Dimensional Analysis

Dimensionless Numbers (or) Non dimension Numbers :-

(i) Nusselt Number :- It is the ratio of heat flow rate by convection process under a unit Temp gradient to heat flow rate by conduction process under a unit temp gradient of stationary thickness 'l' . [convient measure method of convective heat transfer coeff of (h)]

$$Nu = \frac{Q_{conv}}{Q_{cond}} = \frac{h \cdot A \cdot dt}{k \cdot A \cdot \frac{dt}{dx}} = \frac{hL}{k}$$

$\boxed{h \propto k}$ $h \propto \frac{1}{l}$

(ii) Raynold's Number :- It is the ratio of Inertial force to viscous force

$$Re = \frac{F_i}{F_v}$$

Note :- for a laminar flow 'Re' value is less than 2000

For a turbulent flow Re value is 4000 and above.

$$= \frac{\rho v^2 L^2}{\mu v L} = \frac{\rho v L}{\mu} = \frac{vL}{\nu} \quad \left(\because \frac{\rho}{\mu} = \nu\right)$$

(iii) Parndttl number :- It is the ratio of kinematic viscosity to thermal diffusivity

$$Pr = \frac{\nu}{\alpha} = \frac{\nu \cdot \rho \cdot C_p}{k} = \frac{\mu \rho C_p}{\rho k} = \frac{\mu C_p}{k}$$

→ It is a measure of relative effectiveness of the momentum and energy transfer by diffusion

Assignment Questions

IV

1) Determine the heat transfer rate from the rectangular fin of length 20 cm width 40 cm & thickness 2 cm. The tip of fin is not insulated and the fin has thermal conductivity of 150 W/m K the base Temp is 100°c and the fluid is 20°c the heat Transfer coefficient b/w the fin and fluid is 30 W/m²k

2] A Wall of furnance is made up of inside layer of silica brick 120mm thickness provide with a layer of magnesite brick 240 mm Thick. the Temp at the inside surface of a silica brick wall and outside surface of a magnesite brick wall are 725 and 110°c the contact thermal resistence b/w the two walls at the interface is 0.0035 °c per unit wall. per unit wall area. if the thermal conductivity k_s - 1.7 W/m°c, k_m - 5.8 W/m°c, calculate rate of heat loss per unit area of wall., The Temp drop at interface

(iv) Stanton Number:-

It is the ratio of heat transfer coefficient to rate of heat flow per unit Temperature raise due to velocity of fluid

$$St = \frac{h}{\rho v c_p} = \frac{\frac{hL}{k}}{\rho v c_p \left(\frac{L}{k}\right)} = \frac{Nu}{Re \times Pr}$$

$$\times = \rho v L \mu \, c_p / \mu k$$

$$= \left(\frac{\rho v L}{\mu}\right) \cdot \left(\frac{\mu c_p}{L}\right)$$

(v) Peclet Number :-

It is the ratio of mass heat flow rate by convection to mass heat flow rate by conduction for unit Temp gradient for a thickness i.e

$$(P_e) = \frac{Q_{conv}}{Q_{cond.}} = \frac{\rho v C_p}{\frac{K}{L}} \qquad \left(\because \alpha = \frac{K}{\rho C_p}\right)$$

$$(P_e) = \frac{v \cdot L}{\alpha}$$

$$P_e = R_e \times P_r$$

(vi) Graetz Number :-

It is the ratio of heat capacity of the fluid flowing through the pipe per unit length of a pipe to the conductivity of the pipe

$$G = \frac{\frac{m \cdot C_p}{L}}{K} = \frac{\frac{\rho \times V \times C_p}{L}}{K} = \frac{\rho \times V \times C_p}{LK}$$

$$G = \frac{\rho \times \frac{\pi}{4} D^2 \times v \times C_p}{KL}$$

$$G = \frac{\pi}{4} D^2 \cdot v \cdot \rho \cdot \frac{C_p}{KL}$$

$$= \frac{\pi}{4} \rho v D \frac{C_p}{K} \cdot \frac{D}{L} \times \frac{\mu}{\mu}$$

$$= \frac{\pi}{4} \frac{\rho v D}{\mu} \times \frac{\mu C_p}{K} \cdot \frac{D}{L}$$

$$\boxed{G = \frac{\pi}{4} \times R_e \times P_r \times \frac{D}{L}}$$

$$G = \frac{\pi}{4} D^2 \times \frac{P_e}{L}$$

(viii) Grashoff number (Gr)

→ Relates with Natural convection heat transfer

It is the ratio of product of inertia force and buoyant force to square of the viscous force

$$Gr = \frac{[\text{Inertia force}][\text{Buoyant force}]}{[\text{Viscous force}]^2}$$

$$= \frac{[\rho v^2 L^2](\rho \beta g \Delta t L^3)}{[\mu v L]^2}$$

$$= \frac{\rho v^2 L^2 [\rho \beta g \Delta t L^3]}{\mu^2 v^2 L^2}$$

$$\tau = \frac{F}{A}$$

$$= \mu \cdot \frac{du}{dy}$$

$$= \mu L^2 \frac{v}{L}$$

$$= \mu v L$$

$$F_i = ma$$

$$= \rho V \cdot \frac{L}{sec^2}$$

$$= \rho L^2 \cdot L \cdot \frac{L}{sec^2}$$

$$= \rho v^2 L^2$$

m/sec²

Dimension Analysis :-

Dimension Analysis is an experimental investigation to solve simple Engineering problems

length → m → L

Mass → kg → M

Time → sec → T

Temp → °c → θ

Density, Angular velocity, Area, volume, Angular acceleration, Acceleration, Discharge, Kinematic viscosity, Dynamic viscosity, Force, power, Work, specific weight, Thermal conductivity, Heat, specific heat, heat transfer coefficient, Work $\Rightarrow W - F \cdot S \Rightarrow MLT^{-2}L$

$= ML^{2}T^{-2}$

Specific volume $-$ $\frac{\text{Weight}}{\text{Volume}} = \frac{N}{m^3} \Rightarrow \frac{MLT^{-2}}{L^{3}}$

$= ML^{-2}T^{-2}$

Thermal Conductivity $-$ K $-$ W/m°C

$= ML^{2}T^{-3}L^{-1}\theta^{-1}$

$= MLT^{-3}\theta^{-1}$

Heat $\rightarrow$ P.E / K.E.

Specific heat $\rightarrow$ $Q = mc_p\Delta t \Rightarrow c_p = \frac{Q}{m \cdot \Delta t} = L^{2}T^{-2}\theta^{-1}$

heat Transfer coefficient $\rightarrow h \rightarrow W/m^{2}\,°C$

$h = \frac{Q}{A \cdot \Delta t}$

$= \frac{ML^{2}T^{-2}}{L^{2}\theta}$

$= MT^{-2}\theta^{-1}$

$= \cancel{ML^{2}T^{-3}L^{-2}}\theta^{-1}$

$= \cancel{MT^{-3}}\theta^{-1}$

imp

Dimensional homogeneity :-

If the ~~variables~~ dimensions of ~~equation~~ the variables on both sides of equation are equal Then it said to be dimensional homogeneity

Work $= W = F \cdot S \Rightarrow MLT^{-2}L$

$= ML^{2}T^{-2}$

Specific volume $= \frac{\text{Weight}}{\text{Volume}} = \frac{N}{m^3} \Rightarrow \frac{MLT^{-2}L^{-3}}{}$

$= ML^{-2}T^{-2}$

Thermal conductivity $- K - W/m°C$

$= ML^{2}T^{-3}L^{-1}\theta^{-1}$

$= MLT^{-3}\theta^{-1}$

Heat $\rightarrow$ P.E / K.E.

Specific heat $\rightarrow Q = mC_p\Delta t \Rightarrow C_p = \frac{Q}{m \cdot \Delta t} = L^{2}T^{-2}\theta^{-1}$

heat transfer coefficient $\rightarrow h \rightarrow W/m^{2}°C$

$= ML^{2}T^{-3}L^{-2}\theta^{-1}$

$= MT^{-3}\theta^{-1}$

$h = \frac{Q}{A \cdot \Delta t}$

$= \frac{ML^{2}T^{-2}}{L^{2}\theta}$

$= MT^{-2}\theta^{-1}$

Imp

Dimensional homogeneity :-

If the dimensions of the variables on both sides of equation are equal then it said to be dimensional homogeneity

Applications:-

→ It facilitates to determine the dimension of a physical quantity

→ It helps to check whether the eqn is dimensionally homogeneous (or) not.

→ It converts units from one system to another

Dimensional Analysis ~~with the help of~~ :-

With the help of dimensional analysis the equation of physical phenomenona can be developed interms of dimensionless numbers or parameters. There are two methods.

→ Rayleigh's method

→ Buckingham's pie method

Buckingham pie method :-

This method designated /represented dimensionless group has 'π' terms.

Defination :- If there are 'n' variables [dependent [independent] in a dimensionally homogeneous equation, and if these variables contains 'm' fundamental dimensions [M, L, T] then the variables are arranged in (n-m) dimensionless term. These dimensional terms are called as π terms.

Mathematically, If x_1 dependent on $x_2, x_3, x_4, \ldots x_n$ Then the functional eqn can be written as

$$x_1 = f[x_2, x_3, x_4, \ldots x_n] \longrightarrow (i)$$

[ii]

$\Rightarrow f[x_1, x_2, x_3, \ldots\ldots x_n] = 0$

If is a dimensionally homogeneous equation containing 'n' variables

If There are 'm' fundamental dimensions Then According

Buckinghumpie therom

$$f_1[\pi_1, \pi_2, \pi_3 \ldots\ldots \pi_{(n-m)}] = 0$$

Each dimensional less pie Term is formed by combining 'm' variables out of total 'n' variables with one of The remaining (n-1) Term

i.e each pie Term has (m+1) Terms

These 'm' variable which appears repeatedly in a π Term are called as Repeated Variables.

let $x_1, x_2, x_3 \ldots\ldots x_n$ are the variables in a homogeneous equation Assume x_2, x_3, x_4 are the repeated variables and fundamental dimensions are [M, L, T] which is equal to

Then each colum is can be written as

$$\pi_1 = x_2^{a_1} \cdot x_3^{b_1} \cdot x_4^{c_1} \cdot x_1$$

$$\pi_2 = x_2^{a_2} \, x_3^{b_2} \, x_4^{c_2} \, x_5$$

$$\pi_{(n-m)} = x_2^{a_{n-m}} \cdot x_3^{b_{n-m}} \cdot x_4^{c_{n-m}} \cdot \ldots \, x_n$$

Where a_1, b_1, c_1 are Constants That are determined by using dimensional homogenity

(i) These values sub (ii) above eqns. There by $\pi_1, \pi_2, \pi_3 \ldots \pi_{n-1}$ are obtained.

the final general eqn for the phenomenon by expressing any one of π Terms as the function of the other,

$$\therefore \left| \begin{array}{l} \pi_1 = \phi(\pi_2, \pi_3, \pi_4 \cdots \pi_{n-m}) \\ \pi_2 = \phi(\pi_1, \pi_3, \pi_4 \cdots \pi_{n-m}) \end{array} \right.$$

Selection of repeating variables :-

→ m repeating variable must contain jointly all the fundamental dimensions involved in the phenomenona.

→ The repeating variables must not form the non dimensional parameters among themselves,

→ As far as possible the depdant variable shouldn't be selected as repeating variable.

→ No two repeating variables should have same dimensions.

→ The repeating variable should be choosen so that one variable contains geometry property and other variable contains flow property and third variable is a fluid property.

the choice of repeating variables in most of the fluid mechanics problems may be (L, V, ρ); (D, V, ρ); $[L, V, \mu]$: $[D, V, \mu]$

① the resistence are exprienced by a partially submerged body depends upon velocity 'v' length of the body 'l' viscosity of fluid 'μ' density of fluid 'ρ' and gravitational acceleration 'g' obtain a dimensionless expression for 'R'

Step1:- 1) Resistence 'R'

2) velocity 'V'

3) length 'l'

4) viscosity 'μ'

5) density 'ρ'

6) gravitational acceleration 'g'

no.of variables n=6

$$R = f[V, l, \mu, \rho, R, g]$$

$$f_1[R, V, l, \mu, \rho, g] = 0$$

$R - MLT^{-2}$

$V = LT^{-1}$

$L = L$

$\mu = ML^{-1}T^{-1}$ m = fundamental terms = 3 [M, L, T]

$\rho = ML^{-3}$

$g = LT^{-2}$

No. of dimension π Terms $(n-m) = 6-3 = 3$

∴ π terms are π_1, π_2, π_3

$$f_1[\pi_1, \pi_2, \pi_3] = 0.$$

step:-2

Selection of repeating variables

∴ R is depedent variable shouldn't be selected as repeating variable

∴ velocity, length & density are selected as repeating step 3:- $\pi_1 = l^{a_1} V^{b_1} \rho^{c_1} R$

$$\pi_2 = l^{a_2} V^{b_2} \rho^{c_2} \mu$$

$$\pi_3 = l^{a_3} \cdot V^{b_3} \cdot \rho^{c_3} \cdot g$$

step 4:- Each π term is solved by principle of dimensional homogenity

~~For M =~~ $\pi_1 = l^{a_1} \cdot V^{b_1} \cdot \rho^{c_1} \cdot R.$

~~For M →~~ $M^0 L^0 T^0 = [L]^{a_1} \cdot [LT^{-1}]^{b_1} \cdot [ML^{-3}]^{c_1} [MLT^{-2}]$

For M, $\quad 0 = M^{c_1} \cdot M^1$

$$= c_1 + 1$$

$$\boxed{c_1 = -1}$$

For L, $\quad L^0 = L^{a_1} \cdot L^{b_1} \cdot L^{-3c_1} \cdot L^1$

$$0 = a_1 + b_1 - 3c_1 + 1$$

For T, $\quad T^0 = T^{-b_1} \cdot T^{-2}$

$$0 = -b_1 - 2 \Rightarrow \boxed{b_1 = -2}$$

$$0 = a_1 - 2 - 3[-1] + 1$$

$$\boxed{a_1 = 2}$$

$\Pi_2 = l^{a_2} \cdot V^{b_2} \cdot \rho^{c_2} \cdot \mu$

$M^0 L^0 T^0 = L^{a_2} \cdot [LT^{-1}]^{b_2} \cdot [ML^{-3}]^{c_2} ML^{-1}T^{-1}$

For M, $M^0 = M^{c_2} \cdot M^1$

$0 = c_2 + 1$

$\boxed{c_2 = -1}$

For L, $L^0 = L^{b_2} \cdot L^{-3c_2} \cdot L^{-1} \cdot L^{a_2}$

$0 = b_2 - 3c_2 - 1 + a_2$

For T, $T^0 = T^{-b_2} \cdot T^{-1}$

$\boxed{b_2 = -1}$

$0 = \overset{-1}{b_2} - 3(-1) - 1 + a_2$

$\boxed{a_2 = -1}$

$\Pi_3 = l^{a_3} \cdot V^{b_3} \cdot \rho^{c_3} \cdot g$

$M^0 L^0 T^0 = L^{a_3} \cdot [LT^{-1}]^{b_3} \cdot [ML^{-3}]^{c_3} LT^{-2}$

For M, $M^0 = M^{c_3}$

$\boxed{0 = c_3}$

For T, $T^0 = T^{-b_3} \cdot T^{-2}$

$0 = -b_3 - 2$

$\boxed{b_3 = -2}$

For L, $L^0 = L^{a_3} \cdot L^{b_3} \cdot L^{-3c_3} \cdot L^1$

$0 = a_3 + b_3 - 3c_3 + 1$

$0 = a_3 - 2 - 3(0) + 1$

$\boxed{a_3 = 1}$

$\Pi_1 = l^{-2} \cdot V^{-2} \rho^{-1} R$

$\Pi_1 = \dfrac{R}{l^2 V^2 \rho}$

$\Pi_2 = l^{-1} \cdot V^{-1} \cdot \rho^{-1} \mu$

$\Pi_2 = \dfrac{\mu}{l V \rho}$

$\Pi_3 = l^1 V^{-2} g$

$\Pi_3 = \dfrac{lg}{V^2}$

$\Pi_1 = \phi(\Pi_2, \Pi_3)$

$\Pi_2 = \phi(\Pi_1, \Pi_3)$

$\Pi_3 = \phi(\Pi_1, \Pi_2)$

$\dfrac{R}{l^2 V^2 \rho} = \phi\left[\dfrac{\mu}{\rho V l}, \dfrac{lg}{V^2}\right]$

$$\frac{R}{l^2 V^2 \rho} = \phi \left[\frac{\rho V L}{\mu} \frac{V^2}{l g} \right]$$

$$R = \phi\, l^2 V^2 \rho \left[Re, F_r \right]$$

$$f_1 \left[\pi_1, \pi_2, \pi_3 \right] = 0$$

$$f_1 \left[\frac{R}{l^2 V^2 \rho} \cdot \frac{\mu}{\rho V L} \cdot \frac{l g}{V^2} \right] = 0 .$$

Free Convection (Natural) :-

Convection is of two ways

i) free Convection

ii) forced convection

(i) Free Convection in which bulk motion of fluid, is due to density difference of any buoyant force caused by heat transfer b/w fluids

Thermal boundary layer consider with hydro dynamic boundary layer

eg:- Thunderstroms, gliders, cooling fans in a motor, hot coffe in a cup.

(ii) Forced Convection:- Bulk motion of fluid is due to pump

eg:- fluid flow in boilers flow over condensers heat exchangers --etc.

Dimensional Analysis applied to forced convection heat transfer

let us assume that heat transfer coefficient is a function of

ρ, D, V, C_p, μ, K

$$h = f[\rho, D, \mu, K, C_p, V]$$

$$f_1[h, \rho, D, \mu, V, K, C_p] = 0$$

$$n = 7$$

$$h = \frac{W}{m^2\,{}^\circ C} \Rightarrow \frac{ML^2T^{-2}\,T^{-1}}{L^2 \cdot \theta} = MT^{-3}\theta^{-1}$$

$$\rho = \frac{kg}{m^3} = \frac{M}{L^3} = ML^{-3}$$

$$D = L$$

$$V = m/sec \Rightarrow LT^{-1}$$

$$\mu = \frac{MLT^{-2} \cdot L}{L^2 \, LT^{-1}} = ML^{-1}T^{-1}$$

$$C_p = L^2T^{-2}\theta^{-1}$$

$$k = \frac{MT^{-2}\,L^2}{L\,\theta} = MLT^{-3}\theta^{-1}$$

$$n - m = 7 - 4$$

$$= 3$$

$$f_1[\pi_1, \pi_2, \pi_3] = 0.$$

$h, \rho, V, D.$

$\pi_1 = h^{a_1} \rho^{b_1} D^{c_1} V^{d_1} \mu$

$\pi_2 = h^{a_2} \rho^{b_2} D^{c_2} V^{d_2} c_p$

$\pi_3 = h^{a_3} \cdot \rho^{b_3} D^{c_3} \cdot V^{d_3} \cdot K$

$\pi_1 = h^{a_1} \cdot \rho^{b_1} \cdot D^{c_1} \cdot V^{d_1} \cdot \mu$

$M^0 L^0 T^0 \theta^0 = [MT^{-3}\theta^{-1}]^{a_1} [ML^{-3}]^{b_1} \cdot L^{c_1} [LT^{-1}]^{d_1} ML^{-1}T^{-1}$

for M, $0 = M^{a_1} M^{b_1} \cdot M^1$

$0 = a_1 + b_1 + 1$

for L, $0 = L^{-3b_1} \cdot L^{c_1} \cdot L^{d_1} L^{-1}$

$0 = -3b_1 + c_1 + d_1 - 1$

for T, $0 = T^{-3a_1} \cdot T^{-d_1} \cdot T^{-1}$

$0 = -3a_1 - d_1 - 1 \Rightarrow -3(0) - d_1 - 1 = 0$

$\boxed{d_1 = -1}$

for θ, $0 = \theta^{-a_1}$

$\boxed{a_1 = 0}$, $\boxed{b_1 = -1}$

$\boxed{c_1 = -1}$

$\pi_1 = h^0 \cdot \rho^{-1} D^{-1} \cdot V^{-1} \cdot \mu$

$\pi_1 = \dfrac{\mu}{\rho V D}$

π_2 Term : $h^{a_2} \rho^{b_2} \cdot D^{c_2} V^{d_2} \cdot c_p$

$M^0 L^0 T^0 = [MT^{-3}\theta^{-1}]^{a_2} [ML^{-3}]^{b_2} \cdot L^{c_2} [LT^{-1}]^{d_2} \cdot L^2 T^{-2} \theta^{-1}$

M: $0 = a_2 + b_2 - 1$; $\boxed{b_2 = 1}$

L: $0 = -3b_2 + c_2 + d_2 + 2$; $\boxed{c_2 = 0}$

T: $0 = -3a_2 - d_2 - 2$; $\boxed{d_2 = +1}$

θ: $0 = -a_2 - 1$; $\boxed{a_2 = -1}$

$$\pi_2 = h^{a_2} \cdot \rho^{b_2} \cdot D^{c_2} \cdot V^{d_2} \, c_p$$

$$\pi_2 = h^{-1} \rho^{1} \cdot D^{0} \cdot V^{+1} \, c_p = \boxed{\frac{c_p \cdot \rho V}{h}}$$

$$\pi_3 = [ML^{-3}\theta^{-1}]^{a_3} \cdot [ML^{-3}]^{b_3} \cdot (L)^{c_3} \cdot [LT^{-1}]^{d_3} \, MLT^{-3}\theta^{-1}$$

M: $0 = a_3 - 3b_3 + 1$

L: $0 = -3b_3 + c_3 + d_3 + 1$

T: $0 = -3a_3 - d_3 - 3$

θ: $0 = -a_3 - 1$; $\boxed{a_3 = -1}$ $\boxed{b_3 = 0}$, $\boxed{c_3 = -1}$

$\boxed{d_3 = 0}$

$$\pi_3 = h^{-1} D^{-1} k$$

$$\boxed{\pi_3 = \frac{K}{hD}}$$

According to Bucking hung's π Therom $\pi_3 = \phi(\pi_1, \pi_2)$

$$\frac{K}{hD} = \phi\left[\frac{\mu}{\rho V D}, \frac{\rho V c_p}{h}\right]$$

By introducing constants in place of ϕ and powers

$$\frac{K}{hD} = C\left[\frac{\mu}{\rho V D}\right]^m \cdot \left[\frac{\rho V c_p}{h}\right]^n$$

where c, m, n - Constants

if $m > n$ then $\frac{K}{hD} = c\left[\frac{\mu}{\rho V D}\right]^m \left[\frac{\rho V c_p}{h}\right]^n \left[\frac{\rho \mu}{\rho V D}\right]^{m-n}$

① The pre. diff Δp in a pipe of diameter 'D' and length 'l' due to turbulent flow depends on the velocity V, viscosity 'μ', density 'ρ' and roughness 'k' using Buckinghump. Theorm obtain an expression for Δp

Sol →

$$\Delta p = f[D, l, V, \mu, \rho, K]$$

$$f_1[\Delta p, D, l, V, \mu, \rho, K] = 0$$

$\therefore n = 7$

Since the dimensions of h & $\frac{K}{D}$ are same.

$\Delta p - ML^{-1}T^{-2}$

$D - L$

$l - L$

$V - LT^{-1}$

$\mu - ML^{-1}T^{-1}$

$\rho - ML^{-3}$

$K - L$

$$\therefore \pi_2 = \frac{c_p \rho V D}{K}.$$

$$\frac{K}{hD} = C\left[\frac{\mu}{\rho V D}\right]^{n}\left[\frac{\rho V c_p D}{K}\right]^{n}\left[\frac{\mu}{\rho V D}\right]^{m-n}$$

$$\frac{K}{hD} = C\left[\frac{\mu}{\rho V D}\right]^{m-n}\left[\frac{\mu}{\cancel{\rho V D}} \cdot \frac{\cancel{\rho V} c_p \cancel{D}}{K}\right]^{n}$$

$$\frac{K}{hD} = C\left[\frac{\mu}{\rho V D}\right]^{m-n}\left[\frac{\mu c_p}{K}\right]^{n}$$

$$\boxed{Nu = C\cdot[Re]^{m-n}\cdot(P_r)^{n}}$$

$$\boxed{Nu = C\,[Re]^{m-n}\,(P_r)^{n}}$$

$\therefore m = 3$

no. of π Terms $(n-m) = 7-3 = 4$

$$f_1[\pi_1, \pi_2, \pi_3, \pi_4] = 0$$

repeating variables.

$$\pi_1 = D^{a_1}\rho^{b_1}\cdot V^{c_1}\cdot L$$

$$M^0L^0T^0 = L^{a_1}[ML^{-3}]^{b_1}[LT^{-1}]^{c_1}L$$

for M, $M^0 = M^{b_1}$

$$\boxed{b_1 = 0}$$

For T, $T^0 = T^{-c_1}$

$\boxed{c_1 = 0}$

For L, $L^0 = L^{a_1} L^{-3b_1} L^{c_1} . L^1$

$0 = a_1 - 3b_1 + c_1 + 1$

$a_1 - 3(0) + 0 + 1$

$\boxed{a_1 = -1}$

$\pi_1 = D^{-1} \rho^0 V^0 L$

$\boxed{\pi_1 = \frac{L}{D}}$

$\pi_2 = D^{a_2} . \rho^{b_2} V^{c_2} \mu$

$M^0 L^0 T^0 = L^{a_2} [ML^{-3}]^{b_2} [LT^{-1}]^{c_2} . ML^{-1} T^{-1}$

For M, $M^0 = M^{b_2} . M^1$

$\Rightarrow b_2 + 1 = 0$

$\boxed{b_2 = -1}$

For T, $T^0 = T^{-c_2} T^{-1}$

$-c_2 - 1 = 0$ $\boxed{c_2 = -1}$

For L, $L^0 = L^{a_2} L^{-3b_2} L^{c_2} L^{-1}$

$0 = a_2 - 3b_2 + c_2 - 1$

$a_2 - 3[-1] - 1 - 1$

$\boxed{a_2 = -1}$

$\pi_2 = D^{-1} \rho^{-1} . V^{-1} \mu$

$\boxed{\pi_2 = \frac{\mu}{D \rho V}}$

$\pi_3 = D^{a_3} . \rho^{b_3} . V^{c_3} . K$

$M^0 L^0 T^0 = L^{a_3} . [ML^{-3}]^{b_3} [LT^{-1}]^{c_3} . L$

For M, $M^0 = M^{b_3}$

$\boxed{b_3 = 0}$

for T, $T^0 = T^{-c_3}$

$\boxed{c_3 = 0}$

for L: $L^{a_3} L^{-3b_3} L^{c_3} L$

$0 = a_3 - 3b_3 + c_3 + 1$

$a_3 - 3(0) + (0) + 1$

$\boxed{a_3 = -1}$

So

$\pi_3 = D^{-1} \cdot \rho^0 \cdot V^0 \cdot K$

$$\boxed{\pi_3 = \frac{K}{D}}$$

$\pi_4 = D^{a_4} \cdot \rho^{b_4} V^{c_4} \cdot \Delta P$

$M^0 L^0 T^0 = L^{a_4} [ML^{-3}]^{b_4} [LT^{-1}]^{c_4} ML^{-1} T^{-2}$

for $M^0 = M^{b_3} M^1$

$b_4 = -1$

$T^0 = T^{-c_4} \cdot T^{-2}$

$0 = -c_4 - 2 \Rightarrow \boxed{c_4 = -2}$

$L^0 = L^{a_4} L^{-3b_4} \cdot L^{c_4} L^{-1}$

$0 = a_4 - 3b_4 + c_4 - 1$

$a_4 - 3[-1] + (-2) - 1$

$\boxed{a_4 = 0}$

$\pi_4 = D^0 \rho^{-1} V^{-2} \Delta P$

$$\boxed{\pi_4 = \frac{\Delta P}{\rho V^2}}$$

Dimensional Analysis is applied to Natural free Convection :-

The heat transfer coefficient depends upon V, ρ, μ, c_p, L, k in case of free convection velocity depedent variable and Temperarly independent

Since the motion of fluid is due to difference in density b/w the fluid layers due to Temp gradient.

$\therefore$ velocity depends on Δt Temperature diff b/w heated surface and undistrubed fluid. , β - coefficient of vo. expan

g acceleration due to gravity

Thus The heat transfer coefficient 'h' may be expressed as

$$h = f\left[\rho, L, \mu, c_p, k, \beta g \Delta t\right]$$

$$f_1\left[h, \rho, c_p, L, \mu, k, \beta g \Delta t\right] = 0.$$

$n = 7$.

$$h - \frac{Q}{A \cdot \Delta t} = \frac{MLT^{-2} \cdot LT^{-1}}{L^2 \theta} = MT^{-3}\theta^{-1}$$

$$\rho - ML^{-3}$$

$$L = L$$

$$\mu = \frac{MLT^{-2}}{L^2} \cdot \frac{L}{LT^{-1}} = ML^{-1}T^{-1}$$

$$c_p = \frac{Q}{m \cdot \Delta t} = L^2T^{-2}\theta^{-1}$$

Fundamental dimensions $[M, L, \theta, T]$

$\therefore m = 4$.

$$\beta g \Delta t = LT^{-2}$$

$$K = MLT^{-3}\theta^{-1}$$

$m = 4$

no. of π terms = n − m = 7 − 4

no. of π Terms = 3

$\therefore f_1[\pi_1, \pi_2, \pi_3] = 0.$

repeating variables. $\pi_1 = [\rho^{a_1} L^{b_1} \mu^{c_1} k^{d_1} h]$

$\pi_2 = [\rho^{a_2} L^{b_2} \mu^{c_2} k^{d_2} c_p]$

$\pi_3 = [\rho^{a_3} L^{b_3} \mu^{c_3} k^{d_3} \beta g \Delta t]$

$M^0 L^0 T^0 \theta^0 = [ML^{-3}]^{a_1} \cdot [L]^{b_1} \cdot [ML^{-1}T^{-1}]^{c_1} [MLT^{-3}\theta^{-1}]^{d_1} \, MT^{-3}\theta^{-1}$

for M, $M^0 = M^{a_1} M^{c_1} M^{d_1} \cdot M^1$

$0 = a_1 + c_1 + d_1 + 1 \qquad a_1 + 0 + (-1) + 1 = 0$

for L, $L^0 = L^{-3a_1} \cdot L^{b_1} \cdot M^{-c_1} \cdot L^{d_1}$ $\qquad \boxed{a_1 = 0}$

$0 = -3a_1 + b_1 - c_1 + d_1 \qquad -3(0) + b_1 - 0 - 1$

for T, $T^0 = T^{-c_1} T^{-3d_1} T^{-3}$ $\qquad \boxed{b_1 = 1}$

$0 = -c_1 - 3d_1 - 3 \quad \Rightarrow -c_1 - 3[-1] - 3 = 0$

for θ, $\theta^0 = \theta^{-d_1} \cdot \theta^{-1}$ $\qquad \boxed{c_1 = 0}$

$-d_1 - 1 = 0 \qquad \boxed{d_1 = -1}$

$\pi_1 = \rho^0 \cdot L^1 \mu^0 \cdot k^{-1} \cdot h$

$\Rightarrow \boxed{\pi_1 = \frac{hL}{k}}$

$$\pi_2 = [ML^{-3}]^{a_2}\, L^{b_2}\, [ML^{-1}T^{-1}]^{c_2}\, [MLT^{-3}\theta^{-1}]^{d_2}\, L^{2} T^{-3} \theta^{-1}$$

For M, $M^0 = M^{a_2} \cdot M^{c_2} \cdot M^{d_2}$

$0 = a_2 + c_2 + d_2$ $\qquad a_2 + 0 + -1 \quad \boxed{a_2 = 1}$

For L, $L^0 = L^{-3a_2} \cdot L^{b_2}\, L^{-c_2}\, L^{d_2}\, L^{2}$

$0 = -3a_2 + b_2 - c_2 + d_2 + 2$ $\qquad -3(1) + b_2 - 0 - 1 + 2$

$\boxed{b_2 = 2}$

For T, $T^0 = T^{-c_2}\, T^{-3d_2}\, T^{-3}$

$0 = -c_2 - 3d_2 - 3$ $\qquad -c_2 + 3 - 3 = 0$

$\boxed{c_2 = 0}$

For θ, $\theta^0 = \theta^{-d_2}\, \theta^{-1}$

$0 = -d_2 - 1$

$\boxed{d_2 = -1}$

$\pi_2 = \rho^1 L^2 \mu^0 . K^{-1} C_p$

$\pi_2 = \cancel{\frac{\mu C_p}{K}}$ $\qquad \boxed{\pi_2 = \frac{C_p \rho L^2}{K}}$

$$\pi_3 = [ML^{-3}]^{a_3} \cdot L^{b_3}\, [ML^{-1}T^{-1}]^{c_3} \cdot [MLT^{-3}\theta^{-1}]^{d_3} \cdot LT^{-2}$$

For M, $M^0 = M^{a_3}\, M^{c_3} \cdot M^{d_3}$

$0 = a_3 + c_3 + d_3$ $\qquad a_3 - 2 + 0 = 0$

$\boxed{a_3 = 2}$

For L, $L^0 = L^{-3a_3} \cdot L^{b_3}\, L^{-c_3}\, L^{d_3}\, L^{1}$

$0 = -3a_3 + b_3 - c_3 + d_3 + 1$ $\qquad -3(2) + b_3 + 2 + 0 + 1$

$\boxed{b_3 = +3}$

For T, $T^0 = T^{-c_3} \cdot T^{-3d_3} \cdot T^{-2}$

$0 = -c_3 - 3d_3 - 2$ $\qquad -c_3 - 3(0) - 2 = 0$

$\boxed{c_3 = -2}$

For θ, $\theta^0 = \theta^{-d_3}$

$\boxed{d_3 = 0}$

$$\pi_3 = \rho^{a} l^{b} \mu^{-c} K^{d} \, \beta g \Delta t$$

$$\boxed{\pi_3 = \frac{\beta g \Delta t \, \rho^{2} l^{3}}{\mu^{2}}}$$

$$\pi_1 = \phi(\pi_2, \pi_3)$$

$$\frac{hL}{K} = c \left[\frac{\mu c_p}{K} \right]^{m} \left[\frac{\rho^{2} l^{3} \beta g \Delta t}{\mu^{2}} \right]^{n}$$

$$\boxed{Nu = C (Pr)^{n} (Gr)^{m}}$$