -2\pi k_0 L \left[t + \frac{\beta t^2}{2}\right] + C$

Apply Boundary Conditions at $r = r_1$; $t = t_1$

$Q \log r_1 = -2\pi k_0 L\left[t_1 + \frac{\beta t_1^2}{2}\right] + C$

$C = Q\log r_1 + 2\pi k_0 L\left[t_1 + \frac{\beta t_1^2}{2}\right]$

Sub 'c' value in above eqn.

$Q\log r = -2\pi k_0 L\left[t + \frac{\beta t^2}{2}\right] + Q\log r_1 + 2\pi k_0 L\left[t_1 + \frac{\beta t_1^2}{2}\right]$

$Q\log r + 2\pi k_0 L\left(t + \frac{\beta t^2}{2}\right) = Q\log r_1 + 2\pi k_0 L\left[t_1 + \frac{\beta t_1^2}{2}\right]$

$Q\log\left(\frac{r}{r_1}\right) = 2\pi k_0 L\left[\left[t_1 + \frac{\beta t_1^2}{2}\right] - \left(t + \frac{\beta t^2}{2}\right)\right]$

$\frac{Q\log(r/r_1)}{2\pi k_0 L} = t_1 + \frac{\beta t_1^2}{2} - t - \frac{\beta t^2}{2}$

$\frac{\beta t^2}{2} + t - \frac{\beta t_1^2}{2} + Q\frac{\log(r/r_1)}{2\pi k_0 L} - t_1 = 0$

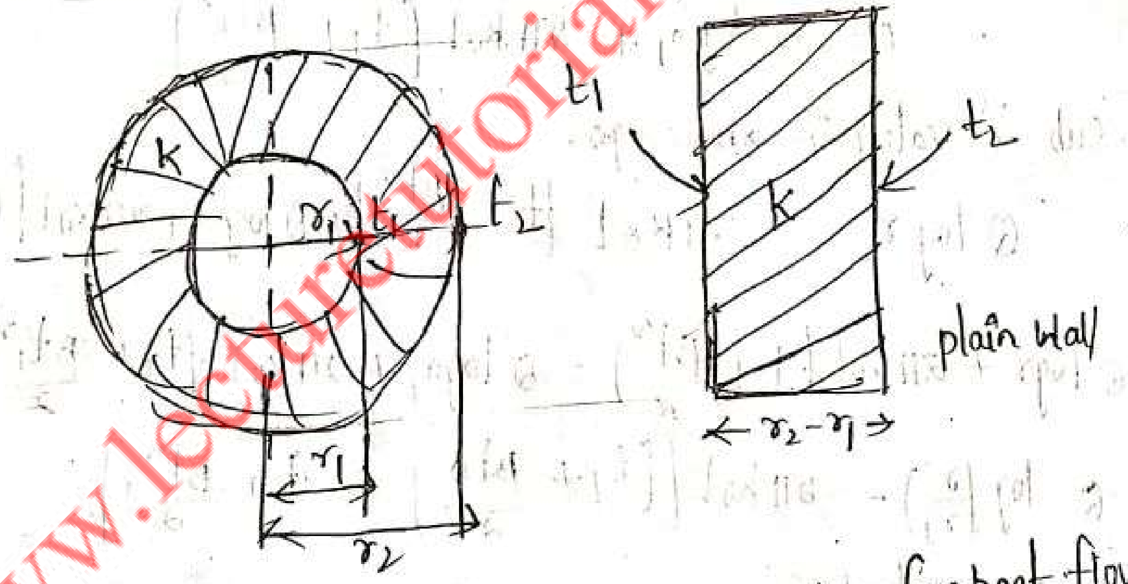
$$t = \frac{-1 \pm \sqrt{1 - 4\left(\frac{\beta}{2}\right)\left[\frac{Q\log(r/r_1)}{2\pi k_0 L} - \left[t_1 + \frac{\beta t_1^2}{2}\right]\right]}}{\cancel{2}\left(\frac{\beta}{\cancel{2}}\right)}$$

$$t = -\frac{1}{\beta} + \sqrt{\frac{1 - 2\beta\left(\frac{Q\log(r/r_1)}{2\pi k_0 L} - \left(t_1 + \frac{\beta t_1^2}{2}\right)\right)}{\beta^2}}$$

$$t = \frac{-1}{\beta} + \sqrt{\frac{1}{\beta^2} - \frac{2}{\beta}\,\frac{Q\log(r/r_1)}{2\pi k_0 L} + \frac{2}{\beta}\left(t_1 + \frac{\beta t_1^2}{2}\right)}$$

$$\boxed{t = \frac{-1}{\beta} + \sqrt{\frac{1}{\beta^2} - \frac{1}{\beta}\,\frac{Q\log(r/r_1)}{\pi k_0 L} + \frac{2}{\beta}\left(t_1 + \frac{\beta t_1^2}{2}\right)}}$$

Logerthermic area in a Hallow cylinder :-



→ It is convenient to have an expression for heat flow through a hallow cylinder is same as that of plain wall

→ $r_2 - r_1$ be the thicknen of plain wall

→ Am is the equivalent of mean area

W.K.T $Q_{cylinder} = \frac{(t_1-t_2)}{\log(r_2/r_1) / 2\pi KL}$

$$Q_{plain\,wall} = \frac{t_1-t_2}{(r_2-r_1/k.A_m)}$$

equating the eqns

$$\frac{\cancel{(t_1-t_2)}}{\left[\frac{\log(r_2/r_1)}{2\pi \cancel{K} L}\right]} = \frac{\cancel{(t_1-t_2)}}{\frac{(r_2-r_1)}{\cancel{K}.A_m}}$$

$$\frac{2\pi L}{\log(r_2/r_1)} = \frac{A_m}{r_2-r_1}$$

$$\Rightarrow A_m = \frac{2\pi L (r_2-r_1)}{\log(r_2/r_1)} = \frac{2\pi L r_2 - 2\pi L r_1}{\log\left(\frac{2\pi L r_2}{2\pi L r_1}\right)}$$

$$\boxed{A_m = \frac{A_0 - A_i}{\log(A_0/A_i)}}$$

Area of cylinder $A = 2\pi r L$

$A_0 = 2\pi r_2 L$

$A_i = 2\pi r_1 L$

Mean radius of a hallow cylinder :-

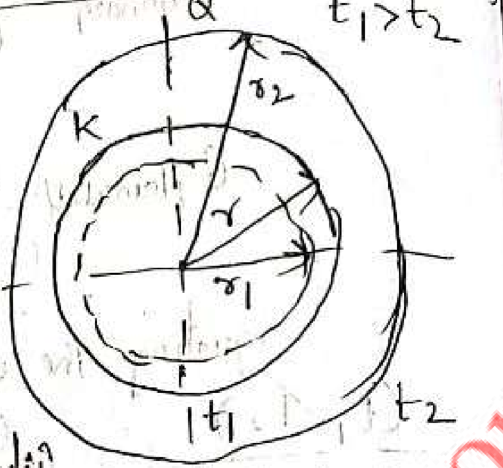
$$A_m = 2\pi r_m L = 2\pi L (r_m)$$

$$A_m = 2\pi L \times \left[\frac{(r_2-r_1)}{\log(r_2/r_1)}\right]$$

$$r_m = \frac{(r_2-r_1)}{\log(r_2/r_1)}$$

Heat conduction through a hollow sphere (heat flows outwards)

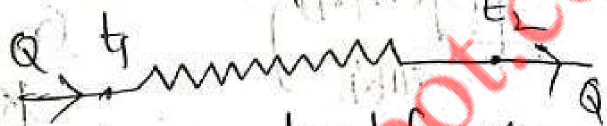
(i) uniform conductivity :-



→ Consider an hollow sphere made of material of uniform conductivity

→ r_1 & r_2 are the outer and inner radii

t_1, t_2 are the Temperatures



K is the thermal conductivity of given material for given Temp

→ the general heat conduction eqn for spherical coordinates is

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left[r^2\frac{\partial t}{\partial r}\right]+\frac{1}{r^2\sin^2\theta}\frac{\partial^2 t}{\partial \phi^2}+\frac{1}{r^2\sin\theta}\cdot\frac{\partial}{\partial\theta}\left(\sin\theta\cdot\frac{\partial t}{\partial\theta}\right)$$

$$+\frac{q_g}{k}=\frac{1}{\alpha}\cdot\frac{\partial t}{\partial T}$$

For a steady state $\frac{\partial t}{\partial T}=0$, unidirectional flow [radial direction] and there is no heat generation inside the system

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left[r^2\frac{\partial t}{\partial r}\right]=0$$

since $\frac{1}{r^2}\neq 0$; $\frac{\partial}{\partial r}\left[r^2\frac{\partial t}{\partial r}\right]=0$

$$\Rightarrow \frac{\partial}{\partial r}\left[r^2\frac{dt}{dr}\right]=0 \longrightarrow (i)$$

Integrating eqn (i)

$$r^2\frac{dt}{dr}=c \Rightarrow dt=\frac{c\cdot dr}{r^2}$$

Again Integrating

$$\int dt = \int c \frac{dr}{r^2}$$

$$t = c\left[\frac{-1}{r}\right] + c_1 \rightarrow \text{(ii)}$$

Apply boundary conditions At $r = r_1$; $t = t_1$

$$t_1 = c\left[\frac{-1}{r_1}\right] + c_1 \Rightarrow t_1 + \frac{c}{r_1} - c_1 = 0 \longrightarrow (3)$$

$$r = r_2,\ t = t_2 \Rightarrow t_2 = c\left(\frac{-1}{r_2}\right) + c_1$$

$$\Rightarrow t_2 + \frac{c}{r_2} - c_1 = 0 \longrightarrow (4)$$

(3) − (4)

$$t_1 + \frac{c}{r_1} - \cancel{c_1} = 0$$

$$t_2 + \frac{c}{r_2} - \cancel{c_1} = 0$$

$$(t_1 - t_2) + c\left[\frac{1}{r_1} - \frac{1}{r_2}\right] = 0$$

$$(t_1 - t_2) = \frac{c}{r_2} - \frac{c}{r_1} \Rightarrow \left[\frac{1}{r_2} - \frac{1}{r_1}\right]c$$

$$(t_1 - t_2) = \left[\frac{r_1 - r_2}{r_1 r_2}\right] c$$

$$\boxed{c = \frac{(t_1 - t_2)\, r_1 r_2}{(r_1 - r_2)}}$$

eqn (iii) $\Rightarrow$ $$t_1 + \frac{\frac{(t_1 - t_2)\, r_1 r_2}{(r_1 - r_2)}}{r_1} - c_1 = 0$$

$$t_1 + \frac{(t_1 - t_2)\, \cancel{r_1} r_2}{(r_1 - r_2)\cancel{r_1}} - c_1 = 0$$

$$\Rightarrow \boxed{c_1 = t_1 + \frac{(t_1-t_2)\, r_2}{(r_1-r_2)}}$$

eqn (ii) $\Rightarrow$

$$t = -\frac{(t_1-t_2)\, r_1 r_2}{r\,(r_1-r_2)} + t_1 + \frac{(t_1-t_2)\, r_2}{(r_1-r_2)}$$

$$\Rightarrow (t-t_1) = \frac{(t_1-t_2)\, r_2}{(r_1-r_2)}\left[-\frac{r_1}{r}+1\right]$$

$$\boxed{\frac{(t-t_1)}{(t_1-t_2)} = \frac{r_2\,(r-r_1)}{r\,[r_1-r_2]}}$$

Heat flux $(Q) = -KA\dfrac{dt}{dr}$

$$Q = -K\cdot 4\pi r^2 \frac{d}{dr}\left[-\frac{(t_1-t_2)\, r_1 r_2}{r\,[r_1-r_2]} + t_1 + \frac{(t_1-t_2)\, r_2}{(r_1-r_2)}\right]$$

$$Q = -K\, 4\pi r^2 \left[\frac{\cancel{-}\,(t_1-t_2)\, r_1 r_2}{\cancel{-}\,(r_2-r_1)}\left(\frac{-1}{r^2}\right) + 0 + 0\right]$$

$$Q = -K\, 4\pi \cancel{r^2}\left[\frac{(t_1-t_2)\, r_1 r_2}{(r_2-r_1)}\left(\frac{-1}{\cancel{r^2}}\right)\right]$$

$$Q = \frac{K\, 4\pi\, (t_1-t_2)\, r_1 r_2}{(r_2-r_1)}$$

$$Q = \frac{(t_1-t_2)}{\left[\dfrac{(r_2-r_1)}{4\pi K r_1 r_2}\right]}$$

$$\boxed{Q = \frac{\Delta t}{[R_{th}]_{overall}}}$$

(a) Variable Thermal Conductivity

Temp distribution for the Interfaces t_1 & t_2

$$Q = -KA \cdot \frac{dt}{dr}$$

$$K = k_0 [1+\beta t]$$

$$Q = -k_0 [1+\beta t] A \frac{dt}{dr} = -k_0 [1+\beta t] 4\pi r^2 \frac{dt}{dr}$$

$$Q \frac{dr}{r^2} = -4\pi k_0 [1+\beta t] \cdot dt$$

Integrating from r_1 to r_2 & t_1 to t_2

$$\int Q \frac{dr}{r^2} = -\int 4\pi k_0 (1+\beta t)\, dt$$

$$\left[\frac{-Q}{r}\right]_{r_1}^{r_2} = -4\pi k_0 \left[t + \frac{\beta t^2}{2}\right]_{t_1}^{t_2}$$

$$Q\left[\frac{-1}{r_2} + \frac{1}{r_1}\right] = -4\pi k_0 \left[(t_2 - t_1) + \frac{\beta}{2}(t_2^2 - t_1^2)\right]$$

$$Q\left[\frac{r_2 - r_1}{r_1 r_2}\right] = -4\pi k_0 \left[(t_2 - t_1)\left[1 + \frac{\beta}{2}(t_2 + t_1)\right]\right]$$

$$Q = \frac{-4\pi k_0 (r_1 r_2)}{(r_2 - r_1)} (t_2 - t_1)\left[1 + \frac{\beta}{2}(t_2 + t_1)\right]$$

$$Q = \frac{4\pi k_0\, r_1 r_2}{(r_2 - r_1)} \left[(t_1 - t_2)\left[1 + \frac{\beta}{2}(t_2 + t_1)\right]\right] \rightarrow (i)$$

Integrating b/w r_1 & r

$$Q = \frac{4\pi k_0\, r_1 r}{(r - r_1)} \left[(t_1 - t)\left[1 + \frac{\beta}{2}(t + t_1)\right]\right] \rightarrow (ii)$$

equating (i) & (ii)

$$\frac{4\pi k_0 r_1 r_2}{(r_2-r_1)}\left[(t_1-t_2)\left[1+\frac{\beta}{2}(t_1+t_2)\right]\right] = \frac{4\pi k_0 r_1 r_0}{(r-r_1)}\left[(t_1-t)\left[1+\frac{\beta}{2}(t+t_1)\right]\right]$$

$$\frac{r_1 r_2}{r_2-r_1}\left[(t_1-t_2)\left[1+\frac{\beta}{2}(t_2+t_1)\right]\right] = \frac{r r_1}{r-r_1}\left[(t_1-t)\left[1+\frac{\beta}{2}(t+t_1)\right]\right]$$

$$\frac{r_2(t_1-t_2)}{r_2-r_1}\left[1+\frac{\beta}{2}(t_2+t_1)\right] = \frac{r(t_1+t_1)}{r-r_1}\left[1+\frac{\beta}{2}(t+t_1)\right]$$

$$t = \frac{1}{\beta}\left[[1+\beta t]^2 - \frac{(r-r_1)}{(r_2-r_1)}\left(\frac{r_2}{r_0}\right)\left[(1+\beta t_1)^2-(1+\beta t_2)^2\right]\right]^{1/2} - \frac{1}{\beta}$$

Temp variation interms of Heat flow $Q = -KA\frac{dt}{dr}$

$$K = k_0[1+\beta t]$$

$$Q = -k_0(1+\beta t)\, 4\pi r^2 \frac{dt}{dr}$$

$$Q \cdot \frac{dr}{r^2} = -k_0(1+\beta t) \cdot 4\pi\, dt$$

Integrating the eqn

$$Q\left[\frac{-1}{r}\right] = -4\pi k_0\left[t+\frac{\beta t^2}{2}\right] + C$$

Apply boundary conditions At $r=r_1$; $t=t_1$

$$Q\left[\frac{-1}{r_1}\right] = -4\pi k_0\left[t_1+\frac{\beta t_1^2}{2}\right] + C$$

$$C = 4\pi k_0\left[t_1+\frac{\beta t_1^2}{2}\right] - \frac{Q}{r_1}$$

Sub 'c' in above eqn

$$-\frac{Q}{r} = -4\pi k_0\left[t+\frac{\beta t^2}{2}\right] + 4\pi k_0\left[t_1+\frac{\beta t_1^2}{2}\right] - \frac{Q}{r_1}$$

$$4\pi k_0\left[t+\frac{\beta t^2}{2}\right] + \frac{Q}{r_1} = 4\pi k_0\left(t_1+\frac{\beta t_1^2}{2}\right) + \frac{Q}{r}$$

$$\frac{Q}{r_1}-\frac{Q}{r}=4\pi k_0\left[t_1\left(1+\frac{\beta t_1^2}{2}\right)-t\left(1+\frac{\beta t^2}{2}\right)\right]$$

$$\frac{Q(r-r_1)}{r r_1 4\pi k_0}=t_1+\frac{\beta t_1^2}{2}-t-\frac{\beta t^2}{2}$$

$$\frac{\beta t^2}{2}+t-t_1-\frac{\beta t_1^2}{2}+\frac{Q(r-r_1)}{r r_1 4\pi k_0}=0$$

$$t=\frac{-1\pm\sqrt{1-\cancel{4}^{2}\left(\frac{\beta}{\cancel{2}}\right)\left[\frac{Q(r-r_1)}{r r_1 4\pi k_0}-t_1-\frac{\beta t_1^2}{2}\right]}}{\cancel{2}(\beta/\cancel{2})}$$

$$t=-\frac{1}{\beta}+\left\{\left[t_1-\frac{1}{\beta}\right]^2-\frac{Q}{\beta k_0}\cdot\frac{1}{2\pi}\left(\frac{1}{r_1}-\frac{1}{r}\right)\right\}^{1/2}$$

Logarithmic mean area of hallow sphere :-

$$Q_{spherical}=\frac{(t_1-t_2)}{\left[\frac{(r_2-r_1)}{4\pi k r_1 r_2}\right]}$$

$$Q_{spherical}=\frac{(t_1-t_2)}{\left(\frac{(r_2-r_1)}{k\cdot A_m}\right)}$$

$$\frac{\cancel{(t_1-t_2)}}{\left(\frac{r_2-r_1}{4\pi k r_1 r_2}\right)}=\frac{\cancel{(t_1-t_2)}}{\frac{(r_2-r_1)}{k A_m}}$$

$$\frac{4\pi k r_1 r_2}{\cancel{(r_2-r_1)}}=\frac{k A_m}{\cancel{(r_2-r_1)}}$$

$$\cancel{k}A_m=4\pi\cancel{k}r_1 r_2$$

$$A_m=4\pi r_1 r_2$$

$$A_m^2=(4\pi r_1^2)\left[4\pi r_2^2\right]$$

$$A_m^2 = A_i A_o$$

$$\boxed{A_m = \sqrt{A_i \cdot A_o}}$$

$$A_m = 4\pi r_1 r_2$$

$$\cancel{4\pi} r_m^2 = \cancel{4\pi} r_1 r_2 \Rightarrow r_m = \sqrt{r_1 r_2}$$

Insulation:- A Material which retards the flow of heat is called as insulation.

It prevents the flow of heat from system to surroundings and viceversa

Ex:- Boilers & steam pipes, air conditioning, Refrigerators, food preservation stores, furnaces -- etc.

Critical Thickness of Insulation:-

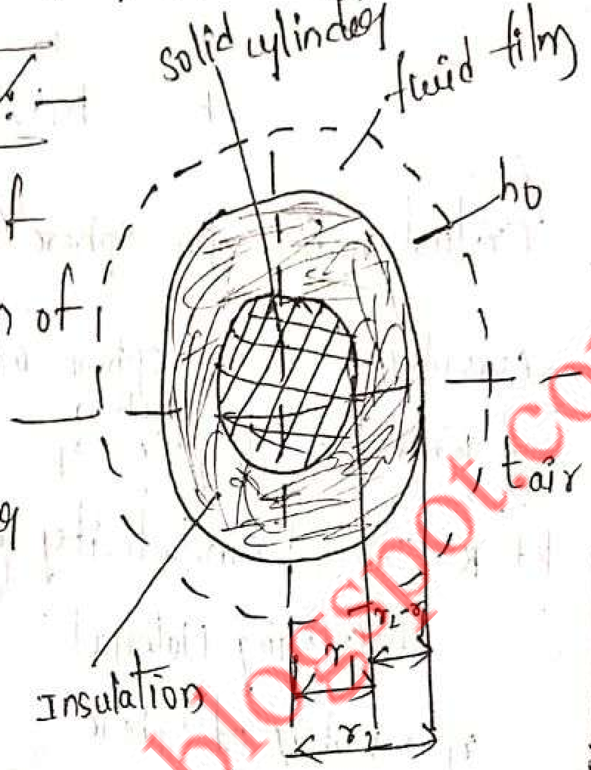
→ Addition of insulation always increases The conductive Thermal resistance but when the total thermal resistence is made of conductive Thermal resistence.

→ The addition of insulation in same cases may reduce The convective thermal resistence due to increase in surface area as in the case of cylinder & sphere and as a result total Thermal resistence decreases resulting increase in heat flow rate.

→ The Thickness upto which Heat flow increase and after which heat flow decreases is called as critical thickness

→ The insulation radius at which resistance in heat flow rate which is min. is called as critical radius

Critical radius for cylinder :-



→ consider a solid cylinder of radius 'r_1' with an insulation of thickness '$r_2 - r_1$'

let L – length of solid cylinder

k – thermal conductivity

t_1 – Temp of solid cylinder

t_{air} – Temp of outer surface of a insulation

h_0 – heat Transfer coefficient of outer surface

∴ Heat Transfer ~~coefficient~~ rate from surface to surrounding is

$$Q = \frac{2\pi L (t_1 - t_{air})}{\frac{\log(r_2/r_1)}{k} + \frac{1}{h_0 r_2}}$$

→ as r_2 increases $\frac{\log(r_2/r_1)}{k}$ also increases and $\frac{1}{h_0 r_2}$ decreases

→ when Q_{max} denominator should be minimum

$$\frac{d}{dr_2}\left[\frac{\log(r_2/r_1)}{k} + \frac{1}{h_0 r_2}\right] = 0$$

$$\frac{1}{k}\left[\frac{1}{r_2} - 0\right] + \frac{1}{h_0}\left[-\frac{1}{r_2^2}\right] = 0$$

$$\frac{1}{k r_2} - \frac{1}{h_0 r_2^2} = 0$$

$$\frac{1}{K\cancel{r_2}} = \frac{1}{h_0 r_2^2}$$

$$\Rightarrow \frac{1}{k} = \frac{1}{h_0 r_2} \Rightarrow \boxed{r_2 = \frac{k}{h_0}}$$

Critical radius for sphere :-

Consider a Hallow sphere of radius 'r_1' with insulating thickness $r_2 - r_1$

let k – Thermal conductivity of Insulating Material

r_1 = radius of sphere

r_2 = outer radius

Heat flow rate $Q = \dfrac{t_1 - t_{air}}{\dfrac{(r_2 - r_1)}{4\pi k r_1 r_2} + \dfrac{1}{4\pi r_2^2 h_0}}$

→ when Q_{max} the denominator should be minimum

$$\frac{d}{dr_2}\left[\frac{r_2 - r_1}{4\pi k r_1 r_2} + \frac{1}{4\pi r_2^2 h_0}\right] = 0$$

$$\frac{d}{dr_2}\left[\frac{\cancel{r_2}}{4\pi k r_1 \cancel{r_2}} - \frac{\cancel{r_1}}{4\pi k \cancel{r_1} r_2}\right] + \frac{d}{dr_2}\left[\frac{1}{4\pi r_2^2 h_0}\right] = 0$$

$$0 - \frac{1}{4\pi k}\left[\frac{-1}{r_2^2}\right] + \frac{1}{4\pi h_0}\left[\frac{-2}{r_2^3}\right] = 0$$

$$\frac{1}{4\pi k r_2^2} - \frac{\cancel{2}}{\cancel{4}\pi h_0 r_2^3} = 0$$