WRE UNIT-VII

Advanced Topics in Hydrology

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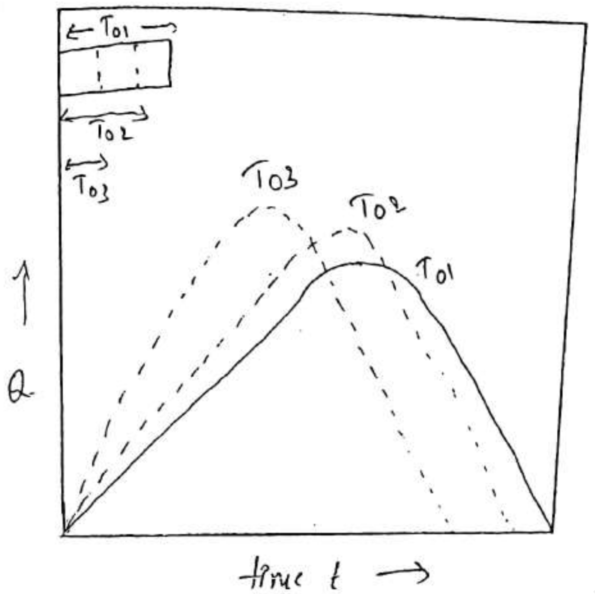
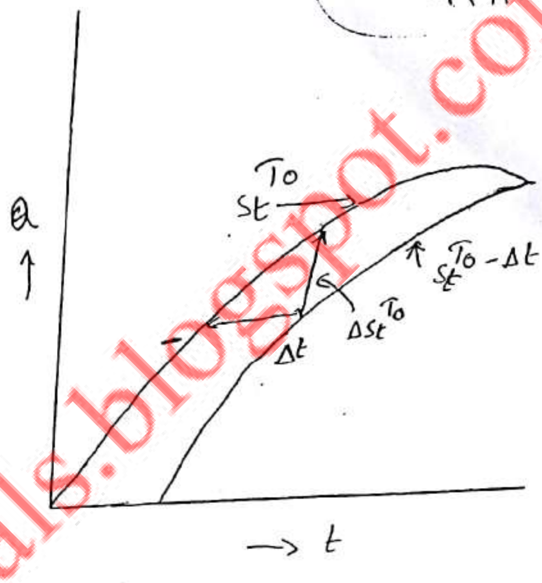
Rainfall - runoff Modelling (Refer pg - 6)

* Instantaneous unit hydrograph (IUH)





→ It is the hydrograph of various intensities and various periods.

→ It is also called as short duration hydrograph

→ The IUH is a unit hydrograph which represents surface runoff from the catchment area. due to an instantaneous precipitation of a rainfall excess volume of 1 cm.

→ The IUH is disignated as $u(t,0)$ (or) $u(t)$

→ this IUH can be derived from S-curve hydrograph which is obtained from UH of duration (T_0)

→ If two S-curves are drawn at a time lack of 't_0' then the ordinate of UH of 't_0' hour unit duration at any time 't' is given by

$$u(t, t_0) = \frac{T_0}{t_0}\left[S_t - S_{t-t_0}\right] \quad \text{--- (1)}$$

where,

$u(t, t_0)$ = ordinate of UH of unit duration 't_0'

T_0 = unit duration of UH

S_t = ordinate of S-curve at any time 't'

$S_t - t_0$ = ordinate of shift S-curve shifted by 't_0'

t_0 = Δt

$$u(t, \Delta t) = \frac{T_0}{\Delta t}\left[S_t^{T_0} - S_{t-\Delta t}^{T_0}\right]$$

(or)

$$u(t, \Delta t) = \frac{T_0}{\Delta t} \cdot \Delta S_t^{T_0}$$

where, $S_t^{T_0}$ = S-curve ordinate derived from the UH T_0 duration

By applying the limit of $\Delta t \to 0$

$$\lim_{\Delta \to 0} u(t, \Delta t) = T_0 \frac{d \Delta S_t^{T_0}}{d \Delta t}$$

(or)

$$u(t) = T_0 \frac{d S_t^{T_0}}{dt}$$

$$= \frac{1}{R_0} \frac{d S_t^{T_0}}{dt} \quad \text{--- (3)}$$

R_0 = Intensity of the rainfall excess given by

$$R_0 = \frac{1}{T_0}$$

By taking $R_0 = 1\,cm$

eq-(3) becomes

$$u(t) = \frac{d\, S_t^{T_0}}{dt} \quad \text{———} (4)$$

Thus the ordinate of IUH at any point time 't' is the slope of S-curve of intensity 1cm per hour)

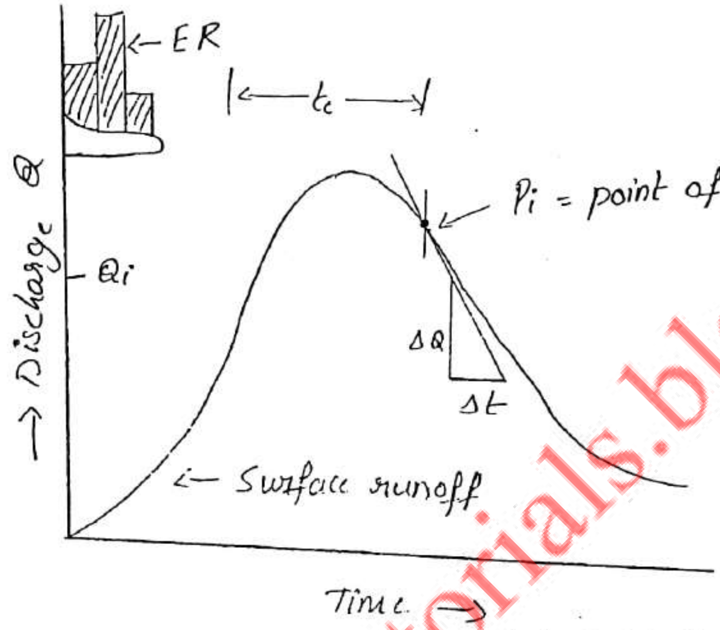
* Conceptual Models:

CLARK'S METHOD for IUH

Clark's method, also known as Time-area histogram method aims at developing an IUH due to an instantaneous rainfall excess over a catchment. It is assumed that the rainfall excess first undergoes pure translation and then attenuation. The translation is achieved by a travel time-area histogram and the attenuation by routing the results of the above through a linear reservior at the catchment outlet.

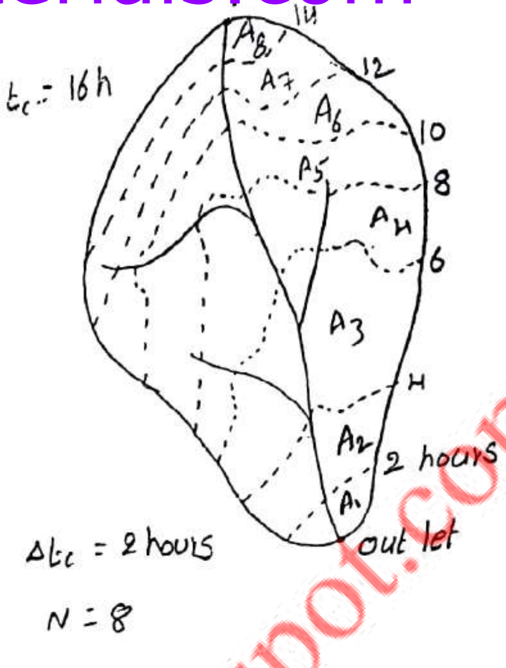
Time-Area Curve

Time here refers to the time of concentration. The time of concentration t_c is the time required for a unit volume of water from the farthest point of catchment to reach the outlet. It represents the maximum time of translation of the surface runoff of the catchment. In gauged areas the time interval between the end of the rainfall excess and the point of inflection of the resulting surface runoff provides a good way of estimating 't_c' from known rainfall - runoff data.



The total catchment area drains into the outlet in t_c hours. If points on the area having equal time of travel, (say t_1 h where $t_1 < t_c$), are considered and located on a map of the catchment, a line joining them is called an Isochrone (or runoff isochrone).

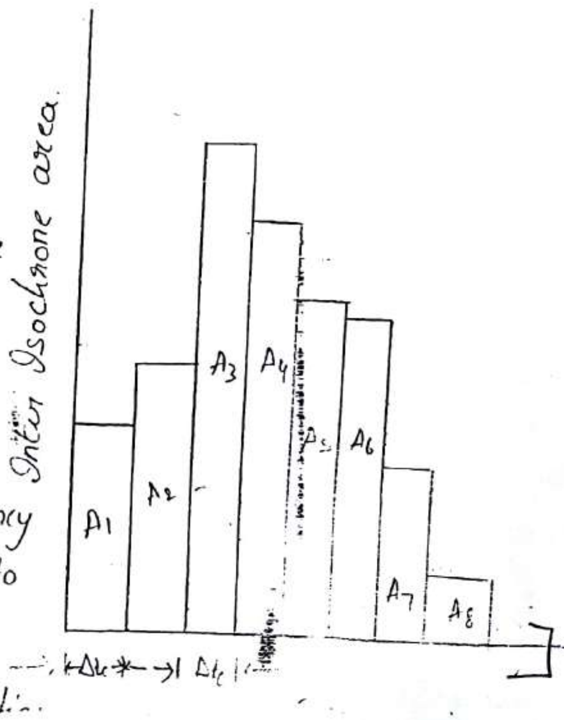
Figure shows a catchment being divided into (N=8) sub areas by Isochrones having an equal time interval.



To assist in drawing Isochrones, the longest water course is chosen and its profile plotted as elevation (vs) distance from the outlet; the distance is then divided into N parts and the elevation of the subparts measured on the profile transfered to the contour map of the catchment.

The inter - Isochrone areas $A_1, A_2 \cdots A_N$ are used to construct a travel time-area histogram. If a rainfall excess of 1 cm occurs instantaneously and uniformly over the catchment area, this time - area histogram represents the sequence in which the volume of rainfall will be moved out of the catchment and arrive at the outlet.

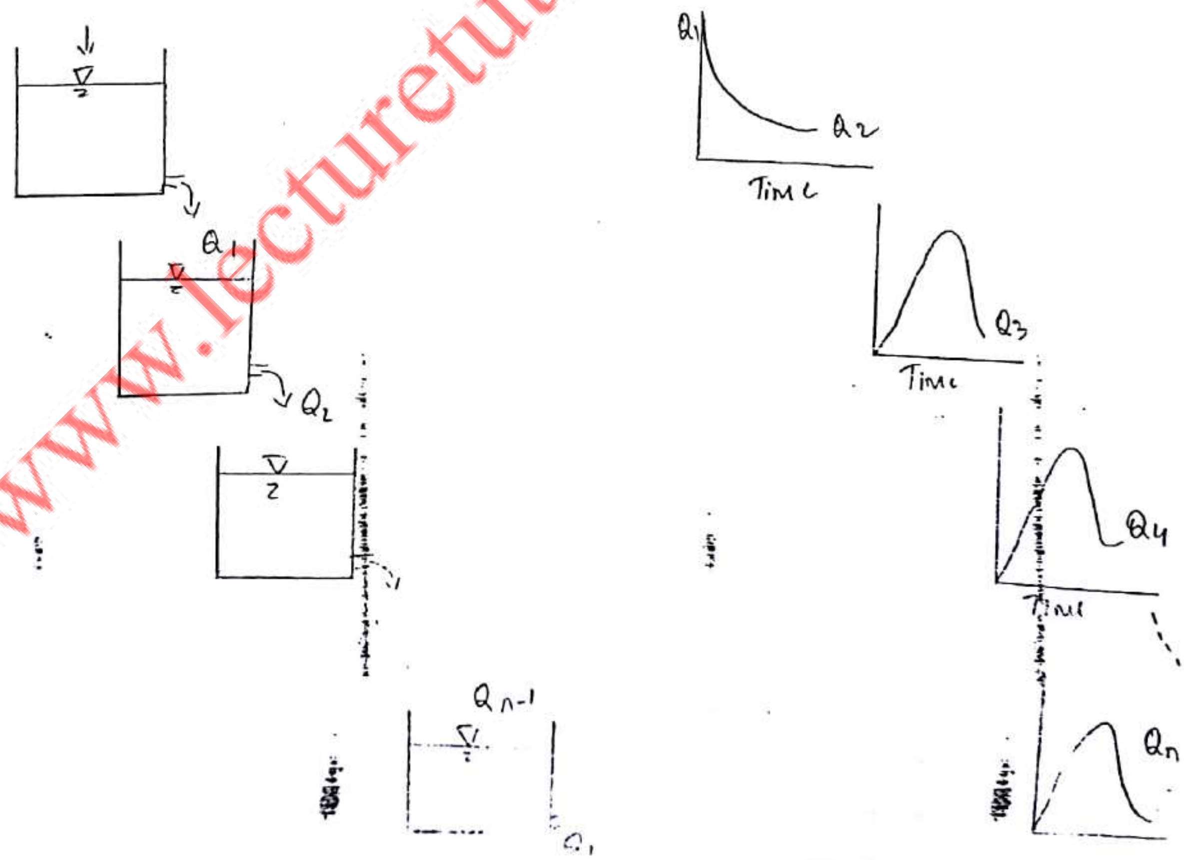
In Fig. a sub area A_r km^2 represent a volume of A_r km^2.cm $= A_r \times 10^4 (m^3)$ moving out in time $\Delta t_c = t_c / N$ hours.



This method do not provide for the storage properties of the catchment. To overcome this deficiency clark assumed a linear reservior to be hypothetically available at the outlet to provide the requisite attenuation.

NASH'S CONCEPTUAL MODEL

Nash (1957) proposed conceptual model of a catchment to develop an equation for IUH. The catchment is assumed to be made up of a series of n identical linear reservoirs each having the same storage constant k. The first reservoir receives a unit volume equal to 1 cm of effective rain from the catchment instantaneously. This inflow is routed through the first reservoir to get the outflow hydrograph. The outflow from the first reservoir is considered as the input to the second; the outflow from the second reservoir is the input to the third and so on for all the 'n' reservoirs. The conceptual cascade of reservoirs as above and the shape of the outflow hydrographs from each reservoir of the cascade is shown in fig. The outflow hydrograph from the n^{th} reservoir is taken as the IUH of the catchment.



For the eq^n of continuity $I - Q = \frac{ds}{dt}$ ——(1)

For linear reservoirs $S = kQ$

$$\frac{ds}{dt} = k \cdot \frac{dQ}{dt}$$

Sub eq—(1) then we get,

$$k\frac{dQ}{dt} + Q = I$$

The solution of the above differential eq^n,

where Q & I are functions of time 't'

i.e., $Q = \frac{1}{k} e^{-t/k} \int e^{t/k} I \, dt$ ——(2)

$t = 0$ & $I = 0$

$\int I \, dt$ = Intantaneous value inflow

= 1cm of effective rain

Hence for the 1st reservoir eq——(2) becomes,

$$Q_1 = \frac{1}{k} e^{-t/k}$$

For 2nd reservoir,

$$Q_2 = \frac{1}{k} e^{-t/k} \int e^{t/k} \cdot \frac{1}{k} e^{-t/k} dt$$

$$\Rightarrow Q_2 = \frac{1}{k^2} t \, e^{-t/k}$$

$$Q_3 = \frac{1}{2} \cdot \frac{1}{k^3} \cdot t^2 e^{-t/k}$$

Similarly for n^{th}

$$Q_n = \frac{1}{(n-1)! \, k^n} t^{n-1} e^{-t/k}$$

. If u(t) is the ordinate of the IUH, then,
IUH of catchment is written as,

$$\boxed{u(t) = \frac{1}{(n-1)!\; k^n}\, t^{n-1} \cdot e^{-t/k}}$$

* Hydrological Models :

There are mainly three deterministic hydrological models, they are,

1) Lumped model
2) Semi distributed model
3) Distributed model.

Lumped Model :

In lumped models, the parameters does not vary spatially with in the basin and it is evaluated only at outlet.

(i) Lumped models are not suitable for event based process
(ii) This model requires simple and minimal data to obtain its parameters

Semi-Distributed Model :

In semi-distributed models, the parameters are allowed partly in space by dividing the basins into sub basins.

There are mainly two types of semi-distributed models
(i) kinematic wave theory
(ii) Probability distributed models.

Kinematic Wave theory: (5)

It is a physically based model in the form of simplified version of surface flow.

Probability Distributed Models:

In this model by using input parameter the spatial resolution is accounted.

Distributed Models:

In distributed models, parameters are fully allowed to vary in space. In this models it requires a large amount of data and obtain results at any location.

* Chow - Kulandaiswamy Model:

let us consider a linear system with constant coefficient $a_1, a_2 - a_n$ and $b_1, b_2 - b_n$

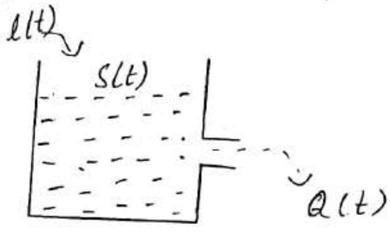


Fig: Hydrologic system

Storage function S is written as

$$S = f\left[I, \frac{dI}{dt}, \frac{d^2I}{dt^2} \cdots Q, \frac{dQ}{dt}, \frac{d^2Q}{dt^2}\right] \quad \text{--- (1)}$$

Equation —(1) can be also written w.r.t constant coefficients

$$\therefore S = a_1 Q + a_2 \frac{dQ}{dt} + a_3 \frac{d^2Q}{dt^2} + \cdots a_n \frac{d^{n-1}Q}{dt^{n-1}} + b_1 I + b_2 \frac{dI}{dt} + b_3 \frac{d^2I}{dt^2} + \cdots + b_m \frac{d^{m-1}I}{dt^{m-1}I} \quad \text{—— (2)}$$

Differentiating eq —(2) w.r.t 't'

$$\frac{ds}{dt} = \frac{d}{dt}\left[a_1 Q + a_2 \frac{dQ}{dt} + a_3 \frac{d^2Q}{dt^2} + \cdots a_n \frac{d^{n-1}Q}{dt^{n-1}} + b_1 I + b_2 \frac{dI}{dt} + b_3 \frac{d^2I}{dt} + \cdots + b_m \frac{d^{m-1}I}{dt^{m-1}I}\right]$$

$$Q_1 + a_1 \frac{dQ}{dt} + a_2 \frac{d^2Q}{dt^2} + \cdots a_{n-1} \frac{d^{n-1}Q}{dt^{n-1}} + a_n \frac{d^nQ}{dt^n} = I - b_1 \frac{dI}{dt} - b_2 \frac{d^2I}{dt^2} - \cdots - b_{m-1}\frac{d^{m-1}}{dt^{m-1}} - b_m \frac{d^mI}{dt^m} \quad (3)$$

By forming differential operation [N(D) & M(D)] eq-(3) written as,

$$ND(Q) = MI(D)$$

$$\boxed{\therefore Q = \frac{M(D)}{N(D)} I}$$

* Rain fall - Runoff Modeling:

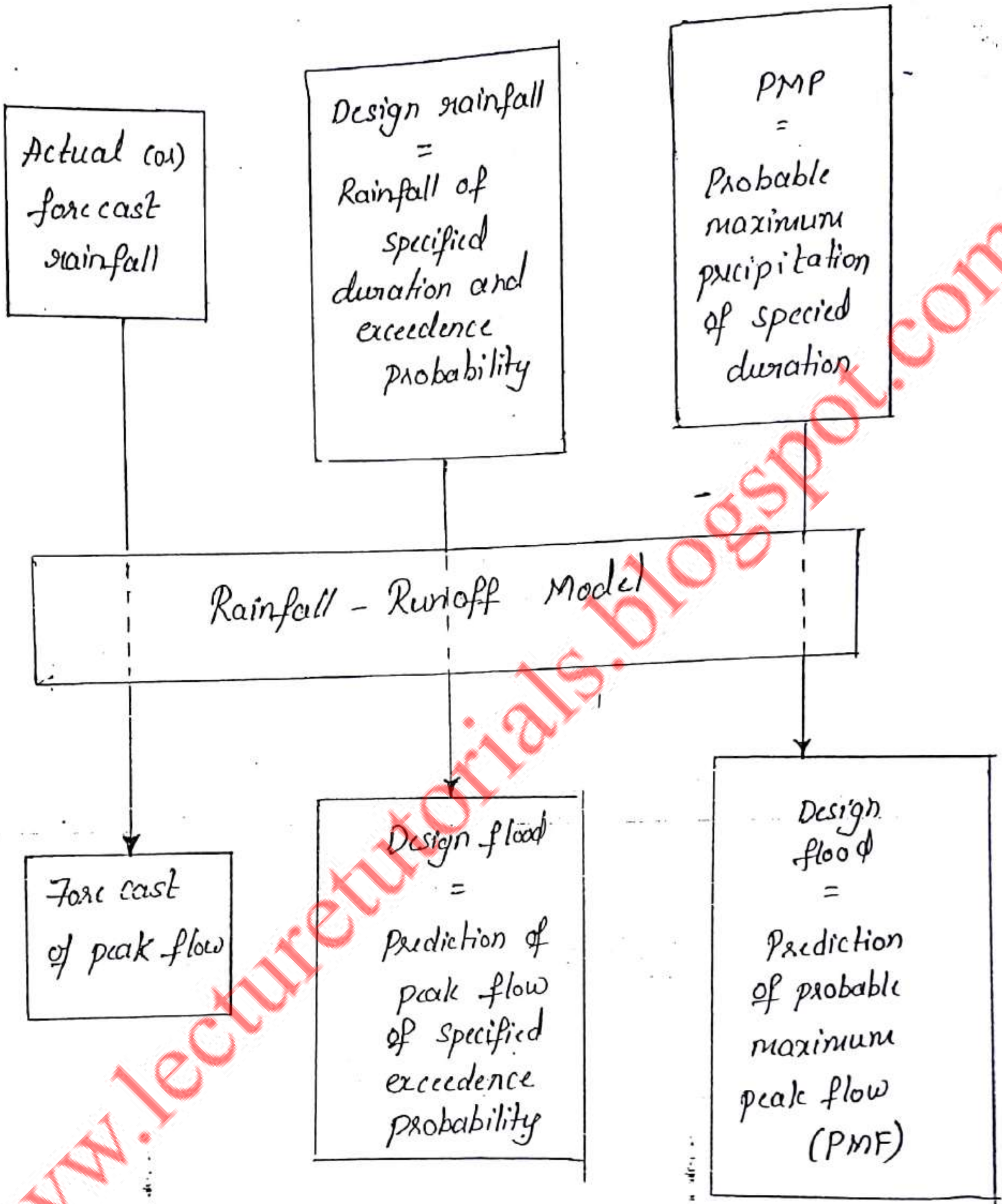
-> The rainfall-runoff modeling was confined mostly to use of empirical formulae or the rational method to transform rainfall into surface runoff.

-> It can be carried out with in a purely analytical framework based on observation of inputs and outputs of a catchment area.

-> For small scale problems i.e for small catchment area the method based on time of concentration was used. But, for larger catchment rational method had to be modified

-> In due course of time the method of Isochrones was introduced. This method was considered as first basic rain fall - runoff method.

-> This rainfall- runoff modeling was used for flood frequency studies, forecasting and prediction of floods.



Rainfall - Runoff Modeling