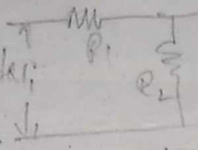
11/12/19

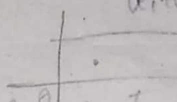
UNIT-3

State Space Analysis

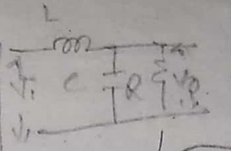
the img parameter used to determine o/p (value)
⇓
represented by variables; state variable



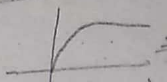
$V_0 \Rightarrow V_0 = \frac{R_2}{R_1+R_2} V_i$



Static
⇓
the change in response is constant



t = ∞
L = S.C, C = O.C
t = 0
L = O.C ; C = S.C



⇒ Dynamic
⇓
the change in response is variable w.r.t to tym.

Here I_L & V_C are state variables

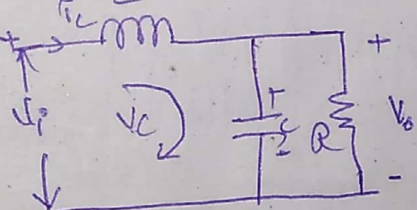
State:- The state of a dynamic system is a smallest set of variables ⇒ The knowledge of this variables at tym $t = T_0$ together with the knowledge of I/p for tym $t \geq T_0$, completely determines the behaviour of the system for any tym $t > T_0$

State variable:- The state variables of a dynamic system are the variables make up smallest set of variables that determine the state of a dynamic system.

State Vector:- If 'n' state variables are needed to cmpltly describe the behaviour of a given system, then this 'n' state variables can be considered the 'n' cmpts of a vector "x" Such a vector is called "State Vector".

State Space:- The 'n' dimensional space whose coordinate axis consist of x_1-axis, x_2-axis - - - x_n- axis is called a State Space. Any state can be represented by a pt in the state space.

* State Space Equations :-



Apply KVL

$$-V_i + L\frac{di_L}{dt} + V_C = 0$$

$$L\frac{di_L}{dt} = V_i - V_C$$

$$\frac{di_L}{dt} = (V_i - V_C)\,1/L \Rightarrow \boxed{\frac{di_L}{dt} = \frac{-1}{L}V_C + \frac{1}{L}V_i}$$

Apply KCL

$$C\cdot\frac{dV_C}{dt} - i_L + \frac{V_C}{R} = 0$$

$$\frac{dV_C}{dt} = \frac{i_L}{C} + \frac{V_C}{RC}$$

$$= \begin{bmatrix} \frac{-1}{RC} & \frac{1}{C} \\ \frac{-1}{L} & 0 \end{bmatrix} \begin{bmatrix} V_C \\ I_L \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{1}{L} \end{bmatrix} V_i$$

State Space Representation :-

1. Controllable Canonical form
2. observable "
3. Diagonal "
4. Jordan "

Difference Eqⁿ: (By using pulse T.F)

$$y(k) + a_1 y(k-1) + a_2 y_2(k-2) + \dots a_n y(k-n) = b_0 u(k) + b_1 u(k-1) + \dots + b_n u(k-n)$$

Apply Z-T/F

$$Y(z) + a_1 z^{-1} Y(z) + a_2 z^{-2} Y(z) + \dots + a_n z^{-n} Y(z) = b_0 u(z) + b_1 z^{-1} u(z) + \dots + b_n z^{-n} u(z)$$

$$Y(z)[1 + a_1 z^{-1} + a_2 z^{-2} + \dots + a_n z^{-n}] = U(z)[b_0 + b_1 z^{-1} + \dots + b_n z^{-n}]$$

$$\frac{Y(z)}{U(z)} = \frac{b_0 + b_1 z^{-1} + b_2 z^{-2} + \dots + b_n z^{-n}}{1 + a_1 z^{-1} + a_2 z^{-2} + \dots + a_n z^{-n}}$$

1. Controllable c.f

$$\frac{Y(z)}{U(z)} = \frac{b_0 z^n + b_1 z^{n-1} + b_2 z^{n-2} + \dots + b_n}{z^n + a_1^{n-1} + a_2 z^{n-2} + \dots + a_n} \quad \text{—} ①$$

matrix-I I/p

$$\underset{n\times1}{\begin{bmatrix} x_1(k+1) \\ x_2(k+1) \\ \vdots \\ x_n(k+1) \end{bmatrix}} = \underset{n\times n}{\overset{G}{\begin{bmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ 0 & 0 & \cdots & 1 & 0 & 0 \\ 0 & 0 & 0 & \cdots & 1 & 0 \\ 0 & 0 & 0 & \cdots & 0 & 1 \\ -a_n & -a_{n-1} & \cdots & -a_1 \end{bmatrix}}} \underset{n\times1}{\begin{bmatrix} x_1(k) \\ x_2(k) \\ \vdots \\ x_n(k) \end{bmatrix}} + \underset{n\times1}{\overset{H}{\begin{bmatrix} 0 \\ \vdots \\ 1 \end{bmatrix}}} \underset{1\times1}{\begin{bmatrix} u(k) \end{bmatrix}}$$

o/p matrix

$$\underset{1\times1}{[y(k)]} = \underset{1\times n}{\overset{C_{1\times n}}{\begin{bmatrix} b_n - a_n b_0 & b_{n-1} - a_{n-1} b_0 & \cdots & b_1 - a_1 b_0 \end{bmatrix}}} \underset{n\times1}{\begin{bmatrix} x_1(k) \\ \vdots \\ x_n(k) \end{bmatrix}} + \underset{1\times1}{\overset{D}{(b_0)}} u(k)$$

2. Observable c.f :-

Transpose of above matrices :-

$$\begin{bmatrix} x_1(k+1) \\ x_2(k+1) \\ \vdots \\ x_n(k+1) \end{bmatrix} = \begin{bmatrix} 0 & 0 & \cdots & -a_n \\ 1 & 0 & \cdots & -a_{n-1} \\ 0 & 1 & \cdots & \\ 0 & 0 & \cdots & \\ 0 & 0 & \cdots & \\ 0 & 0 & & \\ 0 & 0 & \cdots & -a_1 \end{bmatrix} \begin{bmatrix} x_1(k) \\ x_2(k) \\ \vdots \\ x_n(k) \end{bmatrix} + \begin{bmatrix} b_n - a_n b_0 \\ b_{n-1} - a_{n-1} b_0 \\ \vdots \\ b_1 - a_1 b_0 \end{bmatrix} u(k)$$

$$[Y(k)] = [0\ 0\ 0 \cdots 1] \begin{bmatrix} x_1(k) \\ \vdots \\ x_n(k) \end{bmatrix} + [b_0]\, u(k)$$

problems

① $\frac{Y(z)}{U(z)} = \frac{z+1}{z^2+1.3z+0.4}$ ⇒ cmp with stdrd eqⁿ ①

$a_1 = 1.3$

$a_2 = 0.4$

$b_0 = z^n$ value cmp with Den $= 0$

$b_1 = 1$

$b_2 = 1$

I/P (controllable)

$$\begin{bmatrix} x_1(K+1) \\ x_2(K+1) \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -0.4 & -1.3 \end{bmatrix} \begin{bmatrix} x_1(K) \\ x_2(K) \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u(K)$$

O/P

$$[Y(K)] = [1 \quad 1] \begin{bmatrix} x_1(K) \\ x_2(K) \end{bmatrix} + [0] u(t)$$

1-0

observable
⇓
Transpose of controllable matrix

$$[X] = \overset{G}{\begin{bmatrix} 0 & -0.4 \\ 1 & -1.3 \end{bmatrix}} \quad \overset{H}{\begin{bmatrix} 1 \\ 1 \end{bmatrix}}$$

$$[Y] = \overset{C_{1\times n}}{[1 \quad 0]} \quad \overset{D}{[0]}$$

② $\frac{Y(z)}{U(z)} = \frac{z^{-1}+5z^{-2}}{1+4z^{-1}+3z^{-2}} = \frac{z+5}{z^2+4z+3}$

$a_1 = 4$

$a_2 = 3$

$b_0 = 0$

$b_1 = 1$

$b_2 = 5$

Sol:

controllable

I/P.

$$\begin{bmatrix} x_1(K+1) \\ x_2(K+1) \end{bmatrix} = \overset{G}{\begin{bmatrix} 0 & 1 \\ -3 & -4 \end{bmatrix}} \begin{bmatrix} x_1(K) \\ x_2(K) \end{bmatrix} + \overset{H}{\begin{bmatrix} 0 \\ 1 \end{bmatrix}} u(K)$$

O/P

$$[Y(K)] = [5 \quad 1] \begin{bmatrix} x_1(K) \\ x_2(K) \end{bmatrix} + [0] u(t)$$

observable

$$\begin{bmatrix} 0 & -3 \\ 1 & -4 \end{bmatrix} \quad \begin{bmatrix} 5 \\ 1 \end{bmatrix}$$

$$[1 \quad 0] \quad [0]$$

13/12/19 3. Diagonal C.F :-

$$\frac{Y(z)}{U(z)} = \frac{b_0 z^n + b_1 z^{n-1} + \cdots + b_n}{z^n + a_1 z^{n-1} + a_2 z^{n-2} + \cdots + a_n}$$

$$= \frac{b_0 z^n + b_1 z^{n-1} + \cdots + b_n}{(z-P_1)(z-P_2)\cdots(z-P_n)}$$

P.F

$$\frac{Y(z)}{U(z)} = \frac{C_1}{z-P_1} + \frac{C_2}{z-P_2} + \cdots + \frac{C_n}{z-P_n}$$

$$\begin{bmatrix} x_1(K+1) \\ x_2(K+1) \\ \vdots \\ x_n(K+1) \end{bmatrix}_{n\times 1} = \overset{G}{\begin{bmatrix} P_1 & 0 & 0 & 0 & \cdots & 0 \\ 0 & P_2 & 0 & \cdots & \cdots & 0 \\ 0 & 0 & P_3 & \cdots & \cdots & 0 \\ 0 & 0 & 0 & P_4 & \cdots & 0 \\ \vdots & & & & \ddots & \\ 0 & 0 & 0 & 0 & 0 & P_n \end{bmatrix}}_{(n\times n)} \begin{bmatrix} x_1(K) \\ x_2(K) \\ \vdots \\ x_n(K) \end{bmatrix}_{(n\times 1)} + \overset{H}{\begin{bmatrix} 1 \\ 1 \\ \vdots \\ 1 \end{bmatrix}}_{(n\times 1)} \underset{(1\times 1)}{u(K)}$$

$$\underset{(1\times 1)}{y(K)} = \overset{C}{\begin{bmatrix} c_1 & c_2 & \cdots & c_n \end{bmatrix}}_{(1\times n)} \begin{bmatrix} x_1(K) \\ x_2(K) \\ \vdots \\ x_n(K) \end{bmatrix}_{(n\times 1)} + \overset{D}{\underset{(1\times 1)}{[b_0]}} \underset{(1\times 1)}{u(K)}$$

Sum

$$\frac{Y(z)}{U(z)} = \frac{z+1}{z^2+1.3z+0.4}$$

Sol:-

$$= \frac{z+1}{(z+0.8)(z+0.5)}$$

$$z+1 = A(z+0.5)+B(z+0.8)$$

put $z=-0.8$

$$0.2 = -0.3A \Rightarrow A = -2/3 \;;\; B = 5/3$$

$$\frac{Y(z)}{U(z)} = \frac{-2/3}{z+0.8} + \frac{5/3}{z+0.5}$$

$$G = \begin{bmatrix} 0.8 & 0 \\ 0 & 0.5 \end{bmatrix} \quad H = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$C = \begin{bmatrix} \frac{-2}{3} & \frac{5}{3} \end{bmatrix} \quad D = [0]$$

y. Jordon C.F:- (Here roots are repeated)

$$\frac{Y(z)}{U(z)} = \frac{b_0z^n+b_1z^{n-1}+\cdots+b_n}{a_0z^n+a_1z^{n-1}+\cdots+a_n}$$

$$= \frac{b_0z^n+b_1z^{n-1}+\cdots+b_n}{(z-P_1)^4(z-P_2)\cdots(z-P_{n-3})}$$

$$\frac{Y(z)}{U(z)} = \frac{c_1}{z-P_1} + \frac{c_2}{(z-P_1)^2} + \frac{c_3}{(z-P_1)^3} + \frac{c_4}{(z-P_1)^4} + \cdots + \frac{c_n}{(z-P_{n-3})}$$

→ b/c roots are repeating so we get 1 off [illegible]

→ Jordan block

$$\begin{bmatrix} x_1(K+1) \\ x_2(K+1) \\ \vdots \\ x_n(K+1) \end{bmatrix}_{(n\times 1)} = \begin{bmatrix} P_1 & 1 & 0 & 0 & \cdots & 0 \\ 0 & P_1 & 1 & 0 & \cdots & 0 \\ 0 & 0 & P_1 & 1 & \cdots & 0 \\ 0 & 0 & 0 & P_1 & \cdots & 0 \\ 0 & 0 & \cdots & \cdots & \cdots & P_{n-3} \end{bmatrix}_{(n\times n)} \begin{bmatrix} x_1(K) \\ x_2(K) \\ \vdots \\ x_n(K) \end{bmatrix}_{(n\times 1)} + \begin{bmatrix} 1 \\ 1 \\ \vdots \\ 1 \end{bmatrix}_{(n\times 1)} \underset{(1\times 1)}{u(K)}$$

$$y(K)_{(1\times 1)} = \begin{bmatrix} c_1 & c_2 & \cdots & c_n \end{bmatrix}_{(1\times n)} \begin{bmatrix} x_1(K) \\ x_2(K) \\ \vdots \\ x_n(K) \end{bmatrix}_{n\times 1} + \underset{(1\times 1)}{[b_0]} \underset{(1\times 1)}{u(K)}$$

Sum ① $\frac{Y(z)}{U(z)} = \frac{z+1}{z^2-2z+1}$

$= \frac{z+1}{(z-1)^2}$

$z^2 - z - z + 1$

$z(z-1) - 1(z-1)$

$G = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$ $H = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

$[-1 \quad 2]$ $[0]$

P.F

$z+1 = \frac{A}{z-1} + \frac{Bz}{(z-1)^2}$

$z+1 = A(z-1) + Bz$

put $z=1$ $\boxed{B=2}$; $z=0$

$-A = 1 \Rightarrow \boxed{A=1}$

$\frac{Y(z)}{U(z)} = \frac{1}{z-1} + \frac{2z}{(z-1)^2}$

$\frac{-z+1+2z}{(z-1)^2}$ $z+1$

$\frac{1}{z-1} + \frac{2}{(z-1)^2}$

$\frac{z-1+2}{(z-1)^2}$

$= \frac{z+1}{(z-1)^2}$

Sum $\frac{Y(z)}{U(z)} = \frac{z}{z^2+6z+8}$

represent above sum in all forms

Controllable canonical form

$G = \begin{bmatrix} 0 & 1 \\ -8 & -6 \end{bmatrix}$ $H = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$

$C = [0 \quad 1]$ $D = [0]$

Observable

$G = \begin{bmatrix} 0 & -8 \\ 1 & -6 \end{bmatrix}$ $H = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$

$C = [0 \quad 1]$ $D = [0]$

Diagonal

$\frac{Y(z)}{U(z)} = \frac{z}{(z+4)(z+2)}$

$z = A(z+2) + B(z+4)$

$z=-2$ $-2 = B(2) \Rightarrow B=-1$

$z=-4$ $-4 = A(-2) \Rightarrow A=2$

$\frac{Y(z)}{U(z)} = \frac{2}{z+4} \quad \frac{-1}{z+2}$

$G = \begin{bmatrix} 4 & 0 \\ 0 & 2 \end{bmatrix}$ $H = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$ $C = [2 \quad -1]$ $D = [0]$

Here Jordan form is not calculated.

b/c poles are not repeat?

16/12/19 ② $\frac{Y(z)}{U(z)} = \frac{1+0.8z^{-1}}{1-z^{-1}+0.5z^{-2}} = \frac{z^2+0.8z}{z^2 - z+0.5}$

Controllable form

$\begin{bmatrix} 0 & 1 \\ -0.5 & +1 \end{bmatrix}$ $\begin{bmatrix} 0 \\ 1 \end{bmatrix}$

$a_1 = -1$, $a_2 = 0.5$

$b_0 = 1$, $b_1 = 0.8$, $b_2 = 0$

$0.8 - 0.5(1)$ $0 - 0.5$

$0.8 - (-1)(1)$

$[-0.5 \quad 1.8]$ $[1]$

Observable

$\begin{bmatrix} 0 & -0.5 \\ 1 & 1 \end{bmatrix}$ $\begin{bmatrix} -0.5 \\ 1.8 \end{bmatrix}$

$[0 \quad 1]$ $[1]$

Diagonal

$z^2-z+0.5 = z^2-0.5z-0.5z+0.5$

$= z(z-0.5)-0.5(z-0.5)$

$= (z-0.5)^2$

the Diagonal representatⁿ is not possible b/c roots are complex.

Jordan

$\frac{z^2+0.8z}{(z-0.5)^2}$

$\Rightarrow$ By division method.

$\frac{z^2+0.8z}{z^2-z+0.5}$

$$\begin{array}{r} 1 \\ z^2-z+0.5 \overline{)\, z^2+0.8z} \\ -\, z^2 \underset{+}{-} z \pm 0.5 \\ \hline 1.8z-0.5 \end{array}$$

$1+\frac{1.8z-0.5}{z^2-z+0.5}$

$\frac{1.8z-0.5}{(\quad)} = \frac{A}{z-(\frac{1}{2}+i/_2)} + \frac{B}{z-(1/_2-i/_2)}$

$1.8z-0.5 = z(A+B)-(1/_2A+1/_2B)+(-Ai/_2+Bi/_2)$

$A+B=1.8$

$-\frac{1}{2}(1.8)+i/_2(A-B)=-0.5$

$i/_2(A-B)=-0.5+0.9$

$i(A-B)=-0.8i$

$2A=1.8-0.8i$

$A=0.9-0.4i$

$B=0.9+0.4i$

$$\begin{bmatrix} 1/_2+i/_2 & 0 \\ 0 & 1/_2 \end{bmatrix}$$

$$\begin{bmatrix} 0.9-0.4i & 0.9+0.4i \end{bmatrix}$$

③ $\frac{3z^2-4z+6}{(z-1/3)^3} = \frac{3z^2-4z+6}{z^3-z^2+\frac{z}{3}-\frac{1}{27}} = \frac{b_0z^3+b_1z^2+b_2z+b_3}{z^3+a_1z^2+a_2z+a_3}$

controllable

$$\begin{bmatrix} x_1(k+1) \\ x_2(k+1) \\ x_3(k+1) \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1/27 & -1/3 & 1 \end{bmatrix} \begin{bmatrix} x_1(k) \\ x_2(k) \\ x_3(k) \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} u(k)$$

$$[y(k)] = \begin{bmatrix} 6 & -4 & 3 \end{bmatrix} \begin{bmatrix} x_1(k) \\ x_2(k) \\ x_3(k) \end{bmatrix} + [0]\, u(k)$$

observable

$$\begin{bmatrix} 0 & 0 & 1/27 \\ 1 & 0 & -1/3 \\ 0 & 1 & 1 \end{bmatrix} \begin{bmatrix} 6 \\ -4 \\ 3 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 0 & 1 \end{bmatrix} \quad [0]$$

Jordan

$b_3-a_3b_0$ $b_2-a_2b_0$ $b_1-a_1b_0$

$\frac{3z^2-4z+6}{(z-1/3)^3} = \frac{A}{z-1/3}+\frac{B}{(z-1/3)^2}+\frac{C}{(z-1/3)^3}$

$3z^2-4z+6 = A(z-1/3)^2+B(z-1/3)+C$

$A(z^2+1/9-2/3z)$

$z=1/3$ $3(1/9)-4(1/3)+6=C$
$_3$

$\frac{1-4}{3}+6=C$

$\boxed{C=5}$

cmp z^2 z

$\boxed{3=A}$ $-4=\frac{-2}{3}A+B$

$\boxed{-2=B}$

$$= \frac{\quad}{z-1/3} - \frac{2}{(z-1/3)^2} + \frac{8}{(z-1/3)^3}$$

$$\Rightarrow \begin{bmatrix} 1/3 & 1 & 0 \\ 0 & 1/3 & 1 \\ 0 & 0 & 1/3 \end{bmatrix} \quad \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} 3 & -2 & 5 \end{bmatrix} \quad [0]$$

* For the following difference eqⁿ write a State Space representa by all four methods. $y(k+2) + 3y(k+1) + 4y(k) = u(k)$; $y(0)=0$, $y(1)=0$

Sol: Apply z-T/F

$$z^2 Y(z) - z^2 y(0) - z y(1) + 3[z Y(z) - z y(0)] + 4 Y(z) = U(z)$$

$$(z^2+3z+4) Y(z) = \cancel{z^2} U(z) \Rightarrow \frac{Y(z)}{U(z)} = \frac{1}{z^2+3z+4} = \frac{b_0 z^2 + b_1 z + b_2}{z^2 + a_1 z + a_2}$$

Controllable

$$\begin{bmatrix} 0 & 1 \\ -4 & -3 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 \end{bmatrix} [0]$$

$b_2 - a_2 b_0 \quad b_1 - a_1 b_0$

Observable

$$\begin{bmatrix} 0 & -4 \\ 1 & -3 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 1 \end{bmatrix} [0]$$

Diagonal

$$\frac{Y(z)}{U(z)} = \frac{1}{z^2+3z+4}$$

$$= \frac{-b \pm \sqrt{b^2-4ac}}{2a}$$

$$= \frac{-3 \pm \sqrt{9-16}}{2}$$

$$= \frac{-3 \pm \sqrt{-7}}{2}$$

18/12/19 * Solving discrete tym Steady State Eqⁿs :-

Differential Eqⁿs – [Homogeneous → $\frac{dx^2}{dt} + 2\frac{dx}{dx} + k = 0$ → homo → Solⁿ ⇒ CF

Non-homogeneous → " $= \sin x$ → Non ⇒ CF + P.I ↳ by using L.T

We can Solve by using both time & freq domain

→ if Hu(k) is present in diff eqⁿ then it is a non-homo eqⁿ

Steady State eqⁿs are.

$x(k+1) = Gx(k) + Hu(k)$ → State eqⁿ

$y(k) = Cx(k) + Du(k)$ → O/P eqⁿ

To get O/P y(k) we hv to know x(k) it is obtained by solving State e

$k=0$ $x(1) = Gx(0) + Hu(0)$

$k=1$ $x(2) = Gx(1) + Hu(1) = G[Gx(0) + Hu(0)] + Hu(1) = G^2 x(0) + GHu(0) + Hu(1)$

$k=2$ $x(3) = G^3 x(0) + G^2 H u(0) + GHu(1) + Hu(2)$

like wise at $K = K-1$

$x(K) = Gx(K-1) + Hu(K-1)$

$$\boxed{x(K) = \underbrace{G^K x(0)}_{\text{state (C.F)}} + \underbrace{\sum_{j=0}^{K-1} G^{K-j-1} Hu(j)}_{\text{i/p (P.I)}}}$$

G^K = State transition matrix

Consider $x(K+1) = Gx(K)$ —① [from State Eqn] ↳ homo

$x(K) = \varphi(K)\, x(0)$, $\Rightarrow \boxed{\varphi(K) = G^K} \rightarrow$ State transitn matrix

$\varphi(K+1) = G\varphi(K)$ & $\varphi(0) = I$

We see that the soln of eqn ① is simply a transformatn of initial state. Therefore the unique matrix '$\varphi(K)$' is called S.T.M. it is also called fundamental matrix.

→ The S.T.M contains all the i/f about the free motion of the system.

* Z-Transform approach to get the soln of discrete tym state eqn.

$x(K+1) = Gx(K) + Hu(K)$

Apply Z-T/F

$zX(z) - zX(0) = GX(z) + HU(z)$

$zX(z) - GX(z) = zX(0) + HU(z)$; z = value / freq

$(zI - G)X(z) = zX(0) + HU(z)$ → ⓧ

$$\boxed{X(z) = (zI - G)^{-1} z\,x(0) + (zI - G)^{-1} HU(z)}$$

Apply inverse Z-T/F

$z^{-1}\{X(z)\} = z^{-1}\{(zI-G)^{-1} z\,x(0)\} + z^{-1}\{(zI-G)^{-1} HU(z)\}$

$x(K) = z^{-1}\{(zI-G)^{-1} z\}\, x(0) + z^{-1}\{(zI-G)^{-1} HU(z)\}$

$x(K) = G^K x(0) + \sum_{j=0}^{K-1} G^{K-j-1} Hu(j)$

↳ cmp both

$$\boxed{\varphi(K) = G^K = z^{-1}\left[(zI - G)^{-1} z\right]}$$

$$\sum_{j=0}^{K-1} G^{K-j-1} Hu(j) = z^{-1}\{(zI - G)^{-1} HU(z)\}$$

* obtain the State transitn matrix of a following discrete tym system

$x(K+1) = Gx(K) + Hu(K)$

$y(K) = Cx(K) + Du(K)$

where $G = \begin{bmatrix} 0 & 1 \end{bmatrix}$; $H = \begin{bmatrix} 1 \end{bmatrix}$; $C = [1 \;\; 0]$; $D = [0]$

obtain the state $x(k)$ & o/p $y(k)$ when the i/p $u(k) = 1$ for k
assume the initial state $x(0) = \begin{bmatrix} x_1(0) \\ x_2(0) \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$

Sol: find i) STM ii) $x(k)$ iii) $y(k)$

$$\varphi(k) = G^k = Z^{-1}\{(zI-G)^{-1}z\}$$

$$zI - G = z\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} 0 & 1 \\ -0.16 & -1 \end{bmatrix} = \begin{bmatrix} z & -1 \\ 0.16 & z+1 \end{bmatrix}.$$

$$(zI-G)^{-1} = \frac{1}{z(z+1)+0.16}\begin{bmatrix} z+1 & 1 \\ -0.16 & z \end{bmatrix} = \frac{1}{z^2+z+0.16}\begin{bmatrix} z+1 & 1 \\ -0.16 & z \end{bmatrix}$$

$$= \begin{bmatrix} \frac{z+1}{z^2+z+0.16} & \frac{1}{z^2+z+0.16} \\ \frac{-0.16}{z^2+z+0.16} & \frac{z}{z^2+z+0.16} \end{bmatrix} = \begin{bmatrix} \frac{z+1}{(z+0.2)(z+0.8)} & \frac{1}{(z+0.2)(z+0.8)} \\ \frac{-0.16}{(z+0.2)(z+0.8)} & \frac{z}{(z+0.2)(z+0.8)} \end{bmatrix}$$

P.F

i $\frac{z+1}{(z+0.2)(z+0.8)} = \frac{A}{z+0.2} + \frac{B}{z+0.8} \Rightarrow z+1 = A(z+0.8) + B(z+0.2)$

$\Rightarrow -0.8+1 = B(-0.8+0.2)$

$0.2 = B(-0.6) \Rightarrow B = \frac{0.2}{-0.6} = -1/3$

$\Rightarrow -0.2+1 = A(-0.2+0.8)$

$0.8 = A(0.6) \Rightarrow A = \frac{0.8}{0.6} = 4/3$

ii $\frac{1}{(z+0.2)(z+0.8)} = \frac{A}{z+0.2} + \frac{B}{z+0.8}$

$1 = A(z+0.8) + B(z+0.2)$

$1 = B(-0.6)$ | $1 = A(0.6)$

$\boxed{B = -5/3}$ | $\boxed{A = 5/3}$

iii $\frac{-0.16}{(z+0.2)(z+0.8)} = \frac{A}{z+0.2} + \frac{B}{z+0.8}$

$-0.16 = A(z+0.8) + B(z+0.2)$

$+0.16 = B(+0.6)$ | $-0.16 = A(0.6)$

$B = \frac{0.16}{0.6} = \frac{0.8}{0.3}$ | $A = \frac{-0.8}{0.3}$

iv $\frac{z}{(z+0.2)(z+0.8)} = \frac{A}{z+0.2} + \frac{B}{z+0.8}$

$z = A(z+0.8) + B(z+0.2)$

$-0.2 = A(0.6)$ | $-0.8 = B(+0.6)$

$A = -1/3$ | $B = 4/3$

$$(zI-G)^{-1}z = \begin{bmatrix} \frac{\frac{4}{3}z}{z+0.2} - \frac{1/3\,z}{z+0.8} & \frac{5/3\,z}{z+0.2} - \frac{5/3\,z}{z+0.8} \\ \frac{-\frac{0.8}{0.3}z}{z+0.2} + \frac{\frac{0.8}{0.3}z}{z+0.8} & \frac{-1/3\,z}{z+0.2} + \frac{4/3\,z}{z+0.8} \end{bmatrix}$$

$$z^{-1}\left\{(zI-G)^{-1}z\right\} = \begin{bmatrix} \frac{4}{3}z^{-1}\left\{\frac{z}{z+0.2}\right\} - \frac{1}{3}z^{-1}\left\{\frac{z}{z+0.8}\right\} & \frac{5}{3}z^{-1}\left\{\frac{z}{z+0.2}\right\} - \frac{5}{3}z^{-1}\left\{\frac{z}{z+0.8}\right\} \\ -\frac{0.8}{0.3}z^{-1}\left\{\frac{z}{z+0.2}\right\} + \frac{0.8}{0.3}z^{-1}\left\{\quad\right\} & -\frac{1}{3}z^{-1}\left\{\quad\right\} + \frac{4}{3}z^{-1}\left\{\quad\right\} \end{bmatrix}$$

$$\therefore \varphi(K) = \begin{bmatrix} \frac{4}{3}(-0.2)^K - \frac{1}{3}(-0.8)^K & \frac{5}{3}(-0.2)^K - \frac{5}{3}(-0.8)^K \\ \frac{-0.8}{0.3}(-0.2)^K + \frac{0.8}{0.3}(-0.8)^K & -\frac{1}{3}(-0.2)^K + \frac{4}{3}(-0.8)^K \end{bmatrix}$$

ii.

$$X(z) = (zI-G)^{-1}z\,x(0) + (zI-G)^{-1}HU(z)$$

$$X(z) = (zI-G)^{-1}\left[z\,x(0) + HU(z)\right] \qquad \left[Z[u(K)] = U(z) = \frac{z}{z-1}\right]$$

$$= \begin{bmatrix} & \end{bmatrix}\left\{z\begin{bmatrix}1\\-1\end{bmatrix} + \begin{bmatrix}1\\1\end{bmatrix}\left[\frac{z}{z-1}\right]\right\}$$

$$= \begin{bmatrix} & \end{bmatrix}\left\{\begin{bmatrix}z\\-z\end{bmatrix} + \begin{bmatrix}\frac{z}{z-1}\\ \frac{z}{z-1}\end{bmatrix}\right\} = \left[(zI-G)\right]\begin{bmatrix} z+\frac{z}{z-1} \\ -z+\frac{z}{z-1}\end{bmatrix}$$

$$= \begin{bmatrix} \frac{z+1}{(z+0.2)(z+0.8)} & \frac{1}{(z+0.2)(z+0.8)} \\ \frac{-0.16}{(z+0.2)(z+0.8)} & \frac{z}{(z+0.2)(z+0.8)} \end{bmatrix}\begin{bmatrix} \frac{z^2}{z-1} \\ \frac{-z^2+2z}{z-1} \end{bmatrix}$$

$$= \begin{bmatrix} \frac{z^2(z+1)}{(z-1)(z+0.2)(z+0.8)} + \frac{1(-z^2+2z)}{(z-1)(z+0.2)(z+0.8)} \\ \frac{-0.16(z^2)}{(z-1)(z+0.2)(z+0.8)} + \frac{z(-z^2+2z)}{(z-1)(z+0.2)(z+0.8)} \end{bmatrix}$$

$$X(z) = \begin{bmatrix} \frac{z^3+z^2-z^2+2z}{(z-1)(z+0.2)(z+0.8)} \\ \frac{-0.16z^2-z^3+2z^2}{(z-1)(z+0.2)(z+0.8)} \end{bmatrix} = \begin{bmatrix} \frac{z(z^2+2z)}{(z-1)(z+0.2)(z+0.8)} \\ \frac{z\left[-z^2+1.84z\right]}{(z-1)(z+0.2)(z+0.8)} \end{bmatrix}$$

$$\begin{array}{r} 2.00 \\ -0.16 \\ \hline 1.84 \end{array}$$

$$\frac{X(z)}{z} = \begin{bmatrix} \frac{z^2+2z}{(z-1)(z+0.2)(z+0.8)} \\ \frac{-z^2+1.84z}{(z-1)(z+0.2)(z+0.8)} \end{bmatrix}$$

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$$\underline{PF}\quad \frac{z^2+2}{(z-1)(z+0.2)(z+0.8)} = \frac{A}{z-1} + \frac{B}{z+0.2} + \frac{C}{z+0.8}$$

$$z^2+2 = A(z+0.2)(z+0.8) + B(z-1)(z+0.8) + C(z-1)(z+0.2)$$

put $z=1$

$3 = A(1.2)(1.8)$

$A = \frac{3}{(1.2)(1.8)} = \frac{25}{18}$

put $z=-0.2$

$2.04 = B(-1.2)(0.6)$

$B = \frac{2.04}{-1.2\times 0.6} = \frac{-17}{6}$

$z=-0.8$

$C = \frac{22}{9}$

$A+B+C=1$

$\frac{25}{18} - \frac{17}{6} + \frac{22}{9}$ F V

$$X(z) = \begin{bmatrix} \frac{\frac{25}{18}z}{z-1} - \frac{\frac{17}{6}z}{z+0.2} + \frac{\frac{22}{9}z}{z+0.8} \\ \frac{\frac{7}{18}z}{z-1} + \frac{\frac{3.4}{6}z}{z+0.2} - \frac{\frac{17.6}{9}z}{z+0.8} \end{bmatrix}$$

$$x(k) = \begin{bmatrix} \frac{25}{18}u(k) - \frac{17}{6}(-0.2)^k + \frac{22}{9}(-0.8)^k \\ \frac{7}{18}u(k) + \frac{3.4}{6}(-0.2)^k - \frac{17.6}{9}(-0.8)^k \end{bmatrix}$$

iii $y(k) = C\,x(k) = [1 \;\; 0][\quad]$

$$y(k) = \left[\frac{25}{18}u(k) - \frac{17}{6}(-0.2)^k + \frac{22}{9}(-0.8)^k\right]$$

20/12/19 ② Determine the inverse matrix $zI-G$ where $G = \begin{bmatrix} 0.1 & 0.1 & 0 \\ 0.3 & -0.1 & -0.2 \\ 0 & 0 & -0.3 \end{bmatrix}$ also obtain G^k

Sol: $zI-G = z\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} - \begin{bmatrix} 0.1 & 0.1 & 0 \\ 0.3 & -0.1 & -0.2 \\ 0 & 0 & -0.3 \end{bmatrix} = \begin{bmatrix} z-0.1 & -0.1 & 0 \\ -0.3 & z+0.1 & 0.2 \\ 0 & 0 & z+0.3 \end{bmatrix}$

$$(zI-G)^{-1} = \frac{1}{\Delta}\begin{bmatrix} +((z+0.1)(z+0.3)) & -(-0.3(z+0.3)-0) & 0 \\ -((-0.1)(z+0.3)) & +((z-0.1)(z+0.3)-0 & \\ +[(-0.1)(0.2)] & -((z-0.1)(0.2)) & +((z-0.1)(z+0.1)-((-0.1)(-0.3)) \end{bmatrix}^T$$

$-0.01-0.03$

$$= \frac{1}{\Delta}\begin{bmatrix} z^2+0.4z+0.03 & 0.3z+0.09 & 0 \\ 0.1z+0.03 & z^2+0.2z-0.03 & 0 \\ -0.02 & -0.2z+0.02 & z-0.04 \end{bmatrix}^{T}$$

$$\Delta = (z-0.1)(z^2+0.4z+0.03) + 0.1(-0.3z-0.09) + 0$$

$$= z^3+0.4z^2+0.03z-0.1z^2-0.04z-0.003+0.03z+0.009$$

$$\Delta = z^3+0.3z^2+0.04z+0.012 = (z+0.3)(z+0.2)(z-0.2)$$

$$(zI-G)^{-1} = \frac{1}{(z+0.3)(z+0.2)(z-0.2)}\begin{bmatrix} (z+0.1)(z+0.3) & 0.1(z+0.3) & -0.02 \\ 0.3(z+0.3) & (z-0.1)(z+0.3) & -0.2(z+0.1) \\ 0 & 0 & (z+0.2)(z-0.2) \end{bmatrix}$$

$$(zI-G)^{-1} = \begin{bmatrix} \frac{z+0.1}{(z+0.2)(z-0.2)} & \frac{0.1}{(z+0.2)(z-0.2)} & \frac{-0.02}{(z+0.3)(z+0.2)(z-0.2)} \\ \frac{0.3}{(z+0.2)(z-0.2)} & \frac{z-0.1}{(z+0.2)(z-0.2)} & \frac{-0.2(z-0.1)}{(z+0.3)(z+0.2)(z-0.2)} \\ 0 & 0 & \frac{1}{z+0.3} \end{bmatrix}$$

PF :

$$\frac{z+0.1}{(z+0.2)(z-0.2)} = \frac{A}{z+0.2} + \frac{B}{z-0.2}$$

$z+0.1 = A(z-0.2) + B(z+0.2)$

$z=0.2 \Rightarrow 0.3 = B(0.4)$

$\boxed{B = 3/4}$

$z=-0.2$

$-0.1 = A(-0.4)$

$\boxed{A = 1/4}$

y. $0.3 = A(z-0.2)+B(z+0.2)$

$0.3 = 0.4B \Rightarrow \boxed{B = 3/4}$

$0.3 = A(-0.4) \Rightarrow \boxed{A = -3/4}$

3.

$z-0.1 = A(z-0.2)+B(z+0.2)$

$0.1 = B(0.4) \Rightarrow \boxed{B = 1/4}$

$+0.3 = A(-0.4)$

$\boxed{A = 3/4}$

2

$$0.1 = \frac{A}{z+0.2} + \frac{B}{z+0.2} \Rightarrow 0.1 = A(z-0.2) + B(z+0.2)$$

$0.1 = B(0.4) \Rightarrow \boxed{B = 1/4}$

$0.1 = A(-0.4) \Rightarrow \boxed{A = -1/4}$

3.

$$\frac{-0.02}{(z+0.3)(z+0.2)(z-0.2)} = \frac{A}{z+0.3} + \frac{B}{z+0.2} + \frac{C}{z-0.2}$$

$-0.02 = A(z+0.2)(z-0.2) + B(z+0.3)(z-0.2) + C(z+0.3)(z+0.2)$

$z=0.2$

$-0.02 = C(0.4)(0.5)$

$C = \frac{-0.02}{0.2} = -1/10$

$z = -0.2$

$-0.02 = B(0.1)(-0.4)$

$B = \frac{-0.02}{-0.04} = 1/2$

z^2 A $z=-0.3$

$-0.02 = A(-0.1)(-0.5) \Rightarrow A = \frac{0.02}{0.05}$

$\boxed{A = 2/5}$

$$\frac{6}{z}\,\frac{-0.2(z-0.1)}{(z+0.3)(z-0.2)(z+0.2)} = \frac{A}{z+0.3} + \frac{B}{z-0.2} + \frac{C}{z+0.2}$$

$$-0.2(z-0.1) = A(z-0.2)(z+0.2) + B(z+0.3)(z+0.2) + C(z+0.3)(z-0.2)$$

$z=-0.2$

$$-0.2(0.1) = B(0.5)(0.4)$$

$$\frac{-0.02}{0.2} = B \Rightarrow \boxed{B = 1/10}$$

$$-0.2(-0.3) = C(0.1)(-0.04)$$

$$0.06 = C(-0.04) \Rightarrow C = \frac{0.06}{-0.04} = -3/2$$

$z=-0.3$

$$-0.2(-0.4) = A(-0.5)(-0.1)$$

$$0.08 = A(005) \Rightarrow \boxed{A=8/5}$$

$$(zI-G)^{-1}z = \begin{bmatrix} \frac{1/4z}{z+0.2}+\frac{3/4z}{z-0.2} & \frac{-1/4z}{z+0.2}+\frac{1/4z}{z-0.2} & \frac{2/5z}{z+0.3}+\frac{1/2z}{z+0.2}-\frac{1/10z}{z-0.2} \\ \frac{-3/4z}{z+0.2}+\frac{3/4z}{z-0.2} & \frac{3/4z}{z+0.2}+\frac{1/4z}{z-0.2} & \frac{8/5z}{z+0.3}+\frac{1/10z}{z+0.2}-\frac{3/2z}{z-0.2} \\ 0 & 0 & z/z+0.3 \end{bmatrix}$$

$$z^{-1}\{(zI-G)^{-1}z\} = \begin{bmatrix} \frac{1}{4}z^{-1}\left\{\frac{z}{z+0.2}\right\}+\frac{3}{4}z^{-1}\left\{\frac{z}{z-0.2}\right\} & -1/4z^{-1}\{\ \}+\frac{1}{4}z^{-1}\{\ \} & \frac{2}{5}z^{-1}\{\ \}+\frac{1}{2}z^{-1} - 1/10z^{-1}\{\ \} \\ \frac{-3}{4}z^{-1}\{\ \}+\frac{3}{4}z^{-1}\{\ \} & \frac{3}{4}z^{-1}\{\ \}+1/4z^{-1}\{\ \} & \frac{8}{5}z^{-1}\{\ \}+\frac{1}{10}z^{-1}\{\ \}\frac{3}{2}z^{-1} \\ 0 & 0 & z^{-1}\{\ \} \end{bmatrix}$$

$$\varphi(k) = \begin{bmatrix} 1/4(-0.2)^{k}+3/4(0.2)^{k} & -1/4(-0.2)^{k}+1/4(0.2)^{k} & \frac{2}{5}(-0.3)^{k}+\frac{1}{2}(-0.2)^{k}-1/10(0.3)^{k} \\ -3/4(-0.2)^{k}+\frac{3}{4}(0.2)^{k} & \frac{3}{4}(-0.2)^{k}+1/4(0.2)^{k} & \frac{8}{5}(-0.3)^{k}+1/10(-0.2)^{k}-3/2(0.2)^{k} \\ 0 & 0 & (-0.3)^{k} \end{bmatrix}$$

23/12/19 ③ * consider descrete tym cs represented by following T.F $G(z) = \frac{1+0.8z^{-1}}{1-z^{-1}+0.5z^{-2}}$. obtain a State Space representatⁿ of the system in observable canonical form also find its S.T.M.

Sol:-

$$G(z) = \frac{1+0.8z^{-1}}{1-z^{-1}+0.5z^{-2}} = \frac{z^2+0.8z}{z^2-z+0.5}$$

observable

$$\begin{bmatrix} x_1(k+1) \\ x_2(k+1) \end{bmatrix} = \begin{bmatrix} 0 & -0.5 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} x_1(k) \\ x_2(k) \end{bmatrix} + \begin{bmatrix} -0.5 \\ 1.8 \end{bmatrix} u(k)$$

$$y(k) = \begin{bmatrix} 0 & 1 \end{bmatrix} \begin{bmatrix} x_1(k) \\ x_2(k) \end{bmatrix} + [1][u(k)]$$

$$zI - G = \begin{bmatrix} z & 0.5 \\ -1 & z-1 \end{bmatrix} \Rightarrow (zI-G)^{-1} = \frac{1}{z^2-z+0.5} \begin{bmatrix} z-1 & -0.5 \\ 1 & z \end{bmatrix}$$

$$(zI-G)^{-1}z = \begin{bmatrix} \frac{z^2-z}{z^2-z+0.5} & \frac{-0.5z}{z^2-z+0.5} \\ \frac{z}{z^2-z+0.5} & \frac{z^2}{z^2-z+0.5} \end{bmatrix}$$

roots $b^2-4ac = (-1)^2-4(1)(0.5)$
$= 1-2 = -1 < 0$

$$z[a^k \sin(\omega_0 kT)] = \frac{az\sin(\omega_0 T)}{z^2-2az\cos(\omega_0 T)+a^2}$$

$\Rightarrow a^2 = 0.5;\ a = 0.707$
$= 1/\sqrt{2}$

$-2az\cos(\omega_0 T) = -1$

$2\left(\frac{1}{\sqrt{2}}\right)z\cos(\omega_0 T) = 1$

$\cos(\omega_0 T) = 1/\sqrt{2}$

$\omega_0 T = \cos^{-1}(1/\sqrt{2})$

$= \pi/4.$

$z(1/\sqrt{2})^k \sin\left[\frac{\pi}{4}k\right] = \frac{1}{-\sqrt{2}}z$

$$z\left[(1/\sqrt{2})^k \sin\left(\frac{\pi}{4}k\right)\right] = \frac{1/\sqrt{2}\, z\, 1/\sqrt{2}}{z^2-z+0.5}$$

$$z\left[(1/\sqrt{2})^k \cos\left(\frac{\pi}{4}k\right)\right] = \frac{z^2-z/2}{z^2-z+0.5}$$

$$= \begin{bmatrix} \frac{z^2-z/2-z/2}{z^2-z+0.5} & \frac{-z/2}{z^2-z+0.5} \\ \frac{2z/2}{z^2-z+0.5} & \frac{z^2-z/2+z/2}{z^2-z+0.5} \end{bmatrix}$$

$$= \begin{bmatrix} \frac{z^2-z/2}{z^2-z+0.5} - \frac{z/2}{z^2-z+0.5} & \frac{-z/2}{z^2-z+0.5} \\ \frac{2z/2}{z^2-z+0.5} & \frac{z^2-z/2}{z^2-z+0.5} + \frac{z/2}{z^2-z+0.5} \end{bmatrix}$$

$$\phi(k) = \begin{bmatrix} (1/\sqrt{2})^k \cos\frac{\pi}{4}k - (1/\sqrt{2})^k \sin\frac{\pi}{4}k & -1/\sqrt{2}\sin(\pi/4)k \\ 2(1/\sqrt{2})^k \sin(\pi/4 k) & (1/\sqrt{2})^k \cos\left(\frac{\pi}{4}k\right) + (1/\sqrt{2})^k \sin\left(\frac{\pi}{4}k\right) \end{bmatrix}$$

④ for a Homogeneous System given by $x(k+1) = \begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix} x(k)$. Obtain S.T.M $\varphi(k)$

$z^2+2z+z+2$

$(z+1)(z+2)$

Sol: $(zI-G)^{\circ} = \begin{bmatrix} z & 0 \\ 0 & z \end{bmatrix} - \begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix} = \begin{bmatrix} z & -1 \\ 2 & z+3 \end{bmatrix}$

$$(zI-G)^{-1}z = \frac{1}{z^2+3z+2}\begin{bmatrix} z+3 & 1 \\ -2 & z \end{bmatrix} = \begin{bmatrix} \frac{z+3}{(z+2)(z+1)} & \frac{1}{(z+2)(z+1)} \\ \frac{-2}{(z+2)(z+1)} & \frac{z}{(z+2)(z+1)} \end{bmatrix}$$

1

$\frac{z+3}{(z+2)(z+1)} = \frac{A}{z+2} + \frac{B}{z+1}$

$z+3 = A(z+1) + B(z+2)$

$\boxed{2=B}$ | $A+B=1$ $\boxed{A=-1}$

2

$\frac{1}{(z+2)(z+1)} = \frac{A}{z+2} + \frac{B}{z+1} \Rightarrow 1 = A(z+1) + B(z+2)$

$\boxed{1=B}$ | $A+B=0$ $\boxed{A=-1}$

3

$-2 = A(z+1) + B(z+2)$

$\boxed{-2=B} \Rightarrow A+B=0$ $\boxed{A=2}$

4

$z = A(z+1) + B(z+2)$

$\boxed{-1=B}$ | $A+B=1$ $\boxed{A=2}$

$$(zI-G)^{-1}z = \begin{bmatrix} \frac{-1}{z+2} + \frac{2}{z+1} & \frac{-1}{z+2} + \frac{1}{z+1} \\ \frac{2}{z+2} - \frac{2}{z+1} & \frac{2}{z+2} - \frac{1}{z+1} \end{bmatrix}$$

$$z^{-1}\left\{(zI-G)^{-1}z\right\} = \begin{bmatrix} z^{-1}\left\{\frac{-z}{z+2}\right\} + 2z^{-1}\left\{\frac{z}{z+1}\right\} & z^{-1}\left\{\frac{-z}{z+2}\right\} + z^{-1}\left\{\frac{z}{z+1}\right\} \\ 2z^{-1}\left\{\frac{z}{z+2}\right\} - 2z^{-1}\left\{\frac{z}{z+1}\right\} & 2z^{-1}\left\{\frac{z}{z+2}\right\} - z^{-1}\left\{\frac{z}{z+1}\right\} \end{bmatrix}$$

$$\varphi(k) = \begin{bmatrix} -(-2)^k + 2(-1)^k & -(-2)^k + (-1)^k \\ 2(-2)^k - 2(-1)^k & 2(-2)^k - (-1)^k \end{bmatrix}$$

② Find the state model for the following difference eqn and also find its S.T.M. Assume initial condn's are zero

Sol: $y(k+2) + 3y(k+1) + 2y(k) = 2u(k+1) + u(k)$

$[z^2+3z+2]\, Y(z) = U(z)\,[2z+1]$

$$\frac{Y(z)}{U(z)} = \frac{2z+1}{z^2+3z+2}$$

* **PTF: (pulse Transfer Function):**

$x(K+1) = G\,X(K) + H\,U(K)$

$Y(K) = C\,X(K) + D\,U(K)$

$Z\,X(Z) = G\,X(Z) + H\,U(Z)$; $Y(Z) = C\,X(Z) + D\,U(Z)$

$(ZI - G)\,X(Z) = H\,U(Z) \Rightarrow X(Z) = (ZI - G)^{-1} H\,U(Z)$

$Y(Z) = C\left[(ZI - G)^{-1} H\,U(Z)\right] + D\,U(Z)$

$Y(Z) = \left[C(ZI-G)^{-1}H + D\right] U(Z)$; $\dfrac{Y(Z)}{U(Z)} = C(ZI-G)^{-1}H + D$

Solution of C.T SS Eqn's :-

$x = Ax + Bu$ —① ; $y = Cx + Du$ —②

$x - Ax = Bu(t)$

multiply with e^{-At} on B.S

$e^{-At}[x - Ax] = e^{-At}\,Bu(t)$

$\frac{d}{dt}\left[e^{-At}x(t)\right] = e^{-At}\,Bu(t)$

integrate w.r.t "τ" on B.S

$e^{-At}x(t) - x(0) = \int_0^t e^{-A\tau} Bu(\tau)\,d\tau$

$$e^{-At}x(t) = x(0) + \int_0^t e^{-A\tau} Bu(\tau)\, d\tau$$

$$x(t) = e^{At}x(0) + e^{At}\int_0^t e^{-A\tau} Bu(\tau)\, d\tau$$

$$x(t) = e^{At}x(0) + \int_0^t e^{A(t-\tau)} Bu(\tau)\, d\tau$$

Discretization of C.T SSE :-

$$\left.\begin{array}{l} \dot{x} = Ax + Bu \\ y = Cx + Du \end{array}\right\} \Rightarrow \begin{array}{l} x(k+1) = Gx(k) + Hu(k) \\ y(k) = Cx(k) + Du(k) \\ x[(k+1)T] = G(T)x(kT) + H(T)u(T) \end{array}$$

$$x(t) = e^{At}x(0) + \int_0^t e^{A(t-\tau)} Bu(\tau)\, d\tau \quad — ①$$

$$x(kT) = e^{AkT}x(0) + \int_0^{kT} e^{A(kT-\tau)} Bu(\tau)\, d\tau$$

$$x(kT) = e^{AkT}x(0) + e^{AkT}\int_0^{kT} e^{-A\tau} Bu(\tau)\, d\tau \quad — ②$$

$$x((k+1)T) = e^{A(k+1)T}x(0) + e^{A(k+1)T}\int_0^{(k+1)T} e^{-A\tau} Bu(\tau)\, d\tau \quad — ③$$

multiply eqn ② with e^{AT} on B.S

$$e^{AT}x(kT) = e^{AT}e^{AkT}x(0) + e^{AT}e^{AkT}\int_0^{kT} e^{-A\tau} Bu(\tau)\, d\tau$$

$$e^{AT}x(kT) = e^{A(k+1)T}x(0) + e^{A(k+1)T}\int_0^{kT} e^{-A\tau} Bu(\tau)\, d\tau \quad — ④$$

Subtract ③ - ④

$$x[(k+1)T] - e^{AT}x(kT) = 0 + e^{A(k+1)}\left[\int_0^{(k+1)T} e^{-A\tau} Bu(\tau)\, d\tau - \int_0^{kT} e^{-A\tau} Bu(\tau)\, d\tau\right]$$

$$x[(k+1)T) - e^{AT}x(kT) = e^{A(k+1)T}\int_{kT}^{(k+1)T} e^{-A\tau} Bu(\tau)\, d\tau$$

$$= \int_{kT}^{(k+1)T} e^{(A(k+1)T-\tau)} Bu(\tau)\, d\tau$$

put $t-\tau = \lambda \Rightarrow d\tau = d\lambda$

$$= \int_{kT}^{(k+1)T} e^{A((k+1)T-\tau)} Bu(kT)\, d\tau$$

$$x((k+1)T) - e^{AT}x(kT) = \int_0^T e^{A\lambda} Bu(kT)\, d\lambda$$

$$x[(k+1)T] = e^{AT}x(kT) + \int_0^T e^{A\lambda} Bu(kT)\, d\lambda$$

$$x[(k+1)T] = e^{AT}x(kT) + \int_0^T e^{A\lambda} B\,d\lambda \; u(kT)$$

$$x[(k+1)T] = G(T)\,x(kT) + H(T)\,u(kT)$$

$$\boxed{\begin{aligned} G(T) &= e^{AT} \\ H(T) &= \int_0^T e^{A\lambda} B\,d\lambda \end{aligned}}$$

27/12/19

* Controllability :-

A system is said to be completely state controllable if it is possible to transfer the system from any orbitary initial state to any desired state in a finite tym period. that is the c.s is controllable of every state variable can be controlled in a finite tym period by some unconstraint control signal.

$\text{rank}[H \;\; GH \;\; G^2H \;\; ---- \;\; G^{n-1}H] = n \rightarrow$ completely controllable

$< n \rightarrow$ not controllable

* Observability

The system is completely observable if every initial state $x(0)$ can be determined from the observatⁿ of $y(kT)$, over a finite no. of sampling periods. Therefore the system is completely observable if every transition of the state effects every element of the o/p vector. The concept of observability is useful in solving the problem of reconstruct^g unmeasurable state variables

$$\text{rank}\begin{bmatrix} C \\ CG \\ CG^2 \\ \vdots \\ CG^{n-1} \end{bmatrix} \begin{aligned} &= n \rightarrow \text{completely observable} \\ &< n \rightarrow \text{not} \end{aligned}$$

4

① Investigate controllability & observability for the following system

$$\begin{bmatrix} x_1(k+1) \\ x_2(k+1) \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -0.16 & -1 \end{bmatrix}\begin{bmatrix} x_1(k) \\ x_2(k) \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u(k); \quad y(k) = [1 \;\; 0]\begin{bmatrix} x_1(k) \\ x_2(k) \end{bmatrix} + [0]\,u(k)$$

Sol: $G = \begin{bmatrix} 0 & 1 \\ -0.16 & -1 \end{bmatrix}$; $H = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$; $C = [1 \;\; 0]$

Controllability matrix $M = [H \;\; GH]$

$$GH = \begin{bmatrix} 0 & 1 \\ -0.16 & -1 \end{bmatrix}\begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

$M = \begin{bmatrix} 0 & 1 \\ 1 & -1 \end{bmatrix}$ $\quad \det(M) = 0 - 1 = -1 \neq 0$; rank of $M = 2$

∴ System is completely controllable

observability matrix $= \begin{bmatrix} C \\ CG \end{bmatrix}$

$$CG = \underset{(1\times2)}{[1 \quad 0]} \underset{(2\times2)}{\begin{bmatrix} 0 & 1 \\ -0.16 & -1 \end{bmatrix}} = \underset{(1\times2)}{[0 \quad 1]}$$

$$\begin{bmatrix} C \\ CG \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \Rightarrow \det = 1 \neq 0 \Rightarrow \text{rank is '2'}$$

∴ System is completely observable.

② consider the following pulse Transfer function $\frac{Y(z)}{U(z)} = \frac{z+0.2}{(z+0.8)(z+0.2)}$ obtain contr & observ.

Sol. $CG = [0.2 \quad 1] \begin{bmatrix} 0 & 1 \\ -0.16 & -1 \end{bmatrix} = [-0.16 \quad -0.8]$

$$\begin{bmatrix} C \\ CG \end{bmatrix} = \begin{bmatrix} 0.2 & 1 \\ -0.16 & -0.8 \end{bmatrix} \Rightarrow \det = -0.16 - (-0.16) = 0$$

rank < 2 ∴ System is not observabl.

30/12/19

③ For the following system $\frac{Y(z)}{U(z)} = \frac{z^{-1}(1+0.8z^{-1})}{1+1.3z^{-1}+0.4z^{-2}}$ obtain controllable and observable system.

Sol. $G = \begin{bmatrix} 0 & 1 \\ -0.4 & -1.3 \end{bmatrix}$ $H = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$ $C = [0.8 \quad 1]$

controllability matrix $= [H \quad GH]$

$$GH = \underset{(2\times2)}{\begin{bmatrix} 0 & 1 \\ -0.4 & -1.3 \end{bmatrix}} \underset{(2\times1)}{\begin{bmatrix} 0 \\ 1 \end{bmatrix}} = \begin{bmatrix} 1 \\ -1.3 \end{bmatrix}$$

$$M = [H \quad GH] = \begin{bmatrix} 0 & 1 \\ 1 & -1.3 \end{bmatrix} \Rightarrow \det(M) = 0 - 1 = -1 \neq 0$$

Rank of M is 2 ⇒ ∴ System is completely controllable

Observable

$$CG = \underset{(1\times2)}{[0.8 \quad 1]} \underset{(2\times2)}{\begin{bmatrix} 0 & 1 \\ -0.4 & -1.3 \end{bmatrix}} = [-0.4 \quad -0.5]$$

$$\begin{bmatrix} C \\ CG \end{bmatrix} = \begin{bmatrix} 0.8 & 1 \\ -0.4 & -0.5 \end{bmatrix} = -0.4 + 0.4 = 0 < 2$$

∴ System is not observable.

④ Find the State model for the following diff. eqⁿ $y(k+2) + 3y(k+1) + 2y(k) = 2u(k+1) + u(k)$. Determine controllability & observability. Assume initial condⁿs are zero.

Sol. pulse T.F ⇒ $\frac{Y(z)}{U(z)} = \frac{2z+1}{z^2+3z+2} = \frac{2z+1}{(z+1)(z+2)}$

Controllable

$$\begin{bmatrix} x_1(K+1) \\ x_2(K+1) \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix} \begin{bmatrix} x_1(K) \\ x_2(K) \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u(K)$$

$$y(K) = \begin{bmatrix} -1 & -1 \end{bmatrix} \begin{bmatrix} x_1(K) \\ x_2(K) \end{bmatrix} + [1]\, u(K)$$

Observability

$$\begin{bmatrix} x_1(K+1) \\ x_2(K+1) \end{bmatrix} = \begin{bmatrix} 0 & -2 \\ 1 & -3 \end{bmatrix} \begin{bmatrix} x_1(K) \\ x_2(K) \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u(K)$$

$$y(K) = \begin{bmatrix} -1 \\ -1 \end{bmatrix} \begin{bmatrix} x_1(K) \\ x_2(K) \end{bmatrix} + [0]\, u(K)$$

For controllability matrix.

$$GH = \begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ -3 \end{bmatrix}$$

$$M = [H \quad GH] = \begin{bmatrix} 0 & 1 \\ 1 & -3 \end{bmatrix}$$

$$|M| = 0 - 1 \neq 0$$

∴ System is ~~not~~ controllable

For observability matrix :-

$$CG = [-1 \quad -1] \begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix} = [-2 \quad -22]$$

$$\begin{bmatrix} C \\ CG \end{bmatrix} = \begin{bmatrix} -1 & -1 \\ -2 & -22 \end{bmatrix} = 2.2 - 2$$

$$= 0.2 \neq 2$$

∴ System is observable.

⑤ Consider a system described by

$$\begin{bmatrix} x_1(K+1) \\ x_2(K+1) \\ x_3(K+1) \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -1 & -5 & 6 \end{bmatrix} \begin{bmatrix} x_1(K) \\ x_2(K) \\ x_3(K) \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} u(K)$$

$$y(K) = [0 \quad 0 \quad 1] \begin{bmatrix} x_1(K) \\ x_2(K) \\ x_3(K) \end{bmatrix} + [0]\, u(K)$$

. Determine its controllable & observable

Sol:- **Controllable**

$$M = [H \quad GH \quad G^2H]$$

$$G^2H = G(GH) \Rightarrow GH = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -1 & -5 & 6 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ -6 \end{bmatrix}$$

$$G^2H = G(GH) = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -1 & -5 & 6 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \\ -6 \end{bmatrix} = \begin{bmatrix} 1 \\ -6 \\ 31 \end{bmatrix}$$

$$M = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & -6 \\ 1 & -6 & 31 \end{bmatrix} = 0 - 1 = -1$$

$$|M| = -1 \neq 0$$

∴ System is controllable

Observable

$$CG = [-1 \quad -5 \quad -6]$$

$$C^2G = (CG)C = \begin{bmatrix} 0 & 0 & -1 \\ 1 & 0 & -5 \\ 0 & 1 & -6 \end{bmatrix} \begin{bmatrix} -1 & -5 & -6 \end{bmatrix}$$

$$= [6 \quad 29 \quad 31]$$

$$C = \begin{bmatrix} 0 & 0 & 1 \\ -1 & -5 & -6 \\ 6 & 29 & 31 \end{bmatrix}$$

$$|O| = 0 - 0 + 1(-29 + 30) = 1 \neq 0$$

∴ It is not observable