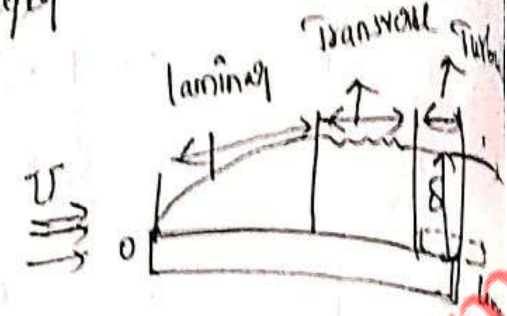
Concepts of hydrodynamics in external force

i) Hydrodynamic boundary layer

ii) Thermal boundary layer



Boundary layer :-

→ the layer adjacent to boundary is known as boundary layer

→ the growth of a boundary layer depends shape of a body

→ leading edge the flow is laminar and the velocity distribution is parallel

→ the Thickness of boundary layer increases with distance from the leading Edge

→ the leading edge ~~the flow is~~ facing The direction of flow

→ The Tracing edge where The flow leaves the edge

Characteristics of boundary layer :-

→ δ (boundary layer) increases as distance from the leading edge is increases

→ δ decreases as 'U' increases

→ δ increases as Kinematic viscosity increases

Turbulent boundary layer:-

→ The laminar boundary layer becomes unstable and motion of fluid with in disturbed and is irregular which leads to Turbulence this called Turbulent boundary layer

Boundary layer thickness :-

Distance measured from the boundary of a solid body measured in y-direction to the point where the velocity is 0.99U (free stream velocity)

Displacement Thickness :-

→ Distance ⊥ar to the boundary, where free stream is displace due to formulation of boundary layer.

$$\boxed{\delta^* = \int_0^\delta \left[1 - \frac{u}{U}\right] dy}$$

dy - Thick of elemental strip

u - velocity of fluid at element.

δ - boundary layer thickness , U - free stream velocity

Momentum thickness :-

Distance measured ⊥ar to the boundary of a solid body by which boundary should be displaced to compensate for momentum reduction of fluid flowing an account of boundary layer formulation

$$\theta = \int_0^\delta \frac{u}{U}\left(1 - \frac{u}{U}\right) dy$$

Energy thickness :-

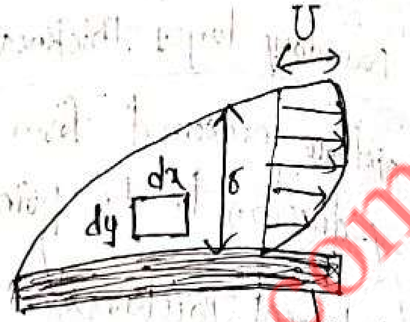
Distance measured ⊥ar to the boundary of a solid body by which boundary to compensate for the reduction in K.E in, an account of boundary layer formation

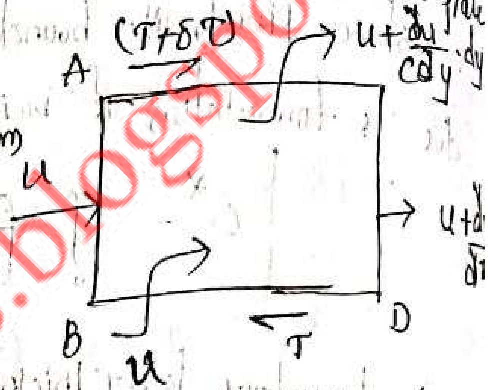
$$\delta^{**} = \int_0^\delta \frac{u}{U}\left[1 - \frac{u^2}{U^2}\right] dy$$

4 Momentum equation for hydrodynamic boundary layer over a flat plate :-

Consider a fluid flowing over a stationary flat plate a hydrodynamic boundary layer is developed.

Inorder to find an differential equation for a boundary layer consider an two dimensional element having volume of $dx \cdot dy$.





Assumptions made for deriving momentum equation

→ flow is steady & incompressible
(velocity doesn't change with time) (ρ = const)

→ viscosity is constant

→ pressure variations in the direction ⊥lar to plate are negligible.

→ fluid is continous both in space & time

→ viscous shear forces in y-direction are neglisible

let u be the velocity of flow at the left ~~phase~~ face AB then

$u + \frac{\partial u}{\partial x} \cdot dx$ at the right face CD

v = fluid velocity in y direction

Mass flow rate along x

$$M_x = \frac{\rho \cdot V}{t} \Rightarrow \rho A \cdot \left(\frac{L}{t}\right) \qquad \frac{L}{t} =$$

$$= \rho(dy \times 1) \cdot u$$

$$M_x = \rho u\, dy$$

change of momentum along x-direction

$$dM_x = \left[\cancel{u} + \frac{\partial u}{\partial x} dx - \cancel{u}\right] M_x$$

$$= M_x \cdot \frac{\partial u}{\partial x} \cdot dx$$

$$\boxed{dM_x = \rho u \frac{\partial u}{\partial x} dx \cdot dy}$$

Mass flow rate along y-direction

$$M_y = \rho \cdot v \cdot dx \cdot 1$$

$$M_y = \rho \cdot v \cdot dx$$

change of momentum along y-direction

dx $$dM_y = M_y\left[\cancel{u} + \frac{\partial u}{\partial y} \cdot dy - \cancel{u}\right]$$

$$= M_y \frac{\partial u}{\partial y} \cdot dy$$

$$\boxed{dM_y = \rho \cdot v \cdot \frac{\partial v}{\partial y} \cdot dx\, dy}$$

Total viscous forces along x-direction

$$F_x = [(\tau + \delta\tau) - \tau] \times \text{area}$$

$$= \left[\mu \cancel{\frac{\partial u}{\partial y}} + \frac{\partial}{\partial y}\left[u \frac{\partial u}{\partial y}\right] dy - \mu \cancel{\frac{\partial u}{\partial y}}\right] dx\, dy$$

$$\boxed{F_x = u \frac{\partial^2 u}{\partial y^2} dx \cdot dy}$$

Assuming that gravitational force are balanced by bouyancy forces for equivibrium of element.

interial forces = viscous force

$$\rho u \frac{\partial u}{\partial x} \cancel{dx\,dy} + \rho v \frac{\partial u}{\partial y} \cancel{dx\,dy} = \mu \frac{\partial^2 u}{\partial y^2} \cancel{dx\,dy}$$

$$\rho u \frac{\partial u}{\partial x} + \rho v \frac{\partial u}{\partial y} = \mu \frac{\partial^2 u}{\partial y^2}$$

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = \frac{\mu}{\rho} \frac{\partial^2 u}{\partial y^2}$$

ν – kinematic viscosity.

$$\boxed{u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = \nu \frac{\partial^2 u}{\partial y^2}}$$

Blasius exact solution for laminar boundary layer:

$$\delta = \sqrt{\frac{\nu x}{U}}$$

δ – boundary layer thickness

ν – kinematic viscosity

U – free stream velocity

streching factor $\eta = y\sqrt{\frac{U}{\nu x}}$

$$\psi = \sqrt{\nu x U} \cdot f(\eta)$$

Van Kerman ~~equation~~ ~~momentum~~ integral eqn :-

(approximate hydrodynamic boundary layer Analysis

Mass flow rate of fluid

$$\boxed{\frac{\tau_0}{\rho U^2} = \frac{d\theta}{dx}}$$

This is Von Kerman momentum integral eqn. for boundary layer and is used to find frictional drag on smooth flat plate for both laminar & turbulent layers

At the surface of a plate $y=0$, $u=0$

at $y=\delta$ then $u=U$

Local coefficient of drag $= \frac{1}{2}\rho u^2$

coefficient of stream friction $C_{fx} = \dfrac{\tau_0}{\frac{1}{2}\rho u^2}$

Avg. coefficient of drag . $\overline{C_f} = \dfrac{F_D}{\frac{1}{2}\rho A U^2}$