4/01/20

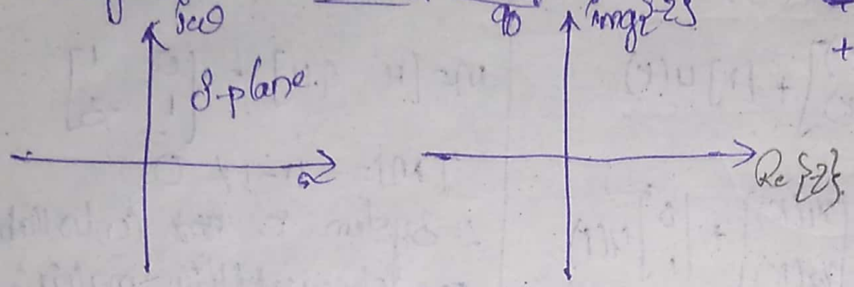
UNIT - 4

STABILITY ANALYSIS

Mapping b/w s-plane & z-plane.

-ve direction is 180, 270

+ve direction is 90, 0



Mathematical expression for x(kT):-

$$x^*(s) = \sum_{k=0}^{\infty} x(k) e^{-skT}$$

$$X(z) = \sum_{k=0}^{\infty} x(k) z^{-k}$$

$$\boxed{z = e^{sT}}$$

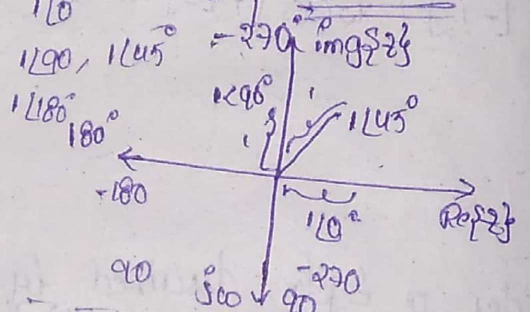
$$= e^{(\sigma + j\omega)T} \quad (\because s = \sigma + j\omega)$$

$$= e^{\sigma T} + e^{j\omega T}$$

$$= e^{\sigma T}(\cos\omega T + j\sin\omega T)$$

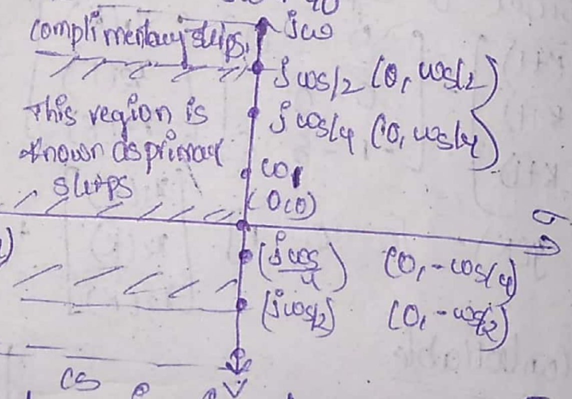
$$z = e^{\sigma T} \angle \omega T \text{ (polar form)}$$

Representation in polar form:



Mapping :- $T = 2\pi/\omega_s$

$(0,0)$	$(0,\omega_s/4)$	$(0,\omega_s/2)$	$(0,-\omega_s/4)$
$= e^0$	$= e^{\frac{j\omega_s}{4}\cdot\frac{2\pi}{\omega_s}}$	$= e^{\frac{-j\omega_s}{2}\cdot\frac{2\pi}{\omega_s}}$	$= e^{-j\pi/2}$
$= 1\angle 0°$	$= e^{j\pi/2}$	$= e^{j\pi}$	$= 1\angle 90$
	$= 1\angle 90$	$= 1\angle 180°$	



* Relation b/w s-plane and z-plane is given by eqn $z = e^{sT}$

$\therefore z = e^{\sigma T} \angle \omega T$

Since $\angle z = \omega T$, the $\angle z$ varies from $-\infty$ to $+\infty$ as ω varies from $-\infty$ to ∞

Consider a representative pt on the $j\omega$ axis in s-plane. ~~[is mapped into a radial line of const. angle $\omega_1 T$ radius in z-plane as shown in fig]~~ As this pt moves from ($-j\omega_s/2$ to $+j\omega_s/2$) on the $j\omega$-axis, we have magnitude of $z = 1$ ($|z| = 1$) & angle of z i.e. $\angle z$ varies from ($-\pi$ to π) in the counter clkwise direction in z-plane. As the representative pt mvs from $j\omega_s/2$ to $j\omega_s/2$ on $j\omega$ axis. The corresponding pt in z plane traces out the unit circle in the counter clkwise direction.

Thus, as pt in s-plane mvs from $-\infty$ to ∞ on the $j\omega$ axis, retrace the unit circle in z plane an infinite no. of times

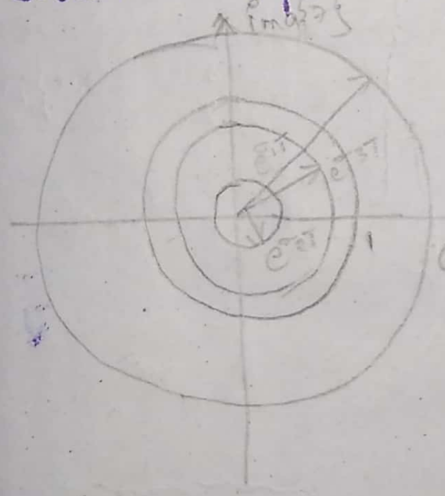
From this, analysis it is clear that each strip of width ω_s in the left half of s-plane maps into inside of unit circle in z-plane.

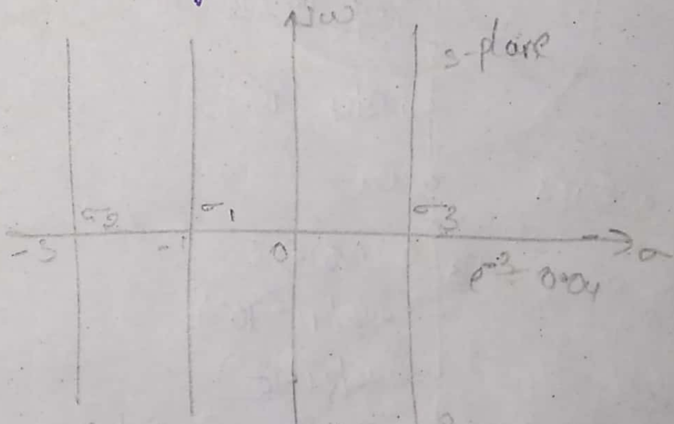
This implies that the left half of the s-plane may be divided into an infinite no. of periodic strips as shown in the fig.

→ The primary strip extends from $j\omega = -\frac{j\omega_s}{2}$ to $+\frac{j\omega_s}{2}$

→ The complimentary strips extends from $\frac{j\omega_s}{2}$ to $\frac{j3\omega_s}{2}$, $\frac{j3\omega_s}{2}$ to $j5\omega_s/2$, $-j\omega_s/2$ to $-\frac{j3\omega_s}{2}$; $-j\frac{3\omega_s}{2}$ to $-j\frac{5\omega_s}{2}$ and so on....

Constant attenuation line:- A constant attenuation line is plotted as σ = constant in the s-plane. Maps to a circle of radius i.e., $z = e^{\sigma T}$ centered at origin in the z-plane as shown in fig.





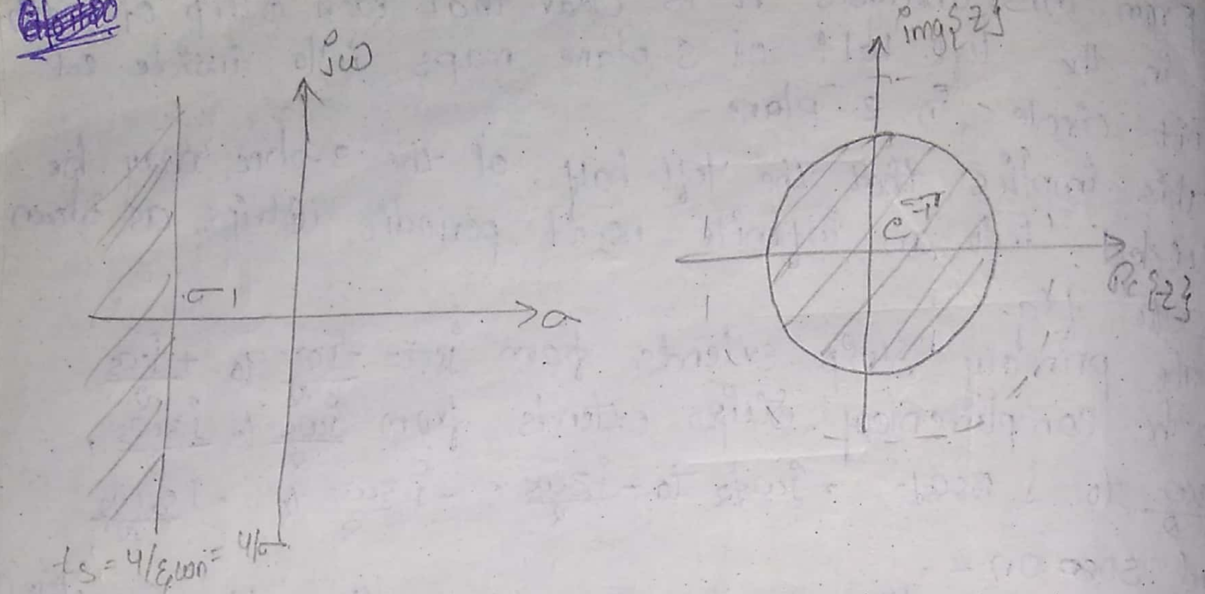
$e^{(\sigma_1 + j\omega)T} = e^{\sigma_1 T} e^{j\omega T} = e^{\sigma_1 T}\angle \omega T$

let T = 1 sec

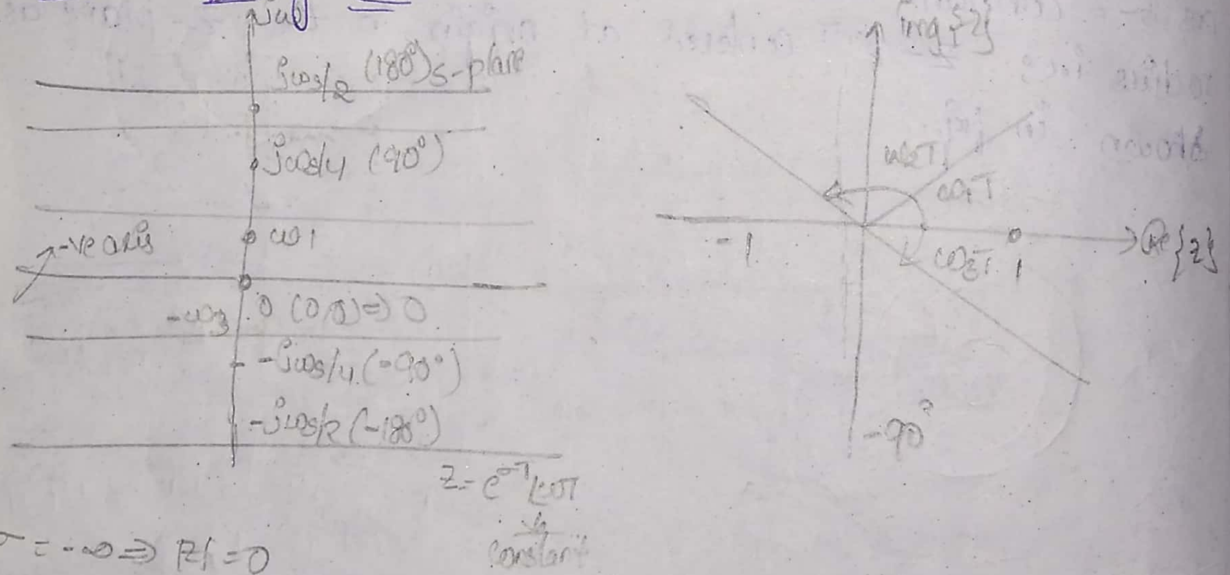
$e^{-1} = 0.3673$

Settling Time (T_s):- The settling time (T_s) is determined by the value of attenuation sigma (σ) of the dominant closed loop poles. If the settling time is specified, it is possible to draw a line $\sigma = -\sigma_1$ in the s-plane corresponding to a given settling time.

The region to the left of the line $\sigma = -\sigma_1$ in the s-plane corresponds to the inside of circle with radius $e^{-\sigma_1 T}$ in the z-plane as shown in the fig.



Constant frequency locus:-



$\sigma = -\infty \Rightarrow |z| = 0$

$\sigma = 0 \Rightarrow |z| = 1$

$\sigma = \infty \Rightarrow |z| = \infty$

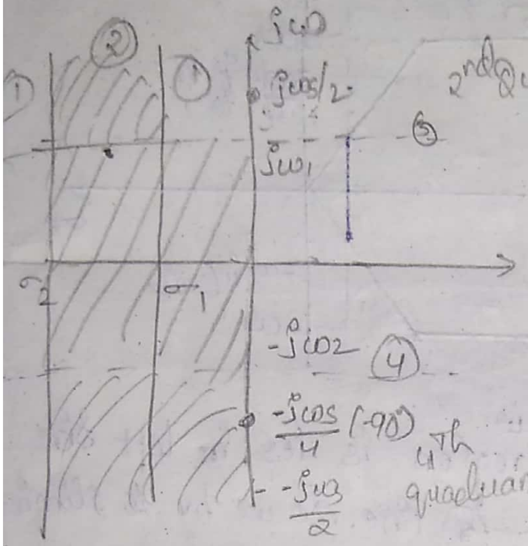
A constant freq. locus $\omega = \omega_1$ in the s-plane is mapped into a radial line of constant angle $\omega_1 T$ radius in the z-plane as shown in fig.

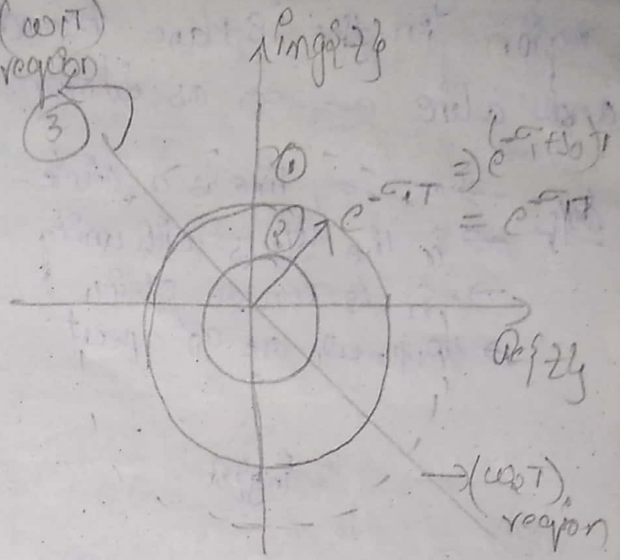
Note:- the constant freq. line $\omega = \pm\omega_s/2$ in the left half of s-plane corresponds to the -ve real axes in the z-plane b/w 0 & -1. Since $\boxed{T(\pm\omega_s/2) = \pm\pi}$ radians.

constant freq. lines at $\omega = \pm\omega$ in the right half of the s-plane corresponds to the -ve real axis in the z-plane b/w -1 and $-\infty$

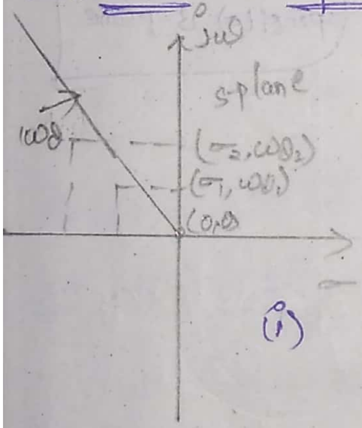
* Map the region shown in the s-plane into z-plane.

Sol:

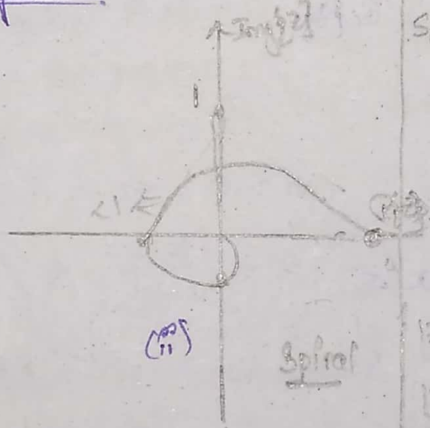




* Constant damping ratio: (ξ)



(i)



(ii) Spiral

$S_1, S_2 = -\xi\omega_n + j\omega_n\sqrt{1-\xi^2}$

$z = e^{(-\xi\omega_n + j\omega_n\sqrt{1-\xi^2})T}$

$= e^{-(\xi\omega_n T + j\omega_n\sqrt{1-\xi^2}\cdot T)}$

$= e^{-\left(\xi\frac{\omega_d}{\sqrt{1-\xi^2}}\cdot\frac{2\pi}{\omega_s} + j\omega_d\frac{2\pi}{\omega_s}\right)}$ $\quad \omega_d = \omega_n\sqrt{1-\xi^2}$, $\omega_n = \frac{\omega_d}{\sqrt{1-\xi^2}}$, $T = \frac{2\pi}{\omega_s}$

$|z| = e^{-\frac{2\pi\xi}{\sqrt{1-\xi^2}}\frac{\omega_d}{\omega_s}}$

$\angle z = \omega_d \cdot \frac{2\pi}{\omega_s}$ → constant

ξ = constant

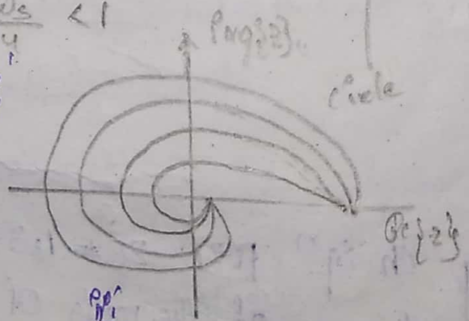
z-plane is varies w.r.t (ω_d)

s-plane

i (0,0) $\Rightarrow e^{(0+j0)\cdot T} = e^0 = 1\angle 0°$

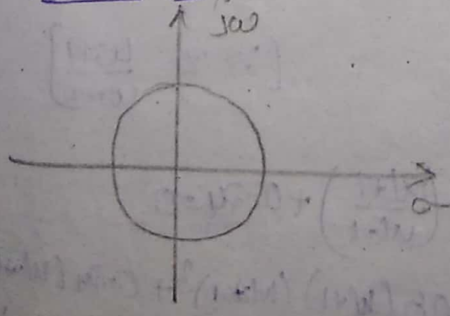
ii $(\sigma_1, \omega_{d_1}) \Rightarrow \sigma_1 = \frac{\omega_s}{4} < 1$

for diff values of ξ



(iii)

* constant ω_n :-

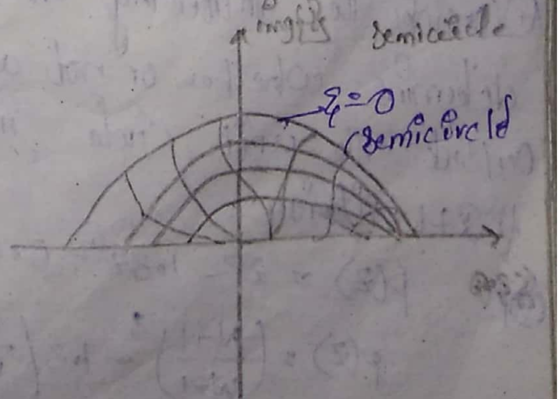


$0.2\left(\frac{\omega_d}{2}\right)$

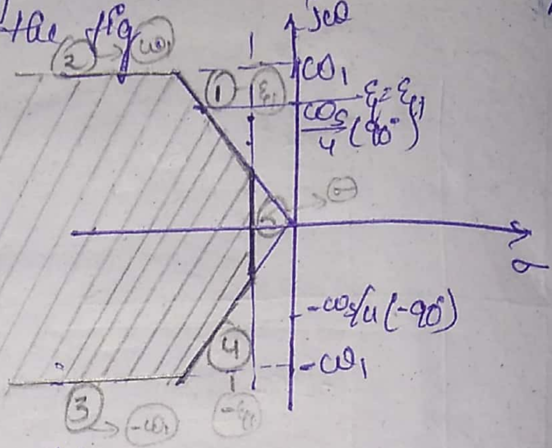
$0.4\left(\frac{\omega_d}{2}\right)$

$0.8\left(\frac{\omega_d}{2}\right)$

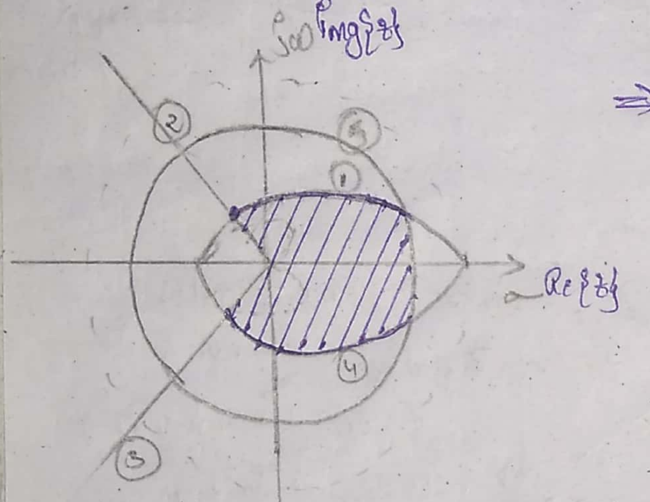
$1\left(\omega_d/2\right)$



* Specify the region in z-plane that corresponds to a desirable region in the s-plane bounded by the line $\omega = \pm\omega_1$, lines $\xi = \xi_1$ and a line $\sigma = -\sigma_1$ as shown in the fig.



Sol: → $\sigma = -\sigma_1$ line is a circle.
→ ξ_1 line starts with unity
→ $-\xi_1$ is reverse of ξ_1
→ ω_1 & $-\omega_1$ are 90° apart



⇒ The entire region is lies in left side of σ (5) line. So we hv to eliminate (1), (5) lines

$\boxed{\text{blue pen} \to z\text{-plane} \quad \text{pencil} \Rightarrow s\text{-plane}}$

* **Modified R-H Criteria:-**

→ No. of sign changes represents no. of poles present in RHS

We can apply RH for z-plane so we hv to convert z-plane → s-plane i.e. "w-plane" z-plane → w-plane "bilinear transformation"

$\boxed{z = \frac{w+1}{w-1}}$

$a_0 s^n + a_1 s^{n-1} + a_2 s^{n-2} + \dots + a_n = 0$ → ch's eqn

RH criterion

a_0	a_2	a_4
a_1	a_3	a_5
$\frac{a_1 a_2 - a_0 a_3}{a_1} = A$	$\frac{a_1 a_4 - a_0 a_5}{a_1} = B$	
$\frac{A a_3 - B a_1}{A}$		
$\vdots$		
a_n		

Sum

(1) Consider the following ch eqn $P(z) = z^3 - 1.3z^2 - 0.08z + 0.24 = 0$ determine whether or not any of the roots of ch's eqn of line outside the unit circle in the z-plane. use bilinear transformation & RH criterion. $\left[\because z = \frac{w+1}{w-1}\right]$

Sol: $P(z) = z^3 - 1.3z^2 - 0.08z + 0.24$

$$P(z) = \left(\frac{w+1}{w-1}\right)^3 - 1.3\left(\frac{w+1}{w-1}\right)^2 - 0.08\left(\frac{w+1}{w-1}\right) + 0.24 = 0$$

$$(w+1)^3 - 1.3(w+1)^2(w-1) - 0.08(w+1)(w-1)^2 + 0.24(w-1)^3 = 0$$

$$w^3 + 1 + 3w^2 + 3w - 1.3(w^2 + 1 + 2w)(w-1) - 0.08(w+1)(w^2 + 1 - 2w)$$

$+0.24(\omega^3+1-3\omega^2+3\omega)=0.$

$\omega^3+3\omega^2+3\omega+1-1.3(\omega^3+\omega+2\omega^2-\omega^2-1-2\omega)-0.08(\omega^3+\omega$

$-2\omega^2+\omega^2+1-2\omega)+0.24(\omega^3-3\omega^2+3\omega-1)=0$

$\omega^3(1-1.3-0.08+0.24)+\omega^2(3+0.18-1.3-0.72)+\omega(3+1.3+0.08+0.72)$

$+1+1.3-0.08-0.24=0$

$\boxed{-0.14w^3+1.06w^2+5.1w+1.98=0}$ $\Rightarrow$ Ch's eq^n

→ 1 sign change is there so one pole is present outside the unit circle. Therefore the system is unstable

sign		
−	-0.14	5.1
+ (sign change)	1.06	1.98
+	$\frac{(1.06)(5.1)-(-0.14\times1.98)}{1.06}=5.36$	0
+	1.98	

Note:
→ If we get any '0' value replace it as 'ϵ' and continue the process. At last equal that eq^n to '0'
→ If any row is entirely '0' then take auxiliary eq^n

(2) $P(z)=z^3-1.1z^2-0.1z+0.2=0$

Sol $P(z)=\left(\frac{w+1}{w-1}\right)^3-1.1\left(\frac{w+1}{w-1}\right)^2-0.1\left(\frac{w+1}{w-1}\right)+0.2=0$

$(w+1)^3-1.1(w+1)^2(w-1)-0.1(w+1)(w-1)^2+0.2(w-1)^3=0$

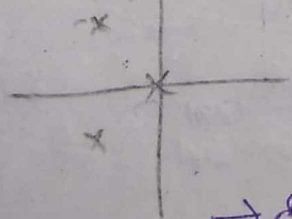
$w^3+3\omega^2+3\omega+1-1.1(w^3+\omega^2-w-1)-0.1(w^3-w^2-w+1)$

$+0.2(\omega^3-3\omega^2+3\omega-1)=0$

$\omega^3(1-1.1-0.1+0.2)+w^2(3-1.1+0.1-0.6)+\omega(3+1.1+0.1+0.6)$

$+1+1.1-0.1-0.2=0$

$1.4w^2+4.8w+1.8=0.$ $\Rightarrow$ ch's eq^n



⇒ No sign change so, poles are in LHS, another pole at origin

1.4	1.8
4.8	0
1.8	

→ So, system is marginally stable/critically stable

22/1/20 (3) check whether or not all the poles of ch's eq^n lies inside the unity circle or not. $5z^2-2z+2=0$

Sol $5\left(\frac{w+1}{w-1}\right)^2 - 2\left(\frac{w+1}{w-1}\right)+2=0$

$5(w^2+2w+1)-2(w^2-1)+2(w^2-2w+1)=0$

$5w^2+10w+5-\cancel{2w^2}+2+\cancel{2w^2}-4w+2=0$

$5w^2+6w+9=0$

+	5	9
+	6	0
+	$\frac{54}{6}=9$	

There are no sign changes in 1st column of RH table.

→ there are zero poles lies to Rt of w-plane

→ All poles are lies inside the unit circle of z-plane. So, the system is "stable"

(4) $z^3-0.2z^2-0.25z+0.05=0$

Sol $\left(\frac{w+1}{w-1}\right)^3 - 0.2\left(\frac{w+1}{w-1}\right)^2 - 0.25\left(\frac{w+1}{w-1}\right)+0.05=0$

$w^3+1+3w^2+3w-0.2(w^2+1+2w)(w-1)-0.25(w+1)(w^2-2w+1)$
$+0.05(w^3-1-3w^2+3w)=0$

$-0.25w^3+0.5w^2-0.25$

$w^3+1+3w^2+3w-0.2(w^3-w^2+w-1+2w^2-2w)-0.25(w^3-2w^2+w$
$-0.2w^3+0.2w^2-0.2w+0.2-0.4w^2+0.4w$ $-0.25w^2+0.5w$
$+w^2-2w+1)+$
$0.05w^3-0.05-0.15w^2+0.15w=0$

a. $0.6w^3+2.9w^2+3.6w+0.9=0$

→ NO sign changes, no pole is outside the circle so system is stable.

0.6	3.6
2.9	0.9
$\frac{(2.9\times3.6)-(0.6\times0.9)}{2.9}$ = 3.41	0
0.9	

(5) $z^4-1.7z^3+1.04z^2-0.268z+0.024=0$

Sol $\left(\frac{w+1}{w-1}\right)^4 - 1.7\left(\frac{w+1}{w-1}\right)^3 + 1.04\left(\frac{w+1}{w-1}\right)^2 - 0.268\left(\frac{w+1}{w-1}\right)+0.024=0$

$(w+1)^2(w+1)^2 - 1.7(w+1)^3(w-1)+1.04(w+1)^2(w-1)^2 - 0.268(w+1)(w-1)^3$
$+0.024(w-1)^2(w-1)=0$

$w^4+4w^3+6w^2+4w+1-1.7(w-1)(w^3+3w^2+3w+1)+1.04(w^2+1+2w)$
$(w^2-2w+1)-0.268(w+1)(w^3-3w^2+3w-1)+0.024($

$+2w^3$

$w^4+4w^3+6w^2+4w+1-1.7(w^4+3w^3+\cancel{3w^2}+w-w^3-\cancel{3w^2}-3w-1)$
$+1.04(w^4-\cancel{2w^3}+w^3+w^2-2w+1+\cancel{2w^3}-4w^2+2w)-0.0268$
$(w^4-3w^3+\cancel{3w^2}-w+w^3-\cancel{3w^2}+3w-1)+0.024(w^4-4w^3+6w^2$
$-4w+1)=$

1.7
1.7
3.4

$-2w^3$

0.096

$w^4(1-1.7+1.04+0.0268+0.024)+w^3(4+3.4+0.536)-(0.024\times4)$
$+w^2(6-(2\times1.04)+(0.024\times6))+1+1.7+1.04-0.268+0.024$
$=0$

0.268
0.268
0.536

4.064 $\quad$ 4.03
$0.096w^4+1.04w^3+\cancel{3.024}\,w^2+\cancel{3.992}+w(4+(2\times1.7)$
$-(0.268\times2)$
$-(4\times0.024))$
$=0$

$0.096w^4+1.04w^3+4.064w^2+7.168w+4.03=0$

0.096	4.064	4.03
1.04	7.168	0
3.56	4.03	
5.99		

No sign changes, system is stable

-0.25

⑥ Find condⁿ on 'm' such that the given system is stable

$z^3+mz^2+3mz+(m-2)=0$

Sol: $\left(\frac{w+1}{w-1}\right)^3+m\left(\frac{w+1}{w-1}\right)^2+3m\left(\frac{w+1}{w-1}\right)+m-2=0$

$w^3+3w^2+3w+1+m(w^3+w^2-w-1)+3m(w^3-w^2-w+1)+$
$(m-2)(w^3-3w^2+3w-1)=0$

$3w^3m.$

$w^3(1+m+3m+m-2)+w^2(3+m-3m-3m+6)+w(3-m-3m$
$+3m-6)+1-m+3m-m$
$+2=0$

$(5m-1)w^3+(9-5m)w^2-w(3+m)+m+3=0$

Condⁿs $5m-1>0 \Rightarrow \boxed{m>1/5}$
$9-5m>0 \Rightarrow m<9/5$
$3+m>0 \Rightarrow m>-3$
$-8m-24/9-5m>0 \Rightarrow -8m-24>0 \Rightarrow -8m>24$, $\boxed{m>-3}$

$5m-1$	$-3-m$
$9-5m$	$m+3$
$\frac{-8m-24}{9-5m}$	
$m+3$	

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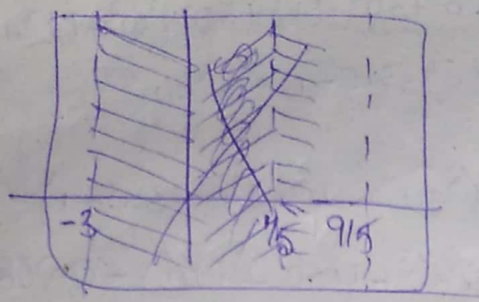
Case-II

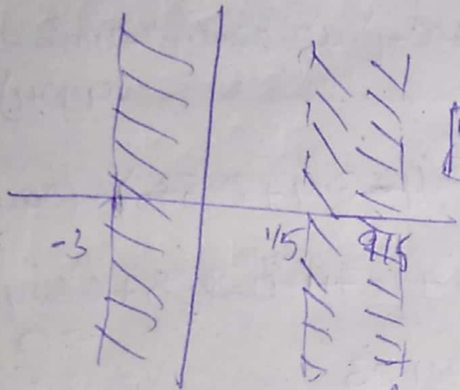


$5m - 1 < 0 \Rightarrow m < 1/5$

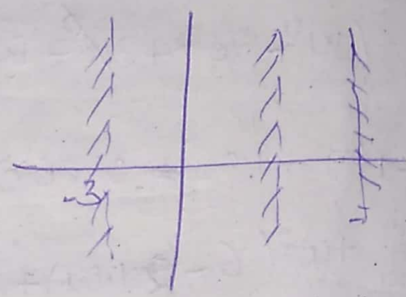
$9 - 5m < 0 \Rightarrow m > 9/5$

$3 + m < 0 \Rightarrow m < -3$

$8m + 24 < 0 \Rightarrow m < -3$



$\boxed{1/5 < m < 9/5}$



(3) Find range of K for which given system is stable, given, pulse T.F of a open loop unity feedback control sys. with sampling time period $T = 1s$ $F(z) = \dfrac{K(0.3679z + 0.2642)}{(z - 0.3679)(z - 1)}$

Soln

$$1 + F(z)H(z) = 0 \qquad [\because H(z) = 0]$$

$$1 + \frac{K(0.3679z + 0.2642)}{(z - 0.3679)(z - 1)} = 0$$

$$(z - 0.3679)(z - 1) + K(0.3679z + 0.2642) = 0$$

$$z^2 - 1.3679z + 0.3679 + K(0.3679z + 0.2642) = 0$$

$$z^2 + z(0.3679K - 1.3679) + 0.2642K + 0.3679 = 0$$

$$\boxed{0 < K < 2.3925}$$

$$z = \frac{w+1}{w-1}$$

$$(w^2 + 2w + 1) + (0.3679K - 1.3679)(w^2 - 1) + (0.3679 + 0.2642K)(w^2 - 2w + 1) = 0$$

$0.3679Kw^2 - 1.3679w^2 - 0.3679K + 1.3679$

$w^2(1 + 0.3679K$

$$w^2 + 2w + 1 + 0.3679Kw^2 - 0.3679K - 1.3679w^2 + 1.3679 + 0.3679w^2 - (2 \times 0.3679)w + 0.3679 + 0.2642Kw^2 - (2 \times 0.2642K)w + 0.2642K = 0$$

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$w^2(K+0.3679K-1.3679+0.3679+0.2642K)+$
$w(2-0.7358-0.5284K)+(1-0.3679K+1.3679+0.3679+0.2642K)=0$

$0.6321Kw^2+(1.2642-0.5284K)w+2.7358-0.1037K=0.$

w^2	$0.6321K$	$2.7358-0.1037K$
w^1	$1.2642-0.5284K$	0
w^0	$2.7358-0.1037K$	

cond^ns

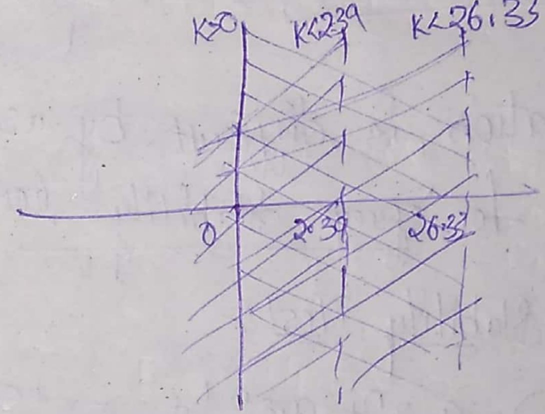
$0.6321K>0 \Rightarrow K>0$

$1.2642-0.5284K>0 \Rightarrow K<2.39$

$2.7358-0.1037K>0$

$\hookrightarrow K<26.33$

$\cancel{0<K<2.39}$



$\boxed{0<K<2.39}$

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⑧ $F(z)=\dfrac{K(z-1)}{(z-0.1)(z-0.8)}$

$1+F(z)H(z)=0$

$1+\dfrac{K(z-1)}{(z-0.1)(z-0.8)}=0$

$(z-0.1)(z-0.8)+K(z-1)=0$

$z^2-0.9z+0.08+Kz-K=0$

$z^2+z(-0.9+K)+0.08-K=0$

$(w^2+2w+1)+(-0.9+K)(w^2-1)+(0.08-K)(w^2-2w+1)=0$

$w^2+2w+1-0.9w^2+0.9+Kw^2-K+0.08w^2-0.16w+0.08-Kw^2$
$+2Kw-K=0$

$w^2(1-0.9+\cancel{K}+0.08-\cancel{K})+w(2-0.16+2K)+(1+0.9-K+0.08-K)$
$=0$

$0.18w^2+(1.84+2K)w+(1.98-2K)=0.$

Cond^n's

$1.98-2K>0$

$\Rightarrow 2K<1.98$

$K<0.99$

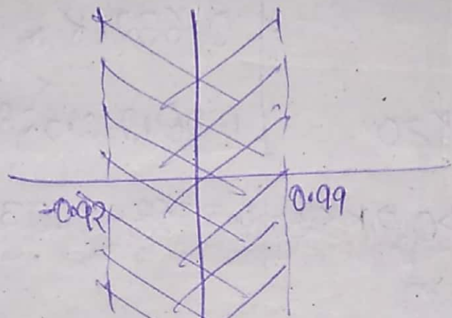
0.18	$1.98-2K$
$1.84+2K$	0
$1.98-2K$	

$1.84+2K>0$

$2K>-1.84$

$K>-0.92$

$\boxed{-0.92<K<0.99}$



* calculation is difficult by using this method then we go for Jury stability test

* Jury Stability Test:-

$P(z) = a_0 z^n + a_1 z^{n-1} + \dots + a_n = 0$

i. $|a_n| < |a_0|$

ii. $P(z)|_{z=1} > 0$

iii. $P(z)|_{z=-1} > 0$ n=even, < 0 n=odd

iv. $|b_{n-1}| > |b_0|$

$|c_{n-2}| > |c_0|$

(n+1) terms	a_n	a_{n-1}	----	a_1	a_0
	a_0	a_1	----	a_{n-1}	a_n
(n) terms	b_{n-1}	b_{n-2}	----	b_0	
	b_0	b_1	----	b_{n-1}	
(n-1) terms	c_{n-2}	c_{n-3}	---	c_0	
	c_0	c_1	----	c_{n-2}	

$a_2 > a_0$

for cal's

$b_{n-1} = \begin{vmatrix} a_n & a_0 \\ a_0 & a_n \end{vmatrix}$; $b_{n-2} = \begin{vmatrix} a_n & a_1 \\ a_0 & a_{n-1} \end{vmatrix}$; $b_0 = \begin{vmatrix} a_n & a_{n-1} \\ a_0 & a_1 \end{vmatrix}$

① $P(z) = z^3 - 1.3z^2 - 0.08z + 0.24 = 0$

0.24	-0.08	-1.3	1
1	-1.3	-0.08	0.24
$(0.24)(0.24)-1 = -0.94$	$(0.24)(-0.08) + (1.3) = 1.28$	$(0.24)(-1.3)+0.08 = -0.28$	

an < a0

i $0.24 < 1$ (True)

ii $1 - 1.3 - 0.08 + 0.24 > 0$

$= -0.14 < 0$ (false)

$\therefore$ System is unstable.

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(2) $P(z) = z^3 - 1.1z^2 - 0.1z + 0.2 = 0$

0.2	-0.1	-1.1	1
1	-1.1	-0.1	0.2
(0.2)(0.2)-1 = -0.96	(0.2)(-0.1)+1.1 = 1.88	(0.2)(-1.1)+0.1 = 0.12	

i $|a_n| < a_0$

$0.2 < 1$ (True)

ii $P(z)|_{z=1} \Rightarrow 1 - 1.1 - 0.1 + 0.2$

$0 > 0$ (Not Satisfied)

$\downarrow$

marginaly Stable (b/c it is 0)

<0 means unstable

$\downarrow$

we hv to check another cond's

iii $P(z)|_{z=-1} \Rightarrow -1 - 1.1 + 0.1 + 0.2 = -1.8 < 0$ (true)

iv $|b_{n-1}| > |b_0| \Rightarrow 0.96 > 0.12$ (true)

$\therefore$ System is marginally stable.

(3) $5z^2 - 2z + 2$

2, -2, 5

Sol: i $|a_n| < |a_0| \Rightarrow 2 < 5$ (true)

ii $P(z)|_{z=1} \Rightarrow 5 - 2 + 2 = 1 > 0$ (true)

iii $P(z)|_{z=-1} \Rightarrow 5 + 2 + 2 = 9 > 0$ (true)

$\therefore$ System is Stable

(4) $z^3 - 0.2z^2 - 0.25z + 0.05$

0.05	-0.25	-0.2	1
1	-0.2	-0.25	0.05
(0.05)(0.05)-1 = -0.99	(0.05)(-0.25) +0.2 = 0.187	(0.05)(-0.2) +0.25 = 0.24	

Sol: i $|a_n| < |a_0| = 0.05 < 1$ (true)

ii $P(z)|_{z=1} = 1 - 0.2 - 0.25 + 0.05$

$\therefore 0.6 > 0$ (true)

iii $P(z)|_{z=-1} = -1 - 0.2 + 0.25 + 0.05$

$= -0.9 < 0$ (true)

$|b_{n-1}| > |b_0| \Rightarrow |b_2| > |b_0| \Rightarrow 0.99 > 0.24$ (true)

$\therefore$ System is Stable.

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(5) $z^3 + Mz^2 + 3Mz + (M-2) = 0$ $\quad (M-2)(3M)) - M)$

$$\begin{array}{cccc} M-2 & 3M & M & 1 \\ 1 & M & 3M & M-2 \\ (M-2)^2-1 & 3M^2-7M & M^2-5M & \\ = M^2-4M+3 & & & \end{array}$$

Sol: i $|a_n| < |a_0| \Rightarrow M-2 < 1 \Rightarrow \boxed{M<3}$

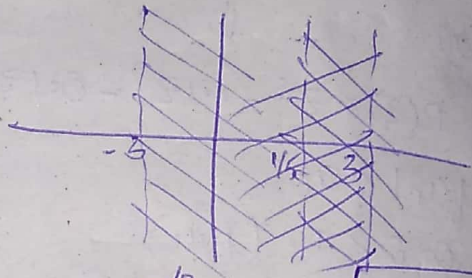
ii $P(z)|_{z=1} = 1+M+3M+M-2$

$= 5M-1 > 0 \Rightarrow \boxed{M > 1/5}$

iii $P(z)|_{z=-1} \Rightarrow -1+M-3M+M-2$

$-M-3 < 0 \Rightarrow$

$\boxed{M>-3}$



iv $|b_{n-1}| > |b_0| \Rightarrow |b_2| > |b_0| \Rightarrow \cancel{M^2}-4M+3 > \cancel{M^2}-5M$ $\quad \boxed{\therefore 1/5 < M < 3}$

$\boxed{M>-3}$

(6)

$$F(z) = \frac{K(0.3679z + 0.2642)}{(z-0.3679)(z-1)}$$

$C.E \Rightarrow 1 + F(z)H(z) = 0$

$1 + F(z) = 0$

$$1 + \frac{K(\ )}{(\ )(\ )} = 0$$

$(z-0.3679)(z-1) + K(0.3679z + 0.2642) = 0$

$z^2 - 1.3679z + 0.3679 + K(\ ) = 0$

$z^2 + z(-1.3679 + 0.3679K) + (0.3679 + 0.2642K) = 0$

i $|a_2| < |a_0|$

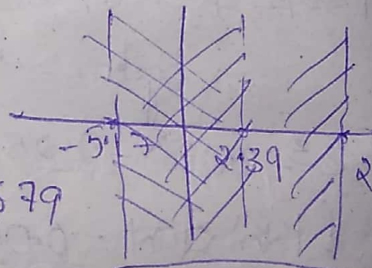
$|0.3679 + 0.2642K| < 1$

$-1 < 0.3679 + 0.2642K < 1$

$-1.3679 < 0.2642K < 1 - 0.3679$

$-5.17 < K < 2.39$

$K < 2.39,\ K > -5.17$



$\boxed{-5.17 < K < 2.39}$

ii $P(z)|_{z=1} = P(1) = 1 - 1.3679 + 0.3679K + 0.3679 + 0.2642K$

$= 0.85K > 0 \Rightarrow K > 0$

iii $P(z)|_{z=-1} = P(-1) = 1 + 1.3679 - 0.3679K + 0.3679 + 0.2642K$

$= 2.7350 - 0.1037K > 0$

$\boxed{K < 26.2}$

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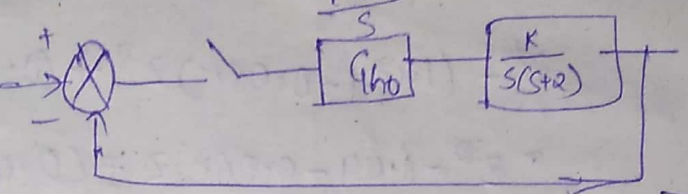
(7) Consider the sample data system shown in fig, assume its sampling period is 0.4 sec. Find the range of K so that the closed loop system is



Soln $G(z) = Z\left[\frac{1-e^{-ST}}{S} \frac{K}{S(S+2)}\right]$

$= (1-z^{-1}) Z\left\{\frac{K}{S^2(S+2)}\right\}$ $\left[\because Z\left[\frac{1-e^{-ST}}{S} G(s)\right] = (1-z^{-1}) Z\left[\frac{G(s)}{S}\right]\right]$

$\frac{1}{S^2(S+2)} = \frac{-1/4}{S} + \frac{1/2}{S^2} + \frac{1/4}{S+2}$

$L^{-1}\left(\frac{1}{S^2(S+2)}\right) = -\frac{1}{4}u(t) + \frac{1}{2}t + \frac{1}{4}e^{-2t}$

$= -\frac{1}{4}u(KT) + \frac{1}{2}KT + \frac{1}{4}e^{-2KT}$

$Z\left[\frac{1}{S^2(S+2)}\right] = -\frac{1}{4}\frac{z}{z-1} + \frac{1}{8}T\left(\frac{z}{z-1}\right)^2 + \frac{1}{4}\frac{z}{z-e^{-2T}}$

$= -\frac{1}{4}\frac{z}{z-1} + \frac{1}{2}(0.4)\left(\frac{z}{z-1}\right)^2 + \frac{1}{4}\cdot\frac{z}{z-e^{-2(0.4)}}$

$= \frac{-\frac{1}{4}[z(z-1)(z-0.44)] + 0.2z(z-0.44) + \frac{1}{4}z(z-1)^2}{(z-1)^2(z-0.44)}$

$= \frac{-\frac{1}{4}[z^3 - 1.44z^2 + 0.44z] + 0.2[z^2 - 0.44z] + \frac{1}{4}[z^3 - 2z^2 + z]}{(z-1)^2(z-0.44)}$

$= \frac{z^2\left[\frac{1.44}{4} + 0.2 - \frac{1}{2}\right] + z\left[-\frac{0.44}{4} - 0.88 + \frac{1}{4}\right]}{(z-1)^2(z-0.44)}$

$= \frac{0.06z^2 - 0.74z}{(z-1)^2(z-0.44)}$

$G(z) = \frac{K(0.06z^2 - 0.74z)}{(z-1)^2(z-0.44)}$

$G(z)_{\text{Gho}} = \frac{z-1}{z} \cdot \frac{K(0.06z^2 - 0.74z)}{(z-1)^2(z-0.44)}$

$= \frac{K(0.06z^2 - 0.74z)}{z(z-1)(z-0.44)}$

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$1+G(z)H(z)=0$

C.E $\Rightarrow z(z-1)(z-0.44)+K(0.06z^2-0.74z)=0$

$z^3-1.44z^2+0.44z+K(\quad)=0$

$z^3-(1.44-0.06K)z^2+(0.44-0.74K)z=0$

$z^2-(1.44-0.06K)z+(0.44-0.74K)=0$

$-0.756<K<0$

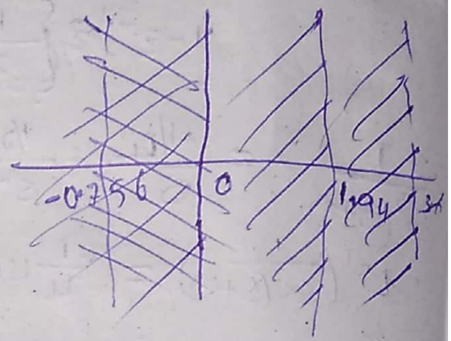
i. $|a_2|<|a_0|$

$|0.44-0.74K|<1$

$-1<0.44-0.74K<1$

$-1.44<-0.74K<0.36$

$\boxed{1.94>K>-0.756}$



ii. $P(1)>0 \Rightarrow 1-(1.44-0.06K)+(0.44-0.74K)>0 \Rightarrow -0.68K>0$

$K<0$

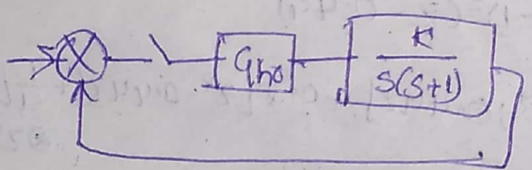
iii. $P(-1)>0 \Rightarrow +1+(1.44-0.06K)+(0.44-0.74K)>0$

$+2.88-0.8K>0 \qquad -0.8K>-2.88$

$\cancel{1.32K>2.88}$

$\underline{K<3.6}$

8



$T=0.2$

Sol $G(z)=Z\left[\frac{1-e^{-ST}}{S}\cdot\frac{K}{S(S+1)}\right]$

$=1-z^{-1}\cdot Z\left[\frac{K}{S^2(S+1)}\right]$

$\frac{1}{S^2(S+1)}=\frac{A}{S}+\frac{B}{S^2}+\frac{C}{S+1}=AS(S+1)+B(S+1)+CS^2$

$1=AS^2+A+BS$ put $\underline{S=-1} \Rightarrow \boxed{1=C}$

$A+C=0 \Rightarrow \boxed{A=-1}$; $A+B=0 \Rightarrow \boxed{B=1}$

$\frac{1}{S^2(S+1)}=\frac{-1}{S}+\frac{1}{S^2}+\frac{1}{(S+1)}$

$L^{-1}\left[\frac{1}{S^2(S+1)}\right]=-u(t)+t+e^{-t}$

$=-u(KT)+KT+e^{-KT}$

$$Z\left[\frac{1}{s^2(s+1)}\right] = -\frac{z}{z-1} + T\cdot\left(\frac{z}{z-1}\right)^2 + \frac{z}{z-e^{-T}}$$

z^2-2z+1

$$= -\frac{z}{z-1} + (0.2)\frac{z}{(z-1)^2} + \frac{z}{z-e^{-0.2}}$$

$z^2-z \qquad z^2-0.818z$

$$= \frac{-z(z-1)(z-0.818) + (0.2)z(z-0.818) + z(z-1)^2}{(z-1)^2(z-0.818)}$$

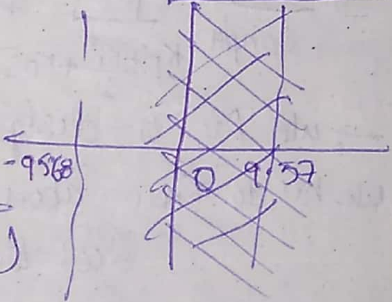
$-z^3+0.818z^2+z^2-0.818z$

$$= \frac{-(z^3-0.818z^2-z^2+0.818z) + 0.2z^2 - 0.1636z + z^3-2z^2+z}{(z-1)^2(z-0.818)}$$

0.818
0.163
0.981
$0.818 \quad 0.019$
0.2
1.018
-1

$$= \frac{z^2(0.818+1+0.2-2) + z(-0.818-0.163+1)}{(\ )^2(\ )}$$

$\boxed{0<K<9.57}$



$$G(z) = \frac{0.018z^2+0.019z}{(z-1)^2(z-0.818)}$$

$$= \frac{z-1}{z}\cdot\frac{0.018z^2+0.019z}{(z-1)^2(z-0.818)}$$

$$G(z) = \frac{0.018z^2+0.019z}{z(z-1)(z-0.818)}$$

$1+G(z)H(z)=0$

(z^2-z)

$z(z-1)(z-0.818) + K(0.018z^2+0.019z)=0$

$z^3-0.818z^2-z^2+0.818z+K0.018z^2+0.019Kz=0$

$z^3-(0.818+0.018K)z^2+(0.818+0.019K)z=0$

$z^2-(1.818-0.018K)z+(0.818+0.019K)=0$

i) $|a_2|<|a_0|$

$|0.818+0.019K|<1$

$-1<0.818+0.019K<1$

$-1.818<+0.019K<0.182$

$-95.68<+K<9.57$

$\boxed{95.68>K>-9.57}$

ii) $P(1)>0$

$1-(1.818-0.018K)+(0.818+0.019K)>0$

$1-1.818+0.018K+0.818+0.019K>0$

$0.037K>0 \Rightarrow \boxed{K>0}$

iii) $P(-1)>0$

$1+1.818-0.018K+0.818+0.019K>0$

$3.636+0.001K>0$

$0.001K>-3.636 \Rightarrow \boxed{K>-3636}$

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